

AN AFFINE PI HOPF ALGEBRA NOT FINITE OVER A NORMAL COMMUTATIVE HOPF SUBALGEBRA

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ABSTRACT. In formulating a generalized framework to study certain noncommutative algebras naturally arising in representation theory, K. A. Brown asked if every finitely generated Hopf algebra satisfying a polynomial identity was finite over a normal commutative Hopf subalgebra. In this note we show that Radford's biproduct, applied to the enveloping algebra of the Lie superalgebra $\mathfrak{pl}(1, 1)$, provides a noetherian prime counterexample.

1. INTRODUCTION

Noncommutative affine PI Hopf algebras arise in fundamental representation-theoretic contexts: as enveloping algebras of finite dimensional restricted Lie algebras in positive characteristic, as group algebras of finitely generated abelian-by-finite groups, and as quantizations of algebraic groups – and their corresponding Lie algebras – at roots of unity. (By *affine* we mean “finitely generated over the ground field as an associative algebra,” and by PI we mean “satisfying a polynomial identity.”) In all of the preceding examples, the Hopf algebra of interest is finitely generated as a module over a normal affine commutative Hopf subalgebra. Therefore, in developing a general structure theory to handle these examples, K. A. Brown asked [3, Question B] whether every affine PI Hopf algebra was finitely generated as a left (or right) module over some normal commutative Hopf subalgebra (cf. [4]). In this note we present a counterexample; we describe a prime affine noetherian PI Hopf algebra not finitely generated as a left or right module over any of its normal commutative Hopf subalgebras. This counterexample also arises in a representation-theoretic context, resulting from an application of Radford's biproduct [12] to the enveloping algebra of the Lie superalgebra $\mathfrak{pl}(1, 1)$. (In the terminology of [10], the counterexample is the “bosonization” of the enveloping algebra of $\mathfrak{pl}(1, 1)$.)

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2. PREPARATORY RESULTS

Let k be an algebraically closed field of characteristic zero. We assume throughout this note that k is the ground field for all of the vector spaces mentioned (explicitly or implicitly). In particular, all algebras (associative and otherwise) are over k . All of the associative algebras discussed below will be assumed to be unital.

The reader is referred to [9] for background on noncommutative rings, to [11] for background on Hopf algebras, and to [13] for background on Lie superalgebras.

2.1. (i) For all of the Hopf algebras H we will encounter, we will use Δ_H to denote the coproduct, ϵ_H to denote the counit, and S_H to denote the antipode. The subscripts will be dropped when the meaning is clear.

(ii) Let H be a Hopf algebra. Recall the left and right adjoint actions of H on itself,

$$(\text{ad}_l a)(b) = \sum a_1 b S(a_2) \quad \text{and} \quad (\text{ad}_r a)(b) = \sum S(a_1) b a_2,$$

for $a, b \in H$, where $\Delta(a) = \sum a_1 \otimes a_2$. Also recall that a Hopf subalgebra A of H is *normal* in H provided $(\text{ad}_l H)(A)$ and $(\text{ad}_r H)(A)$ are contained in A .

2.2. Next we briefly review some superalgebra.

(i) A vector space V equipped with a \mathbb{Z}_2 -grading is termed a *supervector space*; we indicate the grading by writing $V = V_0 \oplus V_1$. The vectors in V_0 are called *even* and those in V_1 are called *odd*. We define the parity of a homogeneous vector $v \in V$ to be $p(v) = 0$ if v is even and $p(v) = 1$ if v is odd.

We will use SuperVect to denote the category of supervector spaces with homogeneous morphisms. For $V, W \in \text{SuperVect}$ the commutativity constraint $V \otimes W \rightarrow W \otimes V$ is different from the classical one and is given by the formula

$$v \otimes w \mapsto (-1)^{p(v)p(w)} w \otimes v,$$

for homogeneous $v \in V$ and $w \in W$.

(ii) One can define Lie algebras, bialgebras, Hopf algebras, etc. in SuperVect by replacing the ordinary structure maps with morphisms in SuperVect . The resulting objects are referred to, respectively, as Lie superalgebras, superbialgebras, Hopf superalgebras, etc.

