SUBGROUP GROWTH IN SOME PRO-$p$ GROUPS

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Abstract. For a group $G$ let $a_n(G)$ be the number of subgroups of index $n$ and let $b_n(G)$ be the number of normal subgroups of index $n$. We show that $a_p^k(\text{SL}_2(\mathbb{F}_p[[t]])) \leq p^{k(k+5)/2}$ for $p > 2$. If $\Lambda = \mathbb{F}_p[[t]]$ and $p$ does not divide $d$ or if $\Lambda = \mathbb{F}_p$ and $d \neq 2$ or $d \neq 2$, we show that for all $k$ sufficiently large $b_p^k(\text{SL}_2(\Lambda)) = b_{p^k+5}(\text{SL}_2(\Lambda))$. On the other hand if $\Lambda = \mathbb{F}_p[[t]]$ and $p$ divides $d$, then $b_n(\text{SL}_2(\Lambda))$ is not even bounded as a function of $n$.

1. Introduction

For a group $G$, let $a_n(G)$ be the number of subgroups of index $n$. Lubotzky and Mann [LM] proved that a pro-$p$ group $G$ is $p$-adic analytic if and only if it has polynomial subgroup growth; that is, there exists a constant $c$ such that $\log_p n$ for some constant $c < \frac{1}{8}$.

Following this result Mann [Ma] asked the following:

**Question.** What is the supremum of the numbers $c$, such that if $G$ is a pro-$p$ group and $a_n(G) < n^{c \log_p n}$ for all large $n$, then $G$ is $p$-adic analytic?

To continue our discussion we need the following definition.

**Definition 1.1.** Let $\Lambda$ be a local ring with a maximal ideal $M$. We define the $n$-congruence subgroup of $\text{SL}_d(\Lambda)$ to be $\text{SL}_d^n(\Lambda) = \ker(\text{SL}_d(\Lambda) \to \text{SL}_d(\Lambda/M^n))$.

The particular examples of local rings we deal with are $\Lambda = \mathbb{Z}_p$, the $p$-adic integers, and $M = p\mathbb{Z}_p$ or $\Lambda = \mathbb{F}_p[[t]]$, formal power series over a field of $p$-elements, and $M = t\mathbb{F}_p[[t]]$. It is well known that for these examples $\Lambda$, $\text{SL}_d(\Lambda)$ is a pro-$p$ group. Moreover, $\text{SL}_d^1(\mathbb{F}_p[[t]])$ is not $p$-adic analytic. In [Sh] it is already shown that $a_p^k(\text{SL}_2(\mathbb{F}_p[[t]])) \leq A p^{2k^2}$ for $p > 2$ and some constant $A$. We show the following:

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Theorem 1.2. Let $G = SL_d^1(F_p[[t]])$ with $p > 2$. Then $a_{p^k}(G) \leq p^{k(k+5)/2}$.

Thus in answer to Mann’s question we show that the supremum is no more than $\frac{1}{2}$ (for $p > 2$).

We now turn our attention to the study of the lattice of normal subgroups of $SL_d^1(\Lambda)$, for $\Lambda = \mathbb{Z}_p$ and $\Lambda = F_p[[t]]$. Let us recall that a group $G$ is called just infinite if its only nontrivial normal subgroups are of finite index. It is well known that if $p \neq 2$ or $d \neq 2$, then $SL_d^1(\Lambda)$ is just infinite (this is actually shown in the proof of Lemma 4.1). In [Yo, Proposition 3.5.1] it is shown that the lattice of normal subgroups of another just infinite pro-$p$ group, $J_p$, the Nottingham group is “periodic” ($p > 3$). In particular for any $k$, $b_{p^k}(J_p) = b_{p^{k+1}}(J_p)$. We show the following:

Theorem 1.3. Suppose $\Lambda = F_p[[t]]$ and $p$ does not divide $d$ or $\Lambda = \mathbb{Z}_p$ and $p \neq 2$ or $d \neq 2$. There is a constant $K = K(p, d)$ such that $b_{p^k}(SL_d^1(\Lambda)) = b_{p^{k+1}}(SL_d^1(\Lambda))$ for all $k > K$.

