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AN AUTOMATIC ADJOINT THEOREM AND ITS APPLICATIONS

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Abstract. In this paper, we prove the following automatic adjoint theorem: For any sequence spaces $E(X)$ and $F(Y)$, if $E(X)$ has the signed-weak gliding hump property and A is an infinite matrix which transforms $E(X)$ into $F(Y)$, then the transpose matrix A' of A transforms $F(Y)^\beta$ into $E(X)^\beta$, and for any $x \in E(X)$ and $T \in F(Y)^\beta$, $[Ax, T] = [x, A'T]$. That is, the adjoint operator of A automatically exists and is just the transpose matrix A' of A. From the theorem we obtain a class of infinite matrix topological algebras (λ, μ), and prove also a λ-multiplier convergence theorem of Orlicz-Pettis type. The theorem improves substantially the famous Stiles' Orlicz-Pettis theorem.

1. Introduction

Let X, Y be Hausdorff topological vector spaces and $L(X, Y)$ the space of all continuous linear operators from X into Y. Let $E(X)$ ($F(Y)$) be a vector space of X-valued (Y-valued) sequences, if $x \in E(X)$, we denote the kth coordinate of x by x_k so $x = (x_k)$. The β-dual of $E(X)$ (with respect to Y), denoted by $E(X)^{\beta Y}$, is the space of all sequences $(T_k) = T$, $T_k \in L(X, Y)$, such that the series $\sum_{k=1}^\infty T_k x_k$ converges for each $x \in E(X)$. If Y is the scalar field, we write $E(X)^{\beta Y} = E(X)^\beta$. If $x \in E(X)$ and $T \in E(X)^{\beta Y}$, we write $[x, T] = \sum_{k=1}^\infty T_k x_k$.

Let $A_{ij} \in L(X, Y)$ for $i, j \in N$, and let A be the operator-valued matrix $[A_{ij}]$. If for each $x \in E(X)$ and $i \in N$, the series $\sum_{j=1}^\infty A_{ij} x_j$ converges and the sequence $\{\sum_{j=1}^\infty A_{ij} x_j\}_{i=1}^\infty \in F(Y)$, then A is said to transform $E(X)$ into $F(Y)$.

The pair (X, Y) is said to have the Banach-Steinhaus property if whenever $T_k \in L(X, Y)$ converges pointwise, that is, $\lim_k T_k x =: T_0 x$ for each $x \in X$, then the limit operator T_0 is continuous. In particular, if X is the scalar field K, then (K, Y) must have the Banach-Steinhaus property.

We say that a sequence $\{z^{(k)}\}$, of X-valued sequences, is a block sequence if there exists a strictly increasing positive integers sequence $\{n_k\}$ such that

$$z^{(k)} = (0, 0, \ldots, 0, z^{(k)}_{n_{k-1}+1}, \ldots, z^{(k)}_{n_k},\ 0, \ldots), \quad \text{where } n_0 := 0.$$

The sequence space $E(X)$ is said to have the signed-weak gliding hump property (Signed-WGHP) if for each $x \in E(X)$ and any block sequence $\{x^{(k)}\}$ with $x =$

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$\sum_{k=1}^{\infty} x^{(k)}$ (coordinatewise sum), then each subsequence of $\{x^{(k)}\}$ has a further subsequence $\{x^{(q_k)}\}$ and a choice of signs $\varepsilon_k \in \{-1, 1\}$ such that $\bar{x} = \sum_{k=1}^{\infty} \varepsilon_k x^{(q_k)} \in E(X)$. The definition of the Signed-WGHP is due to C. Stuart (see [1]).

A sequence space $E(X)$ is said to be a monotone space if $m_0 E(X) = E(X)$, where m_0 is the space of scalar sequences with finite range and $m_0 E(X)$ is the coordinatewise product ([2]).

Any monotone space $E(X)$ has the Signed-WGHP, while the sequence space $bs = \{(t_i) : \sup_n |\sum_{i=1}^{n} t_i| < \infty\}$ has the Signed-WGHP but fails to be monotone (see [3]).