For example, a Hopf superalgebra H is a \mathbb{Z}_2 -graded associative algebra, possessing a coassociative multiplicative (in the super-sense) morphism $\Delta: H \rightarrow H \otimes H$ in SuperVect , and further equipped with a counit ϵ and antipode S satisfying the expected axioms. By “multiplicativity in the super-sense” we mean that Δ satisfies the relation

$$\Delta(ab) = \sum (-1)^{p(a_2)p(b_1)} a_1 b_1 \otimes a_2 b_2$$

for all homogeneous $a, b \in H$ (where $\Delta(a) = \sum a_1 \otimes a_2$, $\Delta(b) = \sum b_1 \otimes b_2$ and the components are homogeneous). This formulation is required because the tensor product of algebras A and B in SuperVect is defined to be the vector space $A \otimes B$ equipped with the multiplication

$$(a \otimes b)(a' \otimes b') := (-1)^{p(a')p(b)} a a' \otimes b b',$$

for homogeneous $a, a' \in A$ and $b, b' \in B$. (Hopf superalgebras are referred to as “graded Hopf algebras” in [7].)

(iii) Henceforth, “graded” will mean “ \mathbb{Z}_2 -graded.”

2.3. (i) The Hopf superalgebras we are most concerned with in this note arise as follows: Let \mathfrak{g} be a Lie superalgebra, and let $U(\mathfrak{g})$ be the enveloping algebra of \mathfrak{g} . Then $U(\mathfrak{g})$ is a Hopf superalgebra with coproduct Δ , counit ϵ , and antipode S determined by

$$\Delta(x) = x \otimes 1 + 1 \otimes x, \quad S(x) = -x, \quad \epsilon(x) = 0,$$

for all $x \in \mathfrak{g}$. Note that $U(\mathfrak{g})$ is supercocommutative with no nontrivial grouplike elements.

(ii) Now suppose that U' is a sub-Hopf superalgebra of $U(\mathfrak{g})$. By a theorem of Kostant [7, Theorem 3] (cf. [1]), there exists a sub-Lie superalgebra \mathfrak{a} of \mathfrak{g} such that $U' = U(\mathfrak{a}) \subseteq U$. (We can apply Kostant's theorem because U , and hence also U' , is supercocommutative with no nontrivial grouplike elements.) Note that \mathfrak{a}_0 is contained in \mathfrak{g}_0 and that \mathfrak{a}_1 is contained in \mathfrak{g}_1 .

2.4. Let U be any Hopf superalgebra.

(i) Recall (see, e.g., [1]) that Radford's biproduct [12] allows us to associate to U a Hopf algebra \overline{U} in the following way. As an algebra,

$$\overline{U} = U \# k[\langle t \rangle] \cong U \# k[\mathbb{Z}/2\mathbb{Z}]$$

is the smash product of U and $k[\langle t \rangle]$, where t is the graded algebra automorphism of U (of order 2) acting on U by parity (i.e., $tat = (-1)^{p(a)}a$ for all homogeneous $a \in U$). The coproduct Δ , counit ϵ , and antipode S of \overline{U} are determined by

$$\Delta(x) = \sum x_1 \# t^{p(x_2)} \otimes x_2 \# 1, \quad \Delta(t) = t \otimes t,$$

and

$$S(x) = t^{p(x)} S_U(x), \quad \epsilon(x) = \epsilon_U(x), \quad S(t) = t, \quad \epsilon(t) = 1,$$

where $x \in U$ is homogeneous and $\Delta_U(x) = \sum x_1 \otimes x_2$ with homogeneous components.

(ii) In particular, $K := k[\langle t \rangle]$ is a Hopf subalgebra of \overline{U} , and there is a unique Hopf algebra projection

$$\overline{U} \xrightarrow{\pi} K,$$

sending $u \# h$ to $\epsilon(u)h$. Observe that

$$U = \{x \in \overline{U} : (\text{id} \otimes \pi)\Delta(x) = x \otimes 1\},$$

where id denotes the identity map, is the algebra of K -coinvariants of \overline{U} (see [12]).

(iii) Also note that U is a subalgebra of \overline{U} , and there is a unique coalgebra projection

$$\overline{U} \xrightarrow{\Pi} U,$$

sending $u \# h$ to $u\epsilon(h)$.

(iv) Lastly, by declaring t to be even, we can extend the grading on U to a grading on \overline{U} . Note that $ta = (-1)^{p(a)}at$ for all homogeneous $a \in \overline{U}$.

2.5. Let \mathfrak{g} be a Lie superalgebra with enveloping algebra $U := U(\mathfrak{g})$.

(i) The Hopf algebra structure on \overline{U} follows from

$$\Delta(x) = x \otimes 1 + t^{p(x)} \otimes x, \quad S(x) = -t^{p(x)}x, \quad \epsilon(x) = 0, \quad \Delta(t) = t \otimes t, \quad S(t) = t, \quad \epsilon(t) = 1,$$

for homogeneous $x \in \mathfrak{g}$ (see, e.g., [5]).

