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CORRIGENDUM TO "WEAKLY ABELIAN LATTICE-ORDERED GROUPS"

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In the proof of Theorem B in [1], I assumed that the Philip Hall collection process actually terminates for all elements of a finitely generated non-Abelian free group G. This is quite false. The proof is corrected as follows:

The notation and references are the same as in the original article [1]. If m is any positive integer, then each non-identity element $g \in G$ can be written uniquely (modulo $\gamma_{m+1}(G)$) in the form

$$(*) \qquad c_1 \cdots c_m$$

where $c_j = a_{j,1}^{\ell_{j,1}} \cdots a_{j,s(j)}^{\ell_{j,s(j)}}$ for $j = 1, \ldots, m$ (please see [2, Chapter 11] or *Nilpotent Groups* (Section 5) in Philip Hall's Collected Works, Clarendon Press, Oxford, 1988). That is, for each $g \in G$ and each positive integer m, we can uniquely write $g = c_1 \cdots c_m w_{m+1}$ with $c_1, \ldots, c_m$ as above and $w_{m+1} \in \gamma_{m+1}(G)$. We say that a weakly Abelian order on the free group G satisfies *the Little Condition* (LC) if the remainder term w_{m+1} eventually becomes negligible (little): for each $g \in G$, there is $r(g) \in \mathbb{N}$ such that $w_{m+1} \ll g$ if $m \geq r(g)$; and write *(LC)-weakly Abelian* for such an order.

By the commutator calculus, each non-identity element g of the free group can be written as a product of conjugates of a finite set $S(g)$ of generators $a_{i,j}$ and their inverses $T(g)$ so that no $a_{i,j}$ occurs in $S(g) \cap T(g)$ (P. Hall, op. cit.). For each $a_{i,j} \in S(g) \cup T(g)$, let $M(i,j)$ be the number of times that a conjugate of $a_{i,j}$ occurs in $S(g)$, and let $m(i,j)$ be the number of times that a conjugate of $a_{i,j}$ occurs in $T(g)$ (where $b^{-1}a_{i,j}^2 b$ corresponds to two occurrences of $a_{i,j}$ in $S(g)$, etc.). Note that $m(i,j) = 0$ if $a_{i,j} \in S(g)$, and $M(i,j) = 0$ if $a_{i,j} \in T(g)$. Let $n(i,j) = M(i,j) - m(i,j)$; so $n(i,j) \neq 0$ for all $a_{i,j} \in S(g) \cup T(g)$. But if the weakly Abelian order is fully tiered, there is a unique index (i_0, j_0) such that $a_{i_0,j_0} \in S(g) \cup T(g)$ with $a_{i_0,j_0} \gg a_{i,j}$ for all $a_{i,j} \in (S(g) \cup T(g)) \backslash \{a_{i_0,j_0}\}$. Then g is Archimedeanly equivalent to $a_{i_0,j_0}^{n(i_0,j_0)}$ and $g a_{i_0,j_0}^{-n(i_0,j_0)} \ll g$. Indeed, for all $m > i_0$ we have $w_{m+1} \ll g$ (so $g > 1$ iff $a_{i_0,j_0}^{n(i_0,j_0)} > 1$ iff $c_1 \cdots c_m > 1$). Consequently, we can take $r(g) = i_0$ in the proof (or just omit it as I inadvertantly did!).

As before, we can obtain surjective *group* homomorphisms from the finitely generated free group (equipped with a weakly Abelian order) onto nilpotent ordered groups. Although these homomorphisms do *not* preserve the order, they do so

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"approximately"; and the "error term" can be eliminated by quotienting out the direct sum, or taking an appropriate ultraproduct. The point is that the image of a positive element g is positive in all $G/\gamma_{m+1}(G)$ for $m \geq r(g)$. This yields Theorem B and a more general result:

Theorem B1. *Every (LC)-weakly Abelian ordered free group on finitely many generators belongs to the variety of lattice-ordered groups generated by all lattice-ordered groups that are nilpotent.*

Caution: There are non-(LC) weakly Abelian orders on F_3 and the technique given to prove Theorem B fails for such examples.

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References

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