LIMITS OF RESIDUALLY IRREDUCIBLE
$p$-ADIC GALOIS REPRESENTATIONS

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Abstract. In this paper we produce examples of converging sequences of
Galois representations, and study some of their properties.

1. Introduction

Consider a continuous representation
\[ \rho : G_L \to GL_m(K) \]
of the absolute Galois group $G_L$ of a number field $L$, with $K$ a finite extension of
$\mathbb{Q}_p$, with $\mathcal{O}$ its ring of integers, $|\cdot|$ its norm, and $k$ its residue field. Then $\rho$ has an
integral model taking values in $GL_m(\mathcal{O})$, and the semisimplification of its reduction
modulo the maximal ideal $m$ of $\mathcal{O}$, denoted by $\overline{\rho}$, is independent of the choice of
integral model. We assume that $\rho$ is absolutely irreducible and, in fact, we assume
that all the $p$-adic representations considered in this paper are residually absolutely
irreducible.

Definition 1. An infinite sequence of (residually absolutely irreducible) continuous
representations $\rho_i : G_L \to GL_m(K)$ tends to $\rho : G_L \to GL_m(K)$, if $|\text{tr}(\rho_i(g)) - \text{tr}(\rho(g))| \to 0$ uniformly for all $g \in G_L$. We also say that the $\rho_i$’s converge to $\rho$, or $\rho$ is their limit point.

By Theorem 1 of [Ca], which we can apply because of our blanket assumption of
residual absolute irreducibility, this is equivalent to saying that given any integer $n$,
for all $i > 0$, the reduction mod $m^n$, $\rho_{i,n}$, of (an integral model of) $\rho_i$ is isomorphic
to the reduction mod $m^n$, $\rho_n$, of (an integral model of) $\rho$. Note that we are not
assuming that the $\rho_i$’s (or $\rho$) are finitely ramified, though we do know by the main
theorem of [KhRa] that the density of primes which ramify in a given $\rho_i$ is 0.

In this paper we study the limiting behavior of the lifts produced in [Ra] and
completely characterize the limit points of these lifts (see Theorem 1 below). This
suggests another approach to certain special cases of the modularity lifting theo-
rems of Wiles, Taylor-Wiles, et al. In the process we construct many sequences of
converging $p$-adic Galois representations (of fixed determinant and fixed ramifica-
tion behaviour at $p$). This raises many questions that can be posed far more easily
than answered.

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Consider $\bar{\rho}: G_{\mathbb{Q}} \to GL_2(k)$ that satisfies the conditions of [R1], namely:

- $\bar{\rho}$ and $\text{Ad}^p(\bar{\rho})$ are absolutely irreducible Galois representations, and the finite field $k$ of characteristic $p$ is the minimal field of definition of $\bar{\rho}$.
- The (prime to $p$) Artin conductor $N(\bar{\rho})$ of $\bar{\rho}$ is minimal amongst its twists. Denote by $S$ the set of primes given by the union of the places where $\bar{\rho}$ is ramified and $\{p, \infty\}$.
- If $\bar{\rho}$ is even, then for the decomposition group $G_p$ above $p$ we assume that $\bar{\rho}|_{G_p}$ is not twist equivalent to $\left( \begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix} \right)$ or twist equivalent to the indecomposable representation $\left( \begin{smallmatrix} \chi & * \\ 0 & 1 \end{smallmatrix} \right)$ where $\chi$ is the mod $p$ cyclotomic character.
- If $\bar{\rho}$ is odd, we assume $\bar{\rho}|_{G_{\mathbb{Q}}} = \text{res} \rho \otimes \chi$ is not twist equivalent to the trivial representation or the indecomposable unramified representation given by $\left( \begin{smallmatrix} 1 & 1 \\ 0 & 1 \end{smallmatrix} \right)$.
- $p \geq 7$ and the order of the projective image of $\bar{\rho}$ is a multiple of $p$.