Further, $c_{00}(X)$ denotes the space of X-valued sequences which are θ eventually. If X is the scalar field we write $c_{00}(X) = c_{00}$.

It is well known that for any two dual pairs $[X_1, X_1']$ and $[Y_1, Y_1']$ and any linear operator $T_0 : X_1 \to Y_1$, the adjoint operator $T_0' : Y_1' \to X_1'$ does not necessarily exist ([4, Lemma 11.1.1]). In this paper, for some sequence spaces and matrix operators, we prove an automatic adjoint theorem. From the theorem, we obtain several new important facts.

2. The main results

In order to prove the automatic adjoint theorem, we at first present a general convergence criterion for double sequences.

Let (G, P) be an abelian quasi-normed group. $s : (G, P) \to (G, P)$ is said to be an additive isometry if for any $x, y \in G$ we have $s(x + y) = s(x) + s(y)$ and $P(s(x)) = P(x)$.

It is clear that the identity mapping and $s(x) = -x$ are both an additive isometry.

Let $p, q \in N$. $[p, q] = \{k : p \leq k \leq q, k \in N\}$ is said to be an interval of N. If $\{\Delta_j\}$ is a sequence of intervals in N with $\max \Delta_j < \min \Delta_{j+1}, j \in N$, then $\{\Delta_j\}$ is said to be a strictly increasing sequence of intervals.

Lemma 1. *Let (G, P) be an abelian quasi-normed group, for $i, j \in N$, $x_{ij} \in G$, and the series $\sum_{j=1}^{\infty} x_{ij}, \sum_{i=1}^{\infty} x_{ij}$ and $\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} x_{ij}$ are convergent. If for each strictly increasing sequence of intervals $\{\Delta_n\}$ in N there is a subsequence $\{\Delta_{n_k}\}$ of $\{\Delta_n\}$ and a sequence of additive isometries $s_k : (G, P) \to (G, P)$ such that the series $\sum_{i=1}^{\infty} \sum_{k=1}^{\infty} \sum_{j \in \Delta_{n_k}} s_k(x_{ij})$ is convergent, then the double series $\sum_{i,j} x_{ij}$ also converges and*

$$\sum_{i,j} x_{ij} = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} x_{ij} = \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} x_{ij}.$$

Proof. At first, we show that the series $\{\sum_{j=1}^{\infty} \sum_{i=1}^{m} x_{ij}\}$ converges uniformly with respect to $m \in N$. If not, there exists $\varepsilon_0 > 0$ such that for each $j_0 \in N$ there exist $j_k \geq j_0$ and m_k satisfying that $P(\sum_{j=j_k}^{\infty} \sum_{i=1}^{m_k} x_{ij}) \geq \varepsilon_0$. Note that the series $\sum_{j=j_k}^{\infty} \sum_{i=1}^{m_k} x_{ij}$ is convergent, so there exists $l_k \in N$ with $l_k \geq j_k$ such that $P(\sum_{j=l_k+1}^{\infty} \sum_{i=1}^{m_k} x_{ij}) < \frac{\varepsilon_0}{2}$. Thus we have

$$P\left(\sum_{j=j_k}^{l_k} \sum_{i=1}^{m_k} x_{ij}\right) \geq \frac{\varepsilon_0}{2}. \tag{1}$$

Let $j_0 = 1$; it follows from inequality (1) that there exist m_1, j_1 and l_1 such that $P(\sum_{j=j_1}^{l_1} \sum_{i=1}^{m_1} x_{ij}) \geq \frac{\varepsilon_0}{2}$. Since the series $\sum_{j=1}^{\infty} \sum_{i=1}^{m} x_{ij}$ is convergent, there exists