Theorem 1.3 and the result for the Nottingham group might suggest that for any just infinite pro-$p$ group a similar phenomenon occurs. The following theorem is thus somewhat surprising, as it shows that there is a big difference in the behavior of $b_n(SL_d^1(F_p[[t]]))$ in the case $p$ divides $d$.

Theorem 1.4. If $p$ divides $d$, then $b_n(SL_d^1(F_p[[t]]))$ is not bounded as a function of $n$.

Our main tool in this paper is Lie methods. It would be interesting to find a proof of Theorem 1.3 in the case $\Lambda = \mathbb{Z}_p$ based on powerful groups. This might help to handle the case where $p = d = 2$.

2. LIE METHODS

Suppose $\Lambda = \mathbb{Z}_p$ or $\Lambda = F_p[[t]]$. Let $G_n = SL_d^1(\Lambda)$. It is straightforward to see that $(G_n, G_m) \subseteq G_{n+m}$ and $G_0 \subseteq G_1$. Thus $G_n/G_{n+1}$ is an elementary abelian $p$-group. It is easy to verify that $|G_n/G_{n+1}| = p^{d^2-1}$ and indeed this quotient is the adjoint module for $SL_d(F_p)$.

The reader is referred to [LSh] for more details on the following construction. Define

$$L(G_1) = \sum G_n/G_{n+1}.$$ 

If $x \in G_n$ and $y \in G_m$, we define the bracket product

$$[xG_{n+1}, yG_{m+1}] = (x, y)G_{n+m+1}.$$ 

Extending this product by linearity gives $L(G_1)$ the structure of a Lie algebra over $F_p$. It is not hard to check that $L(G_1) \cong tsL_d(F_p)[t]$ — the set of polynomials with $0$ constant coefficient over $sL_d(F_p)$.

Let $H$ be a closed subgroup of $G_1$. We define

$$L(H) = \sum (H \cap G_n)G_{n+1}/G_{n+1}.$$ 

The following facts are easy to verify:

1. $L(H)$ is graded subalgebra of $L(G_1)$.
2. If $K \leq H$ are closed subgroups, then $L(K) \subseteq L(H)$ and $\dim(L(H)/L(K)) = \log_p[H:K]$. 


3. If $H$ is normal, then $L(H)$ is an ideal.
4. $G_n \leq H$ if and only if $t^n\mathfrak{s}\mathfrak{l}_d(\mathbb{F}_p)[t] \subseteq L(H)$.
5. If $L(H)$ is generated by $d$ homogeneous elements, then $d(H) \leq d$, where $d(H)$ is the minimal number of elements required to generate $H$ topologically.

Let us remark that one can associate to the group $G_1/G_n$ the Lie algebra $ts\mathfrak{l}_d(\mathbb{F}_p)[t]/(t^n)$. Similar results to the above holds for subgroups and subalgebras.

### 3. THE SUBGROUP GROWTH OF $SL_2^1(\mathbb{F}_p[[t]])$

We first consider a question about generation of Lie subalgebras of $L = ts\mathfrak{l}_2(\mathbb{F}_p)[t]/(t^{n+1})$. There should be an analogous result for other simple Lie algebras. Note that there exist Lie subalgebras of $L$ which require the maximum number of generators given in the result.

**Proposition 3.1.** Let $L = ts\mathfrak{l}_2(\mathbb{F}_p)[t]/(t^{n+1})$ with $p > 2$. If $H$ is a graded subalgebra of $L$ of dimension $d$ and codimension $c$, then $H$ can be generated by $\min\{c+3, d\}$ elements. In particular, $H$ can be generated by no more than $\frac{d}{2}(n+1)$ homogeneous elements.

**Proof.** First note that the second statement follows from the first since $H$ can be generated by $(1/2)(c + d + 3) = (3/2)(n + 1)$ homogeneous elements.

Let $H = H_1t^{+} \cdots \oplus H_n t^n$, where $H_i \subseteq \mathfrak{s}\mathfrak{l}_2(\mathbb{F}_p)$, and let $h_i = \dim H_i$. Similarly, let $H'_i$ denote the degree $i$ component of the derived algebra $[H, H]$ and set $h'_i = \dim H'_i$.