Let $Q = \{q_1, \ldots, q_a\}$ be a finite set of primes such that $q_i \neq \pm 1 \mod p$, unramified in $\bar{\rho}$, and the ratio of the eigenvalues of $\bar{\rho}(\text{Frob}_{q_i})$ equal to $q_i^{k-1}$. We will call the primes as in $Q$ above Ramakrishna primes for $\bar{\rho}$ or $R$-primes for short (suppressing the $\bar{\rho}$ which is fixed). We consider the deformation ring $R_{Q,-new}^{S,\mathbb{Q}}$ of [KR] (see Definition 1 of loc. cit.). To orient the reader we recall the definition of $R_{Q,-new}^{S,\mathbb{Q}}$. For this we need:

**Definition 2.** If $q$ is a prime, $G_{\mathbb{Q}_q}$ the absolute Galois group of $\mathbb{Q}_q$ and $R$ a complete Noetherian local ring with residue field $k$, a continuous representation $\rho : G_{\mathbb{Q}_q} \to GL_2(R)$ is said to be special if $\rho$ is even and a continuous representation $\varepsilon \in G_{\mathbb{Q}_q}$ such that $\rho|_{G_q} \otimes \varepsilon$ is a continuous character.

A continuous representation $\tilde{\rho} : G_{\mathbb{Q}} \to GL_2(R)$, is said to be special at a prime $q$ if $\tilde{\rho}|_{D_q}$, with $D_q$ a decomposition group at $q$, is special.

Then $R_{Q,-new}^{S,\mathbb{Q}}$ is the universal ring that parametrizes deformations of $\bar{\rho}$ that are minimally ramified at $S$ (in the sense of [R1]), and when $\bar{\rho}$ is even there is no condition at $p$ and such that at primes $q \in Q$ these deformations are special. The ring $R_{Q,-new}^{S,\mathbb{Q}}$ is a complete Noetherian local $W(k)$-algebra, with $W(k)$ the Witt vectors of $k$. The deformation rings considered here are for the deformation problem with a certain fixed (arithmetic) determinant character, and all the deformations of $\bar{\rho}$ we consider will have this fixed determinant character.

**Definition 3.** A finite set of $R$-primes $Q$ is said to be auxiliary if $R_{Q,-new}^{S,\mathbb{Q}} \simeq W(k)$.

In [R1] auxiliary sets $Q$ of the above type were proven to exist. The representation corresponding to $R_{Q,-new}^{S,\mathbb{Q}} \simeq W(k)$ is denoted by $\rho_{Q,-new}^{S,\mathbb{Q}}$. We will call these lifts Ramakrishna lifts of $\bar{\rho}$ or $R$-lifts for short (suppressing the $\bar{\rho}$ which is fixed).

**Theorem 1.** A continuous representation $\rho : G_{\mathbb{Q}} \to GL_2(W(k))$ that is a deformation of $\bar{\rho}$, is a limit point of distinct $R$-lifts, if and only if $\rho$ is unramified outside $S$ and the set of all $R$-primes, and minimally ramified at primes of $S$.

**Remark.** Thus, we have a complete description of the “$p$-adic closure” of $R$-lifts. Note that, in particular, each $R$-lift is a limit point of other $R$-lifts. Note also that any deformation $G_{\mathbb{Q}} \to GL_2(K)$ of $\bar{\rho}$ that is a limit point of $R$-lifts has a model that takes values in $GL_2(W(k))$. The above theorem can be viewed in a sense as producing an “infinite fern” structure (in the sense of Mazur) in the set of all $R$-lifts.
of a given \( \mathfrak{p} \) as above (see the picture below). From the proof of Theorem 1 above, we in fact can deduce that each \( R \)-lift gives rise to infinitely many “splines” passing through it, where a “spline” consists of a sequence of \( R \)-lifts converging to it, and each element in a spline gives rise to its own infinitely many splines. Missing from the picture are the limit points of \( R \)-lifts which themselves are not \( R \)-lifts and which the theorem above characterizes completely.

In Section 2 we prove Theorem 1 which is a simple consequence of the methods of \( [R1] \) and \( [T1] \). In Section 3 we prove a result about converging sequences of representations arising from newforms, and point out a possible approach to the lifting theorems of Wiles, et al. that is suggested by the work here. In Section 4 we raise questions about rationality and motivic properties of converging sequences of \( p \)-adic Galois representations.