$j_{00} \in N$ with $j_{00} > l_1$, such that whenever $j_k \geq j_{00}$ and $l_k \geq j_{00}$, for each $m \leq m_1$ we have $P(\sum_{j=j_k}^{l_k} \sum_{i=1}^{m} x_{ij}) < \frac{\varepsilon_0}{2}$. It follows from (1) that for j_{00}, there exist $m_2, j_2 \geq j_{00}$ and l_2 such that

$$P\left(\sum_{j=j_2}^{l_2} \sum_{i=1}^{m_2} x_{ij}\right) \geq \frac{\varepsilon_0}{2}.$$

That $m_2 > m_1$ and $j_2 > l_1$ is obvious. Continuing this process we can obtain two increasing sequences of positive integers $j_1 \leq l_1 < j_2 \leq l_2 < \cdots$ and $m_1 < m_2 < m_3 < \cdots$ such that

$$P\left(\sum_{j=j_n}^{l_n} \sum_{i=1}^{m_n} x_{ij}\right) \geq \frac{\varepsilon_0}{2}, \qquad n \in N. \tag{2}$$

Let $\Delta_n = \{j : j_n \leq j \leq l_n, j \in N\}$; then $\{\Delta_n\}$ is a strictly increasing sequence of intervals in N and (2) can be written as follows:

$$P\left(\sum_{i=1}^{m_n} \sum_{j \in \Delta_n} x_{ij}\right) \geq \frac{\varepsilon_0}{2}, \qquad n \in N. \tag{3}$$

Consider the infinite matrix $[\sum_{i=1}^{m_p} \sum_{j \in \Delta_n} x_{ij}]_{pn}$. For each $n \in N$, we have

$$\lim_p \sum_{i=1}^{m_p} \sum_{j \in \Delta_n} x_{ij} = \sum_{i=1}^{\infty} \sum_{j \in \Delta_n} x_{ij}.$$

For each strictly increasing positive integers sequence $\{n_q\}$, it follows from the hypothesis that there exist a subsequence $\{n_{q_k}\}$ of $\{n_q\}$ and a sequence of additive isometries $s_k : (G, P) \to (G, P)$ such that the series $\sum_{i=1}^{\infty} \sum_{k=1}^{\infty} \sum_{j \in \Delta_{n_{q_k}}} s_k(x_{ij})$ is convergent, so we have

$$\lim_p \sum_{i=1}^{m_p} \sum_{k=1}^{\infty} \sum_{j \in \Delta_{n_{q_k}}} s_k(x_{ij}) = \sum_{i=1}^{\infty} \sum_{k=1}^{\infty} \sum_{j \in \Delta_{n_{q_k}}} s_k(x_{ij}).$$

It follows from [5, Theorem 2.7] that

$$\lim_p \sum_{i=1}^{m_p} \sum_{j \in \Delta_p} x_{ij} = 0.$$

This contradicts (3). So the series $\{\sum_{j=1}^{\infty} \sum_{i=1}^{m} x_{ij}\}_m$ converges uniformly with respect to $m \in N$.

Now, we can prove that the double series $\sum_{i,j} x_{ij}$ converges and

$$\sum_{i,j} x_{ij} = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} x_{ij} = \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} x_{ij}.$$

In fact, for any $\varepsilon > 0$, since the series $\{\sum_{j=1}^{\infty} \sum_{i=1}^{m} x_{ij}\}_m$ converges uniformly in $m \in N$, there exists $n_1 \in N$ such that for each $m \in N$ and $n \geq n_1$ we have

$$P\left(\sum_{j=n+1}^{\infty} \sum_{i=1}^{m} x_{ij}\right) = P\left(\sum_{i=1}^{m} \sum_{j=n+1}^{\infty} x_{ij}\right) = P\left(-\sum_{i=1}^{m} \sum_{j=n+1}^{\infty} x_{ij}\right) < \frac{\varepsilon}{2}. \tag{4}$$

On the other hand, note that the series $\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} x_{ij}$ is convergent, so there exists $m_1 \in N$ such that whenever $m \geq m_1$ we have

$$P\left(-\sum_{i=m+1}^{\infty}\sum_{j=1}^{\infty} x_{ij}\right) = P\left(\sum_{i=m+1}^{\infty}\sum_{j=1}^{\infty} x_{ij}\right) < \frac{\varepsilon}{2}. \tag{5}$$