We recall that if $M$ is a nilpotent Lie algebra and $S$ is a subalgebra, then $S = M$ if and only if $S + [M, M] = M$ [3, Exercise I.10]. In particular, this implies that if $M$ is a finite dimensional graded nilpotent Lie algebra, then $M$ can be generated by $\dim(M/[M,M])$ homogeneous elements (of course, this is also the minimum number of generators required).

We also recall that $\mathfrak{s}\mathfrak{l}_2(\mathbb{F}_p)$ is a simple Lie algebra and therefore a perfect Lie algebra, namely equals its derived subalgebra. Let $V, U$ be subspaces of $\mathfrak{s}\mathfrak{l}_2(\mathbb{F}_p)$. As $\dim \mathfrak{s}\mathfrak{l}_2(\mathbb{F}_p) = 3$, it is easy to verify the following facts:

1. If $\dim V = 2$, then $[V, \mathfrak{s}\mathfrak{l}_2(\mathbb{F}_p)] = \mathfrak{s}\mathfrak{l}_2(\mathbb{F}_p)$.
2. If $\dim V = 1$, then $\dim[V, \mathfrak{s}\mathfrak{l}_2(\mathbb{F}_p)] = 2$.
3. If $\dim V = 2$, then $\dim[V, V] = 1$.
4. If $V \neq U$ and $\dim V = \dim U = 2$, then $[V, U] = \mathfrak{s}\mathfrak{l}_2(\mathbb{F}_p)$
5. If $\dim V = 2$ and $\dim U = 1$, then $1 \leq \dim[V, U] \leq 2$.

Of course, $H$ can always be generated by $d$ homogeneous elements.

We use an induction on $n$. For $n = 1, 2$, the result is clear. If $h'_n = 0$, then for all $1 \leq i < n$, $[H_i, H_{n-i}] \subseteq H'_i$; therefore $h_i + h_{n-i} \leq 3$. Thus, $c \geq (3/2)(n-1)$ and $d \leq c + 3$. Note in fact this argument is valid under the weaker assumption that $h_i + h_{n-i} \leq 3$ for all $1 \leq i < n$.

So we assume that $h_i + h_{n-i} \geq 4$ for some $i$. Let $j$ be the smallest positive integer such that $h_j + h_{n-j} \geq 4$.

If $h_i + h_{n-i} \geq 5$ for some $i$, then $h'_n = 3$. If $h'_n = 3$, then by induction $H/H_nt^n$ can be generated by at most $c + 3$ homogeneous elements. Since $H_nt^n \subseteq [H, H]$, this implies the same for $H$.

So we may assume that $h_i + h_{n-i} \leq 4$ for all $i$ and that $h'_n \leq 2$.

Let $\Delta$ denote the set of integers with $h_i + h_{n-i} = 4$. Set $e = |\Delta|$. We notice that $d \leq 3 + (3/2)(n-1) + e/2$ and $c \geq (3/2)(n-1) - e/2$. Thus $d - e \leq c + 3$. Since $j$ is minimal in $\Delta$, $n - j$ is maximal in $\Delta$ and so $i + j \leq n$ for all $i \in \Delta$. 

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First suppose that \( h_j \geq 2 \). Then \([H_j t^i, H_i t^j] \neq 0\) for any \( i \in \Delta \) and since these spaces are independent, it follows that \( \dim [H, H] \geq e \). Thus, \( \dim H/[H, H] \leq d - e \leq c + 3 \) as required.

Finally, consider the case that \( h_j = 1 \) (and so \( h_{n-j} = 3 \) and \( h_n = 2 \)). Let \( i \in \Delta \). So either \( h_i = h_{n-i} = 2 \) and \([H_j t^i, H_m t^m] \neq 0\) for \( m = i, n - i \) or exactly one of \( h_i, h_{n-i} = 3 \). Thus, \([H_j t^i, \bigoplus \Lambda \in \Delta H_i t^i] \) has dimension at least \( e \). So \([H, H] \) has dimension at least \( e \) and as in the previous paragraph, we deduce that \( \dim H/[H, H] \leq e + 3 \).