2. CONVERGING SEQUENCES OF GALOIS REPRESENTATIONS

We now prove Theorem 1 which follows from the methods of \( [R1] \) and \( [T1] \). For the proof we need the following lemma which follows from the methods of \( [R1] \) (see also Lemma 1.2 of \( [T1] \)) and Lemma 8 of \( [KR] \).

Lemma 1. Let \( \rho_n : G \mathbb{Q} \to \text{GL}_2(W(k)/(p^n)) \) be a lift of \( \mathfrak{p} \) that is unramified outside \( S \) and the set of all \( R \)-primes, minimally ramified at primes of \( S \), and special at all the primes outside \( S \) at which it is ramified. Let \( Q'_n \) be any finite set of primes that includes the primes of ramification of \( \rho_n \), such that \( Q'_n \setminus S \) contains only \( R \)-primes and such that \( \rho_n|_{D_q} \) is special for \( q \in Q'_n \setminus S \). Then there exists a finite set of primes \( Q_n \) that contains \( Q'_n \), such that \( \rho_n|_{D_q} \) is special for \( q \in Q_n \setminus S \), \( Q_n \setminus S \) contains only \( R \)-primes and \( Q_n \setminus S \) is auxiliary.

Proof. We use \( [R1] \) and Lemma 8 of \( [KR] \) to construct an auxiliary set of primes \( T_n \) such that \( \rho_n|_{D_q} \) is special for \( q \in T_n \). Then as \( Q'_n \setminus S \) contains only \( R \)-primes, it follows (using notation of \( [R1] \)) from Proposition 1.6 of \( [W] \) that the kernel and cokernel of the map

\[
H^1(G_{S \cup T_n \cup Q'_n}, \text{Ad}^0(\mathfrak{p})) \to \bigoplus_{v \in S \cup T_n \cup Q'_n} H^1(G_v, \text{Ad}^0(\mathfrak{p}))/N_v
\]
have the same cardinality. Then using Proposition 10 of \[\text{[KR]}\], or Lemma 1.2 of \[\text{[TT]}\], and Lemma 8 of \[\text{[KR]}\], we can augment the set \(S \cup T_n \cup Q'_n\) to get a set \(Q_n\) as in the statement of the lemma.

We are now ready to prove Theorem \[\text{[H]}\]. If \(\rho : G_Q \to GL_2(W(k))\) is a limit point of \(R\)-lifts, then it is clear that \(\rho\) is unramified outside \(S\) and the set of all \(R\)-primes, and minimally ramified at primes of \(S\). We prove the converse. So let \(\rho\) satisfy the conditions of Theorem \[\text{[H]}\] and recall that we denote by \(\rho_n\) the reduction modulo \(p^n\) of \(\rho\). It is easily checked that if \(q\) is an \(R\)-prime, then any deformation of \(\overline{\tau}|_{D_q}\) to a ramified \(p\)-adic representation is special; this follows from the structure of tame inertia and the fact that \(q^2 \not\equiv 1 \pmod{p}\). Furthermore, from the method of proof of Proposition 1 of \[\text{[KhRa]}\], we easily deduce that the set of primes \(q\) for which \(\rho|_{D_q}\) is special is of density 0. Thus using Cebotarev and the assumptions on \(\overline{\tau}\) in the introduction, we choose a finite set of primes \(Q'_n\) such that

- \(Q'_n\) consists of \(R\)-primes and \(\rho_n|_{D_q}\) is special for \(q \in Q'_n\).
- \(Q'_n\) contains all the ramified primes of \(\rho_n\).
- For some prime \(q \in Q'_n\), \(\rho|_{D_q}\) is not special.

Using Lemma \[\text{[T]}\] we complete \(Q'_n\) to a set \(Q_n\) such that \(Q_n|_S\) is auxiliary and \(\rho_n|_{D_q}\) is special for \(q \in Q_n\). Then we claim \(\rho_{Q_n|_S - new} \equiv \rho \pmod{p^n}\). The claim is true as there is a unique representation \(G_Q \to GL_2(W(k)/(p^n))\) (with the determinant that we have fixed) that is unramified outside \(S \cup Q_n\), minimal at \(S\) and special at primes of \(Q_n|_S\) (as \(R_{Q_n|_{S - new}} \cong W(k)\)). By construction the sets \(Q_n\) contain at least one prime at which \(\rho\) is not special. Thus, we see that we can pick a subsequence of mutually distinct representations \(\rho_i\) from the \(\rho_{Q_n|_{S - new}}\) such that \(\rho_i \to \rho\).