(ii) Now choose homogeneous $a, b \in \mathfrak{g}$. Write $\Delta_U(a) = \sum a_1 \otimes a_2$. Then

$$\begin{aligned} (\text{ad}_{\mathfrak{g}} a)(b) &= [a, b] = ab - (-1)^{p(b)p(a)}ba = \sum a_1 b (-1)^{p(b)p(a_2)} S_U(a_2) \\ &= \sum (a_1 \# 1)(b \# 1) (-1)^{p(b)p(a_2)} (1 \# t^{p(a_2)}) S(a_2 \# 1) \\ &= \sum (a_1 \# t^{p(a_2)})(b \# 1) S(a_2 \# 1) = (\text{ad}_l a)(b), \end{aligned}$$

where ad_l denotes the left adjoint action of \overline{U} on itself, and $\text{ad}_{\mathfrak{g}}$ denotes the adjoint (graded) representation of \mathfrak{g} in itself. Consequently, if A is a normal Hopf subalgebra of \overline{U} such that $A \cap U = U(\mathfrak{a})$, for some Lie superalgebra \mathfrak{a} of \mathfrak{g} , then \mathfrak{a} is an ideal of \mathfrak{g} .

(iii) Note that $U(\mathfrak{g}_0)$ is both a sub-Hopf superalgebra of U and a Hopf subalgebra of \overline{U} .

2.6. Lemma. *Let U be any Hopf superalgebra, and let A be a Hopf subalgebra of \overline{U} containing t . Then*

(i) $A \cap U$ is a sub-Hopf superalgebra of U . In particular, if U is the enveloping algebra of a Lie superalgebra \mathfrak{g} , then $A \cap U = U(\mathfrak{a})$ for some sub-Lie superalgebra \mathfrak{a} of \mathfrak{g} .

(ii) $A = (A \cap U) \# K$.

Proof. Set $\Delta := \Delta_{\overline{U}}$ and $S := S_{\overline{U}}$. To begin, $\pi(A) = K$ since $t \in A$. Also, the composition of Hopf algebra homomorphisms

$$K \xrightarrow{i} A \xrightarrow{i} \overline{U} \xrightarrow{\pi} K,$$

where i denotes the inclusion map, reduces to the identity map on K . Hence, by Theorem 3 in [12], we obtain a biproduct decomposition

$$A = \tilde{A} \# K,$$

where, by construction, $\tilde{A} = \Pi(A)$ is a Hopf superalgebra. Therefore, it is straightforward to verify that

$$\tilde{A} = \{x \in A : (\text{id} \otimes \pi)\Delta(x) = x \otimes 1\} = A \cap U,$$

is a subalgebra of U . Moreover, since A and U are stable under the (left or, equivalently, right) adjoint action of t , we see that $A \cap U$ is stable under the (left or, equivalently right) adjoint action of t . Therefore, $A \cap U$ is a graded subalgebra of U . Also, since Π is a coalgebra map, we can conclude that $A \cap U = \tilde{A} = \Pi(A)$ is a subcoalgebra of U .

Finally, if a is a homogeneous element of $A \cap U$, then $S(a) = t^{p(a)} S_U(a) \in A$. Hence $S_U(a) = t^{p(a)} S(a) \in A$, and we see that $S_U(A \cap U) \subseteq A \cap U$. In view of (2.3ii), the lemma now follows. \square

3. THE EXAMPLE

3.1. In this section we let $\mathfrak{g} = \mathfrak{g}_0 \oplus \mathfrak{g}_1$ denote the Lie superalgebra $\text{pl}(1, 1)$ of 2×2 matrices. Fix a basis for \mathfrak{g} ,

$$x = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad y = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad u = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad v = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}.$$

Recall that \mathfrak{g}_0 is generated by x and y , while \mathfrak{g}_1 is generated by u and v . Set $U := U(\mathfrak{g})$. As in (2.4i), let $\overline{U} := U \# k[\langle t \rangle]$. The relations defining \overline{U} as an algebra are

$$\begin{aligned} xh = hx \quad \text{for all } h \in \overline{U}, \quad yu - uy = u, \quad yv - vy = -v, \quad uv + vu = x, \\ u^2 = v^2 = 0, \quad tx - xt = ty - yt = 0, \quad tu = -ut, \quad tv = -vt, \quad t^2 = 1. \end{aligned}$$

We can now state the main result of this paper.