It follows from (4) and (5) that for $m \geq m_1$ and $n \geq n_1$, we have

$$\begin{aligned} &P\left(\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} - \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} x_{ij}\right) \\ &\quad \leq P\left(\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij} - \sum_{i=1}^{m}\sum_{j=1}^{\infty} x_{ij}\right) + P\left(\sum_{i=1}^{m}\sum_{j=1}^{\infty} x_{ij} - \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} x_{ij}\right) \\ &\quad = P\left(-\sum_{i=1}^{m}\sum_{j=n+1}^{\infty} x_{ij}\right) + P\left(-\sum_{i=m+1}^{\infty}\sum_{j=1}^{\infty} x_{ij}\right) \\ &\quad < \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon. \end{aligned}$$

Thus, the double series $\sum_{i,j} x_{ij}$ converges and $\sum_{i,j} x_{ij} = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} x_{ij}$. Furthermore, it follows from

$$\begin{aligned} P\left(\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} x_{ij} - \sum_{j=1}^{n}\sum_{i=1}^{\infty} x_{ij}\right) &= \lim_{m} P\left(\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} x_{ij} - \sum_{j=1}^{n}\sum_{i=1}^{m} x_{ij}\right) \\ &= \lim_{m} P\left(\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} x_{ij} - \sum_{i=1}^{m}\sum_{j=1}^{n} x_{ij}\right) \end{aligned}$$

that whenever $n \geq n_1$ we have

$$P\left(\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} x_{ij} - \sum_{j=1}^{n}\sum_{i=1}^{\infty} x_{ij}\right) < \varepsilon.$$

So,

$$\sum_{i,j} x_{ij} = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} x_{ij} = \sum_{j=1}^{\infty}\sum_{i=1}^{\infty} x_{ij}.$$

The lemma is proved. □

Let (G, τ) be a Hausdorff abelian topological group. Then the topology τ can be generated by a family of quasi-norms (see [6]). Thus, from Lemma 1 we have:

Corollary 1. *Let (G, τ) be a Hausdorff abelian topological group, for $i, j \in N$, $x_{ij} \in G$ and the series $\sum_{j=1}^{\infty} x_{ij}, \sum_{i=1}^{\infty} x_{ij}$ and $\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} x_{ij}$ are convergent. If for each strictly increasing sequence of intervals $\{\Delta_n\}$ in N there exist a subsequence $\{\Delta_{n_k}\}$ of $\{\Delta_n\}$ and a sequence $\{\varepsilon_k\}$ with $\varepsilon_k = 1$ or $\varepsilon_k = -1$ such that the series $\sum_{i=1}^{\infty}\sum_{k=1}^{\infty}\sum_{j \in \Delta_{n_k}} \varepsilon_k x_{ij}$ is convergent, then the double series $\sum_{i,j} x_{ij}$ is* *also convergent, and we have*

$$\sum_{i,j} x_{ij} = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} x_{ij} = \sum_{j=1}^{\infty}\sum_{i=1}^{\infty} x_{ij}.$$

Now, we can prove the following automatic adjoint theorem:

Theorem 1. *Let $E(X)$ have the Signed-WGHP and contain $c_{00}(X)$. If (X,Y) has the Banach-Steinhaus property and the matrix $A = [A_{ij}]$ transforms $E(X)$ into $F(Y)$, then the transpose matrix A' must transform $F(Y)^{\beta Y}$ into $E(X)^{\beta Y}$ and for each $x \in E(X)$ and $T \in F(Y)^{\beta Y}$, $[Ax, T] = [x, A'T]$, where $Ax = \{\sum_{k=1}^{\infty} A_{ik}x_k\}_{i=1}^{\infty}, A'T = \{\sum_{i=1}^{\infty} T_i A_{ij}\}_{j=1}^{\infty}$.*