**Remark.** It is of vital importance that \(\rho\) is \(GL_2(W(k))\)-valued since, otherwise, we would not be able to invoke the disjointness results that are used in the proof of Lemma \[\text{[T]}\] (Lemma 8 of \[\text{[KR]}\]).

**Remark.** Theorem \[\text{[H]}\] can be applied in practice to give many examples of converging sequences of \(p\)-adic representations: for a non-CM elliptic curve \(E/Q\) for most primes \(p\) the mod \(p\) representation satisfies the conditions given in the introduction, and the corresponding \(p\)-adic representation is minimally ramified and \(GL_2(Z_p)\) valued.

We end this section with a result that refines the main result of \[\text{[KhRa]}\].

**Proposition 1.** If \(\rho_i : G_L \to GL_m(K)\) is a sequence of (residually absolutely irreducible) continuous representations that converges to \(\rho\), then the set of primes where any of the \(\rho_i\)’s is ramified (i.e., \(\bigcup \text{Ram}(\rho_i)\) where \(\text{Ram}(\rho_i)\) is the set of primes at which \(\rho_i\) is ramified) is of density zero.

**Proof.** Denote by \(\rho_{i,n}\) (resp., \(\rho_n\)) the reduction mod \(m^n\) of an integral model of \(\rho_i\) (resp., \(\rho\)). The proof consists of applying Theorem 1 of \[\text{[KhRa]}\] twice; more precisely, first its statement, and then its proof. By an application of its statement we conclude that the density of \(\bigcup_{i=1}^n \text{Ram}(\rho_i)\) is 0 for any \(n\). Now applying the proof of Theorem 1 of \[\text{[KhRa]}\], we define \(\epsilon_{\rho,n}\) to be the upper density of the set \(S_{\rho,n}\) of primes \(q\) of \(L\) that

- lie above primes which split in \(L/Q\),
- are unramified in \(\rho_i\) and \(\neq p\),
Lemma 2. Given any \( \varepsilon > 0 \), there is an integer \( N_\varepsilon \) such that \( c_{p,n} < \varepsilon \) for \( n > N_\varepsilon \).

To prove Proposition 3 it is enough to show that given any \( \varepsilon > 0 \), the upper density of the set \( \bigcup \text{Ram}(\rho_i) \) is \( < \varepsilon \). Since \( \bigcup_{i=1}^{\infty} \text{Ram}(\rho_i) \) has density 0 for (the finite) \( n \) that is the supremum of the \( i \)'s such that \( \rho_{i,N_\varepsilon} \) is not isomorphic to \( \rho_{N_\varepsilon} \), and \( \rho_{N_\varepsilon} \) is finitely ramified, it follows from the lemma above that the upper density of \( \bigcup \text{Ram}(\rho_i) \) is \( < \varepsilon \). Hence Proposition 3.

Remark. One can ask for more refined information about the asymptotics of ramified primes in (limits of) residually absolutely irreducible \( p \)-adic Galois representations. For instance, in Theorem 1 of [KhRa] one can ask (clued by Theorem 10 of [S1]) if the order of growth of ramified primes can be proved to be bounded by \( O(x^{1-\frac{1}{p^{n+\epsilon}}}) \), where \( N \) is the \( p \)-adic analytic dimension of \( \text{im}(\rho) \), for any \( \epsilon > 0 \). Such quantitative refinements asked for by Serre in an e-mail message to the author are difficult and will require a new idea (that goes beyond [KhRa]) and a strong use of effective versions of the Cebotarev density theorem.

3. Finite and infinite ramification

Let \( L \) be a number field and \( K \) a finite extension of \( \mathbb{Q}_p \) as before.

Definition 4. We say that a residually absolutely irreducible continuous representation \( \rho : G_L \to GL_n(K) \) is motivic if \( \rho \) arises as a subquotient of the \( i \)th étale cohomology \( H^i(X \times_L \overline{T}, K) \) of a smooth projective variety \( X \) defined over a number field \( L \).