**Proof of Theorem 1.2.** Let \( G_i = SL_d^i(F_p[[t]]) \). The \( G_i \) are a base for the neighborhoods of the identity. As \( G \) is finitely generated for any given \( k \), there is \( n \) big enough such that \( G_m \) is contained in all subgroups of index \( p^k \) (actually Shalev [Sh, Theorem 4.1] proved that \( m = k + 1 \) is sufficient). Therefore \( a_{p^k}(G) = a_{p^k}(G/G_m) \). For any group \( H \) let \( g_n(H) \) be the supremum on the number of generators of subgroups of index \( n \). From [LSh, Lemma 4.1] we see that

\[
a_{p^k}(G) = a_{p^k}(G/G_m) \leq p^g(G/G_m) + q_0(G/G_m) + \cdots + q_{p-1}(G/G_m).
\]

By fact 5 in Section 2, the remark following it and Proposition 5.1 we deduce that

\[
a_{p^k}(G) \leq p^{0+1+2+\cdots+(k-1)+3k} = p^{k(k+5)/2}.
\]

\[\square\]

4. The normal subgroup growth of \( SL_d^i(\Lambda) \)

Suppose \( \Lambda = \mathbb{Z}_p \) or \( \Lambda = F_p[[t]] \). Let \( G = SL_d^i(\Lambda) \) and \( G_n = SL_d^i(\Lambda) \).

**Lemma 4.1.** Suppose \( \Lambda = F_p[[t]] \) and \( p \) does not divide \( d \) or \( \Lambda = \mathbb{Z}_p \) and \( p \neq 2 \) or \( d \neq 2 \). Then there is a constant \( f = f(p, d) \) such that, for any normal subgroup \( N \) of \( G \) there exists an \( n \) such that \( G_n + f < N \leq G_n \).

**Proof.** Let \( \Lambda = F_p[[t]] \) or \( \Lambda = \mathbb{Z}_p \). Let \( n \) be maximal such that \( N \leq G_n \). Therefore we can find \( x \in N \) such that \( x \notin G_{n+1} \). Passing to \( L(N) = \sum_{i \geq 1} L_i t^i \), we can find a homogeneous element \( \bar{x} \in L(N) \), where \( \bar{x} \neq \bar{e} \in sl_d(F_p) \). Define \( U_1 = \langle \bar{x}, sl_d(F_p) \rangle \) and by induction \( U_m+1 = \langle U_m, sl_d(F_p) \rangle \). Define \( U = \cup U_m \). This is a nontrivial ideal of \( sl_d(F_p) \). If \( p \) does not divide \( d \), then \( sl_d(F_p) \) is a simple Lie algebra. Therefore \( U = sl_d(F_p) \). As \( sl_d(F_p) \) is perfect, we see that \( U_m \subseteq U_{m+1} \) for all \( m \) and moreover, equality holds if and only if \( U_m = sl_d(F_p) \). As \( \text{dim}(sl_d(F_p)) = d^2 - 1 \), we deduce that \( U_{d+1} = sl_d(F_p) \).

Since \( N \) is normal, \( L(N) \) is an ideal and thus \( [L(N), sl_d(F_p)] \subseteq L(N) \). Therefore \( \langle \bar{x} \rangle \subseteq L_{d+1} \) and \( sl_d(F_p) = L_{d^2+j} \) for \( j = 0, 1, \ldots \). We now use fact 4 from section two to deduce that \( G_{n+d^2-1} \leq N \).

Suppose now that \( \Lambda = \mathbb{Z}_p \) and \( p \) divides \( d \geq 2 \). Let \( s \) be the largest positive integer such that \( p^s \) divides \( d \). Since \( p \neq 2 \) or \( d \neq 2 \), \( sl_d(F_p) \) is perfect Lie algebra and its only non-trivial ideal is the center. Hence if \( U \) is not central, we can argue as above. Suppose that \( \bar{x} \) is a scalar. Let \( x = I + A \), where \( A \in p^n M_d(\mathbb{Z}_p) \).