3.2. Theorem. *The Hopf algebra \overline{U} is a noetherian prime affine PI Hopf algebra not finitely generated, as a right or left module, over any of its normal commutative Hopf subalgebras.*

The remainder of this section is devoted to a proof of the theorem

3.3. (i) Note that u is an $\text{ad}_{\mathfrak{g}}(y)$ -eigenvector with eigenvalue 1 and that v is an $\text{ad}_{\mathfrak{g}}(y)$ -eigenvector with eigenvalue -1 .

(ii) Let w be an $\text{ad}_{\mathfrak{g}}(y)$ -eigenvector of \mathfrak{g}_1 . By (i), the corresponding eigenvalue is ± 1 . Hence, in \overline{U} , for all positive integers n , either $y^n w = w(y+1)^n$ or $y^n w = w(y-1)^n$.

3.4. Let F be an algebra containing a subalgebra E , and suppose that F is finitely generated on the right (or left) as an E -module. Noting that F is a ring subquotient of a matrix ring over E , we see that E is PI if and only if F is PI. Next, we will also require some basic and well-known properties of Gelfand-Kirillov dimension (abbreviated to “GK dimension” or “GKdim”); see [8] or [9, Chapter 8] for background. In particular, $\text{GKdim } E = \text{GKdim } F$ (see, e.g., [8, 5.5]). The GK dimension of the commutative polynomial algebra $k[x_1, \dots, x_n]$ is equal to n , and the enveloping algebra of an n -dimensional Lie algebra has GK dimension equal to n .

3.5. Note that $k[x, y] = k\langle x, y \rangle \subset \overline{U}$ is a commutative polynomial ring in x and y . Moreover, \overline{U} is finitely generated as a left and a right $k[x, y]$ -module. Therefore, by (3.4), \overline{U} is an affine noetherian PI algebra of GK dimension 2.

3.6. Suppose that \mathfrak{a} is a sub-Lie superalgebra of \mathfrak{g} such that $\mathfrak{a}_0 \neq \mathfrak{g}_0$. Then $\dim \mathfrak{a}_0 \leq 1$, and it follows that $\text{GKdim } U(\mathfrak{a}_0) \leq 1$. It follows from the Poincaré-Birkhoff-Witt Theorem for Lie superalgebras (PBW Theorem) that $U(\mathfrak{a})$ is finitely generated as a left $U(\mathfrak{a}_0)$ -module. Hence, by (3.4), we see that $\text{GKdim } U(\mathfrak{a}) \leq 1$.

3.7. Let C be a commutative normal Hopf subalgebra of \overline{U} . We now prove that \overline{U} cannot be finitely generated as a left or right module over C .

Case 1. Assume that $t \notin C$.

Set $A := C\langle t \rangle$, and observe that A is a Hopf subalgebra of \overline{U} . Since C is stable under the left and right adjoint actions of t , we see that C is a graded associative subalgebra of \overline{U} . Since t is homogeneous, the grading on C extends to a grading on A . For $a, b \in W := C_0 \cup C_1 \cup \{t\}$, we further see that $ab = \pm ba$. Also, A is generated as a k -algebra by W .

Now choose $a_1, \dots, a_n \in W$. Set $A' := k\langle a_1, \dots, a_n \rangle$, and set $Z' := k\langle a_1^2, \dots, a_n^2 \rangle$. Then Z' is central in A , and A' is generated as a left (or right) Z' -module by the products $a_1^{i_1} \cdots a_n^{i_n}$, for $i_1, \dots, i_n \in \{0, 1\}$. In particular, A' is finitely generated as

a left (or right) module over the commutative noetherian ring Z' . It follows that A is integral over its center, in the sense of [9, 5.3.2].

Since $t \in A$, it follows from (2.6i) that $A \cap U = U(\mathfrak{a})$ for some sub-Lie superalgebra \mathfrak{a} of \mathfrak{g} .

Subcase 1a. Assume that $\mathfrak{a}_0 = \mathfrak{g}_0$ (and so $y \in A$).