A motivic representation is finitely ramified. In [R] examples of residually irreducible representations \( \rho : G_{\mathbb{Q}} \to GL_2(K) \) were constructed that were infinitely ramified (see also the last section of [KR]). Infinitely ramified \( p \)-adic representations cannot be motivic. But they can arise as limits of \( p \)-adic representations that are motivic. Fix an embedding \( \mathbb{Q} \to \overline{\mathbb{Q}}_p \). Then as in [R] (and the last section of [KR]), there is a sequence of eigenforms \( f_i \in S_2(\Gamma_0(N_i)) \), for a sequence of square-free integers \( N_i \) such that \( N_i \to \infty \) and \( (p,N_i) = 1 \), new of level \( N_i \) such that the corresponding \( p \)-adic representations \( \rho_{f_i} : G_{\mathbb{Q}} \to GL_2(\mathbb{Z}_p) \) have a \( p \)-adic limit \( \rho \), with \( \rho \) infinitely ramified. Such a \( \rho \) is non-motivic, but is the limit of motivic \( p \)-adic representations. Such limits of eigenforms (in the works of Serre and Katz; for instance, cf., [Ka]) have been considered when varying weights or varying the \( p \)-power level, while fixing the prime-to-\( p \) part of the level.

Proposition 2. Let \( f_i \in S_2(\Gamma_0(N_i)) \) be a sequence of eigenforms with coefficients in a finite extension \( K \) of \( \mathbb{Q}_p \) with \( (N_i,p) = 1 \) and \( p \geq 3 \), that in the \( p \)-adic \( q \)-expansion topology tend to an element \( f \in K[[q]] \), such that the corresponding residual representation \( \overline{\mathfrak{f}} \) satisfies the conditions in the introduction. The element \( f \), that gives rise naturally to a Galois representation \( \rho_f : G_{\mathbb{Q}} \to GL_2(K) \), is the \( q \)-expansion of a classical eigenform (of weight 2) if and only if \( \rho_f \) is finitely ramified.
Proof. The only if part is clear. The if part follows from the methods of Wiles (see Chapter 3 of [W] and also [TW]) and their refinements: note that $\rho_f$ is finite flat at $p$. □

Remark. Applying Theorem 1 when $\rho$ is odd and finite flat at $p$, in which case the $R$-lifts are modular by Theorem 1 of [K], we can construct systematically many examples of sequences of eigenforms $f_i \to f$ ($f \in K[[q]]$), with the levels of $f_i$ unbounded and such that $\rho_f$ is finitely ramified ($f$, in fact, is then a classical eigenform as above). On the other hand, as recalled above in [R] (see also last section of [KR]), we have examples of situations as above with $\rho_f$ infinitely ramified.

It will be of interest to see if Proposition 2 could be proved in a more self-contained manner. The proof above does not use seriously the fact that one does know that $f$ arises as a limit of the classical forms $f_i$. If such a proof could be devised, in conjunction with Theorem 1 above and Theorem 1 of [K] (which is due to Ravi Ramakrishna) it would give, in special cases, a simpler approach using $R$-primes to the modularity lifting theorems of Wiles, et al. (see also [K]) that directly works with the $p$-adic Galois representation that needs to be proved modular, and if it could be implemented, would avoid (albeit in special cases) the sophisticated deformation theoretic approach of [W].

We elaborate on this: Assume that $\rho$ is modular. In Theorem 1 we have characterized the limit points of $R$-lifts. By Theorem 1 of [K] which proves that the representation corresponding to $R_{S,\mathbb{Q}}^{\text{new}}$ is modular as a consequence of the isomorphism $R_{S,\mathbb{Q}}^{\text{new}} \simeq T_{S,\mathbb{Q}}^{\text{new}}$ (using notation of [K]), we know that $R$-lifts are modular. Hence, limits of $R$-lifts do arise as limits of $p$-adic representations arising from classical newforms. It only (!) remains to prove that a limit of a converging sequence of $p$-adic representations arising from newforms (say of weight 2 and level prime to $p$ to avoid delicate considerations at $p$) that is finitely ramified itself arises from a newform (i.e., prove Proposition 2 without appealing directly to [W]). Note that for a semistable elliptic curve $E$, for all large enough primes $p$ (bigger than 3 for the methods here to directly work unfortunately!), $T_p(E)$ is a limit point of $R$-lifts.