As \( \bar{x} \) is a scalar we can write \( A = p^n \lambda I + B \), where \( \lambda \) is an invertible element of \( \mathbb{Z}_p \), \( B \in p^r M_d(\mathbb{Z}_p), r > n \), and \( B \) mod \( p^{r+1} \) is not a scalar. Hence we can write \( x = (1 + p^n \lambda)I + C \) where \( C \in p^r M_d(\mathbb{Z}_p) \). We note that

\[
\det((1 + p^n \lambda)I) = 1 + \sum_{i \geq 1} \binom{d}{i} p^{ni} \lambda^i.
\]
Let \( t \) be maximal such that, for all \( i \geq 1 \), \( (d^i)p^n \mod p^t = 0 \). We notice that when \( n > s \), \( t = n + s \). Hence \( t - n \) is bounded by a function of \( p \) and \( d \). As \( 1 = \det(x) = \det((1 + p^n\lambda)I) \det(I + C) \) and \( \det((1 + p^n\lambda)I) \mod p^{t+1} \neq 0 \), one can deduce that \( C \mod p^{t+1} \neq 0 \); moreover as \( p \) divides \( d \), \( C \mod p^{t+1} \) is not a scalar.

It is not hard to find an element \( y \in G \) which has the form \( y = I + D \), where \( D \in \text{pM}_d(\mathbb{Z}_p) \) and \( [D,C] \mod p^{t+2} \) is not a scalar. Set \( z = (x,y) \in G_{t+1} \). We leave to the reader to verify that \( z = I + E \), where \( E \in p^{t+1}nM_d(\mathbb{Z}_p) \), and \( E \mod p^{t+2} = [D,C] \). From here the proof goes as in the case where \( p \) divides \( d \), where we replace \( x \) by \( z \), and noticing that \( t + d^2 - n \) is bounded in terms of \( p \) and \( d \).

**Remarks.** 1. The case where \( p \) does not divide \( d \) already appeared in an unpublished preprint by Aner Shalev.

2. In the course of the proof we actually showed that if \( \Lambda = \mathbb{F}_p[[t]] \) and \( p \) does not divide \( d \) or \( \Lambda = \mathbb{Z}_p \) and \( p \neq 2 \) or \( d \neq 2 \), then \( G \) is just infinite.

3. A similar argument to the case where \( \Lambda = \mathbb{Z}_p \) and \( p \) divides \( d \) can be used to show that \( G \) is just infinite even when \( \Lambda = \mathbb{F}_p[[t]] \) as long as \( p \neq 2 \) or \( d \neq 2 \).

We note that conjugation in \( G \) induces a structure of \( G \)-set on \( G_n/G_{n+d^2-1} \) for all \( n \).

**Lemma 4.2.** Suppose \( \Lambda = \mathbb{F}_p[[t]] \) and \( p \) does not divide \( d \) or \( \Lambda = \mathbb{Z}_p \) and \( p \neq 2 \) or \( d \neq 2 \). Let \( f \) be as in the previous lemma. Then there is one to one correspondence between the set of normal subgroups of \( G \) and the pairs \((n,H)\) such that \( H \) is a subgroup of \( G_n/G_{n+f} \) which is not contained in \( G_{n-1}/G_{n+f} \) and \( H \) is \( G \)-invariant.

**Proof.** Let \( N \) be a normal subgroup of \( G \). By Lemma 4.1 we can find \( n \) such that \( G_{n+f} < N \leq G_n \). We choose \( n \) to be maximal. Let \( H = N/G_{n+f} \). Since \( n \) is maximal, \( H \) is not contained in \( G_{n-1}/G_{n+f} \). As \( N \) is normal, \( H \) is \( G \)-invariant. On the other hand, given a pair \((n,H)\), we take \( N \) to be the pre-image of \( H \) under the quotient map from \( G_n \) onto \( G_n/G_{n+f} \). It is easy to verify that these maps are the inverses of each other. \( \square \)

**Lemma 4.3.** Let \( f \) be some constant. Then for \( n > f \) there is a map

\[
\varphi : G_n/G_{n+f} \to G_{n+1}/G_{n+f+1}
\]

such that \( \varphi \) is an equivariant group isomorphism and

\[
\varphi(G_{n+1}/G_{n+f}) = G_{n+2}/G_{n+f+1}.
\]

**Proof.** First let us deal with the case \( \Lambda = \mathbb{F}_p[[t]] \). Notice that every element of \( G_n \) has the form \( I + A \), where \( A \in t^nM_d(\mathbb{F}_p[[t]]) \). We leave to the reader to check that if \( n > f \), then the fact that the determinants of elements in \( G \) are one implies that \( \text{Trace}(A) \mod t^{n+f} = 0 \). On the other hand if \( \text{Trace}(A) \mod t^{n+f} = 0 \), then one can construct (using induction) an element in \( G_n \) of the above form.