Proof. Let $T = (T_i) \in F(Y)^{\beta Y}$, and let A_i be the ith row of A so

$$[Ax, T] = \sum_{i=1}^{\infty} T_i[x, A_i] = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} T_i A_{ij} x_j$$

for $x \in E(X)$. Thus, for $j \in N$ and $x_0 \in X$, let $e_j \otimes x_0$ be the sequence with an x_0 in the jth coordinate and θ elsewhere. It follows from $E(X) \supseteq c_{00}$ and since (X,Y) has the Banach-Steinhaus property that, for each $j \in N$, the series $\sum_{i=1}^{\infty} T_i A_{ij}$ converges in the strong operator topology of $L(X,Y)$. Since $E(X)$ has the Signed-WGHP, by Corollary 1, if we set $C_j = \sum_{i=1}^{\infty} T_i A_{ij}$ and $C = (C_j)$, then

$$[Ax, T] = \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} T_i A_{ij} x_j = \sum_{j=1}^{\infty}\sum_{i=1}^{\infty} T_i A_{ij} x_j = [x, C] = [x, A'T]. \tag{6}$$

From (6) we get that $C = (C_j) = A'T \in E(X)^{\beta Y}$. Thus the theorem holds. □

3. THE TOPOLOGICAL ALGEBRAS (λ, μ)

In this section, we present the first application of Theorem 1.

Let λ and μ be scalar-valued sequence spaces and have both the Signed-WGHP and $\lambda \supseteq \mu \supseteq c_{00}$, (λ, μ) be all scalar matrices A such that A transforms λ into μ.

It is obvious that (λ, μ) is a vector space under the usually matrix addition and matrix scalar multiplication operations.

If $A = (a_{ij})$ and $B = (b_{ij}) \in (\lambda, \mu)$, it is clear that for each $i \in N$, $\{a_{ij}\}_{j=1}^{\infty} \in \lambda^{\beta}$. For each $j \in N$, let e_j be the scalar sequence with a 1 in the jth coordinate and 0 elsewhere. It follows from $e_j \in \lambda$ and $Be_j \in \mu$ that $\{b_{ij}\}_{i=1}^{\infty} \in \mu$. Note that $\lambda^{\beta} \subseteq \mu^{\beta}$, so for $i, j \in N$, the series $\sum_{k=1}^{\infty} a_{ik}b_{kj}$ is convergent. Thus, we can define the matrix multiplication of A and B by $(\sum_{k=1}^{\infty} a_{ik}b_{kj})$.

Theorem 2. *(λ, μ) is an algebra. In fact, let $A \in (\lambda, \mu)$, and let A_i be the ith row of A; then $A_i \in \lambda^{\beta} \subseteq \mu^{\beta}$. If $x \in \lambda$, it follows from Theorem 1 and $\lambda^{\beta} \subseteq \mu^{\beta}$ that for each $i \in N$, $[Bx, A_i] = [x, B'A_i]$, so we have $(AB)x = A(Bx)$. Since $B \in (\lambda, \mu)$, $x \in \lambda$, it follows that $A(Bx) \in A(\mu) \subseteq A(\lambda) \subseteq \mu$, i.e., $AB \in (\lambda, \mu)$; thus (λ, μ) is closed with respect to matrix multiplication. Similarly, we can prove that the matrix multiplication is also associative. Thus (λ, μ) is an algebra.*

Example 1. Since bv_0 and bs both have Signed-WGHP, so (bv_0, bv_0) and (bs, bs) are both algebras (see [7]).

Now, we equip (λ, μ) with the strong topology and the weak topology as follows:

The strong topology T_s is determined by all the following neighbourhoods $V_s(\theta, M, D)$ of θ in (λ, μ):

$$V_s(\theta, M, D) = \{A \in (\lambda, \mu) : \sup_{u \in D, x \in M} |[Ax, u]| \leq 1\},$$

here D is a bounded subset of $(\mu^\beta, \sigma(\mu^\beta, \mu))$ and M is a bounded subset of $(\lambda, \sigma(\lambda, \lambda^\beta))$.