Note. In recent work we have indeed been able to give a self-contained approach to a result such as Proposition 2 above under some technical restrictions; see [K1].

4. Questions

Proposition 2 suggests that a representation that arises as a limit of motivic representations (of “bounded weights”; see Definition 3 below) is finitely ramified if and only if it is motivic. We first recall one of the main conjectures in [FM] in a form that is most pertinent for the considerations here.

Conjecture 1 (Fontaine-Mazur). Consider a continuous residually absolutely irreducible representation $\rho : G_L \to GL_m(K)$ that is potentially semistable at places above $p$. Then the following are equivalent:

1. $\rho$ is motivic,
2. $\rho$ is finitely ramified.
From our earlier considerations it is natural to ask the following weaker question.

**Question 1.** Consider a continuous residually absolutely irreducible representation \( \rho : G_L \to GL_m(K) \) that is potentially semistable at places above \( p \) and arises as the limit of motivic representations \( \rho_i \). Then if \( \rho \) is finitely ramified, is \( \rho \) motivic?

It seems unlikely that the infinitely ramified representations produced in [R] are algebraic (see definition below). This motivates the following considerations.

**Definition 5.** A continuous (residually absolutely irreducible) representation \( \rho : G_L \to GL_m(K) \) is said to be algebraic if there is a number field \( F \) such that the characteristic polynomial of \( \rho(Frob_q) \) has coefficients in the ring of integers of \( F \) for all primes \( q \) which are unramified in \( \rho \). The minimal such field is the field of definition of \( \rho \).

As by the main theorem of [KhRa], the set of primes at which \( \rho \) ramifies is of density 0, the definition above is a sensible one.

**Definition 6.** A continuous (residually absolutely irreducible) algebraic representation \( \rho : G_L \to GL_m(K) \) is said to be of weight \( \leq t \) (\( t \in \mathbb{Z} \)) if for primes \( q \) that are unramified in \( \rho \), any root \( \alpha \) of the characteristic polynomial of \( \rho(Frob_q) \) satisfies \( |\iota(\alpha)| \leq |k_q|^{t-1} \) for any embedding \( \iota : \overline{Q} \to C \), with \( k_q \) the residue field at \( q \).

**Question 2.** If \( \rho_i : G_L \to GL_m(K) \) is an infinite sequence of (residually absolutely irreducible) distinct algebraic representations, all of weight \( \leq t \) for some fixed integer \( t \), converging to \( \rho : G_L \to GL_m(K) \), and \( K_i \) the field of definition of \( \rho_i \), does \( [K_i : Q] \to \infty \) as \( i \to \infty \)?

**Remark.**
- It is observed in [R] (this is a remark of Fred Diamond) that in the situation of Question 2 only finitely many of the \( \rho_i \)'s can arise from elliptic curves; this is a consequence of the Mordell conjecture which gives that suitable twists of the classical modular curves \( X(p^n) \) for \( n \gg 0 \) have finitely many \( L \)-valued points for a given number field \( L \).
- If Question 2 has a negative answer, using Proposition 1 we deduce that for a set of primes \( \{r\} \) of density one, the characteristic polynomials of \( \rho_i(Frob_r) \) are eventually constant. Hence, we deduce that the characteristic polynomials of \( \rho(Frob_r) \) are defined and integral over a fixed number field \( F \), i.e., \( \rho \) is algebraic (in the case when \( \rho \) is infinitely ramified this is linked to the questions below).

**Question 3.** Let \( \rho : G_L \to GL_m(K) \) be a continuous, residually absolutely irreducible representation that is potentially semistable at places above \( p \). Then are the following equivalent:

1. \( \rho \) is motivic,
2. \( \rho \) is finitely ramified,
3. \( \rho \) is algebraic?

In the question above, the equivalence of 1 and 2 is the Fontaine-Mazur conjecture recalled above: the possible equivalence of 3 to 1 and 2 is the main thrust of the question. One might even ask the stronger question: If \( \rho : G_L \to GL_m(K) \), a continuous, residually absolutely irreducible representation, is algebraic, then is \( \rho \) forced to be both finitely ramified and potentially semistable at places above \( p \)?

All the questions of this section have a positive answer when \( m = 1 \).
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