We define a map

\[
\varphi : G_n/G_{n+f} \to G_{n+1}/G_{n+f+1}.
\]

By

\[
\varphi((I + A)G_{n+f}) = (I + tA)G_{n+f+1}
\]

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(this is a slight abuse of notation as $I + tA$ does not necessarily have determinant 1). It is easy to check that $\varphi$ satisfies the required conditions.

For $A = \mathbb{Z}_p$, the argument is very similar when we replace $t$ by $p$. \hfill \Box

Proof Theorem 1.3 Let $f$ be as in Lemma 1.1. For $n > f$ we define $c(s)$ to be the number of $G$-invariant subgroups of $G_n/G_{n+f}$ which are not contained in $G_{n+1}/G_{n+f}$ and have index $p^s$. By Lemma 4.3 this is well defined and in particular does not depend on $n$.

Let us define $b_{k,n}(G)$ to be the number of normal subgroups of index $p^k$ of $G$ which contain $G_{n+f}$ and are contained in $G_n$ and $n$ is maximal under this property. If $H$ is a normal subgroup of index $p^k$, then by Lemma 1.1 there is $n$ such that $G_{n+f} < H \leq G_n$. We deduce that $p^{(n+f-1)(d^2-1)} > p^k \geq p^{(n-1)(d^2-1)}$; thus \( \frac{k}{d^2-1} + 1 - f < n \leq \frac{k}{d^2-1} + 1 \). We note that if $k > (2f-1)(d^2-1)$, then $n > f$.

By Lemma 1.2 and the above argument we see that for $n > f$ the following is true:

$$b_{k,n}(G) = \begin{cases} 0 & \text{if } n \leq \frac{k}{d^2-1} + 1 - f, \\ 0 & \text{if } \frac{k}{d^2-1} + 1 < n, \\ c(k - (n-1)(d^2-1)) & \text{otherwise}. \end{cases}$$

By Lemma 1.2 for $k > (2f-1)(d^2-1)$

$$b_{p^k}(G) = \sum_{n \geq 1} b_{k,n}(G) = \sum_{\frac{k}{d^2-1} + 1 - f < n \leq \frac{k}{d^2-1} + 1} c(k - (n-1)(d^2-1)).$$

Thus $b_{p^k}(G)$ depends only on $k \mod d^2-1$ for $k > (2f-1)(d^2-1)$. \hfill \Box

Proof of Theorem 1.4. We note that $G_n/G_{2n}$ is an elementary abelian $p$-group; moreover $G_n/G_{2n}$ is a $G$-module. Let $x \in G_n$ and let us write $x = I + A$, where $A \in t^n M_d(\mathbb{F}_p[t])$. We note that $\det(x) = 1$ implies that $\text{Trace}(A) \mod t^{2n} \equiv 0$. On the other hand suppose $\text{Trace}(A) \mod t^{2n} \equiv 0$; then one can construct (using induction) an element in $G_n$ of the above form. As $p$ divides $d$ if $A \mod t^{2n}$ is a scalar, then $\text{Trace}(A) \mod t^{2n} \equiv 0$. We also note that if $A \mod t^{2n}$ is a scalar, then $G$ acts trivially on $\langle xG_{2n} \rangle$. Let $N_x$ be the pre-image of $\langle xG_{2n} \rangle$ in $G_n$. This is a normal subgroup of $G$ of index $p^{(2n-1)(d^2-1)-1}$. Of course the number of such subgroups is equal to the number of $A \in t^n M_d(\mathbb{F}_p[t])$ such that $A \mod t^{2n}$ are nonzero scalars divided by $p-1$. Therefore $b_{p^{(2n-1)(d^2-1)-1}}(G) \geq (p^n - 1)/(p - 1)$. \hfill \Box

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