The weak topology T_w is determined by all the following neighbourhoods $V_w(\theta, G, Q)$ of θ in (λ, μ):

$$V_w(\theta, G, Q) = \{A \in (\lambda, \mu) : \sup_{u \in Q, x \in G} |[Ax, u]| \leq 1\};$$

here Q is a finite subset of μ^β and G is a finite subset of λ.

It is easy to prove that $((\lambda, \mu), T_s)$ and $((\lambda, \mu), T_w)$ are both locally convex topological algebras (see [8]). Thus, we have obtained a class of concrete topological algebras (λ, μ).

We will discuss further properties of (λ, μ) in other papers.

4. On the λ-multiplier convergent series

Now, we present the second application of Theorem 1.

In this section, we assume that (E_0, τ) is a Hausdorff topological vector space with a basis $\{b_i\}$ and coordinate functionals $\{f_i\}$, $F_0 = \{f_i : i \in N\}$; $\sigma(E_0, F_0)$ is the weak topology on E_0 from the duality between E_0 and F_0; and $\lambda \supseteq c_{00}$ is a scalar-valued sequence space. A series $\sum_{j=1}^\infty x_j$ in E_0 is said to be τ-subseries convergent if for each subsequence $\{x_{n_j}\}$ of $\{x_j\}$, the series $\sum_{j=1}^\infty x_{n_j}$ is τ-convergent. A series $\sum_{j=1}^\infty x_j$ in E_0 is said to be $\tau - \lambda$-multiplier convergent if for each $(t_j) \in \lambda$, the series $\sum_{j=1}^\infty t_j x_j$ is τ-convergent. The famous Stiles' Orlicz-Pettis theorem shows that (see [9, 10]): Any $\sigma(E_0, F_0)$-subseries convergent series must also be τ-subseries convergent, or equivalently, if $\lambda \supseteq c_{00}$ is a monotone space, then any $\sigma(E_0, F_0) - \lambda$-multiplier convergent series must also be $\tau - \lambda$-multiplier convergent. Now, we present a general λ-multiplier convergence theorem.

Theorem 3. *If λ has the Signed-WGHP and contains c_{00}, then any $\sigma(E_0, F_0) - \lambda$-multiplier convergent series must also be $\tau - \lambda$-multiplier convergent.*

Proof. Let $E = \lambda, F = \{(x, x, \dots) : x \in E_0\}$, $A = \begin{pmatrix} x_1 & x_2 & \cdots \\ x_1 & x_2 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$, and $t = (t_j) \in \lambda$, and let $x_0 = \sum_{j=1}^\infty t_j x_j$ be the $\sigma(E_0, F_0)$ sum of this series. Let K be the scalar field. Then (K, E_0) has the Banach-Steinhaus property. For $j \in N$ and $x \in E_0$, denote $f_j \otimes b_j(x) = f_j(x) b_j$. It follows from Theorem 1 that for each $T \in F^{\beta E_0}$, $[At, T] = [t, A'T]$. On the other hand, it is clear that $(f_i \otimes b_i) \in F^{\beta E_0}$. Thus,

$$\begin{aligned} \sum_{i=1}^\infty f_i \left(\sum_{j=1}^\infty t_j x_j \right) b_i = \sum_{i=1}^\infty f_i(x_0) b_i = x_0 = \sum_{j=1}^\infty t_j \left(\sum_{i=1}^\infty f_i \otimes b_i(x_j) \right) \\ = \sum_{j=1}^\infty t_j \sum_{i=1}^\infty f_i(x_j) b_i = \sum_{j=1}^\infty t_j x_j, \end{aligned}$$

and the theorem is proved. □

Example 2. Each $\sigma(E_0, F_0) - bs$-multiplier convergent series must be also $\tau - bs$-multiplier convergent.

Since bs has the Signed-WGHP, but bs is not a monotone space, Theorem 3 substantially improved the Stiles' Orlicz-Pettis theorem.

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