First suppose that $\mathfrak{a}_1 \neq 0$, ensuring that \mathfrak{a}_1 contains an $\text{ad}_{\mathfrak{g}}(y)$ -eigenvector. We can impose a nonnegative \mathbb{Z} -grading on \mathfrak{g} by setting $\deg(y) = 0$, $\deg(u) = \deg(v) = 1$, and $\deg(x) = 2$. This \mathbb{Z} -grading restricts to \mathfrak{a} since \mathfrak{a} must be generated by \mathbb{Z} -homogeneous elements. Moreover, by declaring $\deg(t) = 0$, we obtain a \mathbb{Z} -grading on A . Note that the 0-degree \mathbb{Z} -homogeneous component of A is $k[y, t]$, and also note that the \mathbb{Z} -grading on A restricts to the center of A . It now follows from the PBW Theorem, and (3.3ii), that the 0-degree \mathbb{Z} -homogeneous component of the center is k . However, we learned above that y must be integral over the center of A , and so y must be integral over the 0-degree \mathbb{Z} -homogeneous component of the center of A . Because y cannot be integral over k , we have arrived at a contradiction. Therefore, $\mathfrak{a}_1 = 0$.

It now follows that $A \cap U = U(\mathfrak{g}_0)$, and it follows from (2.6ii) that $A = U(\mathfrak{g}_0) \# K$. In particular, A is a pointed cocommutative Hopf algebra, and the set of grouplike elements of A is $\{1, t\}$. Also, \mathfrak{g}_0 is the Lie algebra of primitive elements of A . Now, since $t \notin C$, we see that C is a cocommutative connected Hopf algebra, and again by a theorem of Kostant (see, e.g., [11, 5.6.5]), we find that $C = U(\mathfrak{c})$, where \mathfrak{c} is the Lie algebra of primitive elements of C . Of course, $\mathfrak{c} \subseteq \mathfrak{g}_0$, and so $\mathfrak{c} = \mathfrak{c}_0$ is an even sub-Lie superalgebra of \mathfrak{g} . Moreover, it follows from (2.5ii) that \mathfrak{c} is an ideal of \mathfrak{g} . Now, any ideal of \mathfrak{g} containing y must also contain u and v , and so we see that $y \notin \mathfrak{c}$. Thus $\mathfrak{c} \neq \mathfrak{g}_0$. Therefore, $\text{GKdim } C \leq 1$, by (3.6), and so \overline{U} cannot be a finitely generated left or right C -module, since $\text{GKdim } \overline{U} = 2$.

Subcase 1b. Assume that $\mathfrak{a}_0 \neq \mathfrak{g}_0$.

It now follows from (3.6) that $\text{GKdim } A \cap U \leq 1$. By (2.6ii), A is finitely generated as a left $A \cap U$ -module, and so by (3.4), $\text{GKdim } A \leq 1$. Therefore, \overline{U} cannot be finitely generated as a left or right A -module. Hence \overline{U} cannot be finitely generated as a left or right C -module.

Case 2. Assume that $t \in C$.

By (2.6i), it follows that $C \cap U = U(\mathfrak{c})$, for some sub-Lie superalgebra \mathfrak{c} of U . Moreover, it follows from (2.5ii) that \mathfrak{c} is an ideal of \mathfrak{g} . Consequently, if $\mathfrak{c}_0 = \mathfrak{g}_0$, then $\mathfrak{c} = \mathfrak{g}$. But $\mathfrak{c} \neq \mathfrak{g}$, since otherwise U is equal to the commutative algebra $C \cap U$. So $\mathfrak{c}_0 \neq \mathfrak{g}_0$, and we deduce from (3.6) that $\text{GKdim } C \cap U \leq 1$. Hence, by (2.6ii) and (3.4), $\text{GKdim } C \leq 1$. We conclude that \overline{U} cannot be a finitely generated left or right C -module.

3.8. It only remains to prove that \overline{U} is prime. Let R be a prime noetherian ring, and let τ be an automorphism of R of order 2. Let T denote the Ore extension $R[Y; \tau]$. It follows, for example, from [6, 2.3i–iii] that $T/\langle Y^2 - 1 \rangle$ is prime. As noted in [2], U is prime. Therefore, since \overline{U} is isomorphic as a ring to $U[Y; t]/\langle Y^2 - 1 \rangle$, we see that \overline{U} is prime.

3.9. Remark. Let \mathfrak{b} be the sub-Lie superalgebra of \mathfrak{g} generated by y and u . Using the methods of this note, one can show that $U(\mathfrak{b}) \# k[\langle t \rangle]$ is a noetherian Hopf PI algebra, of GK dimension 1, not finitely generated as a right or left module over any

of its normal commutative Hopf subalgebras. However, the ideal $\langle u \rangle$ of $U(\mathfrak{b})\#k\langle t \rangle$ is nilpotent, and so $U(\mathfrak{b})\#k\langle t \rangle$ is not semiprime. Now recall, as a consequence of the Small-Warfield Theorem [14], that a prime affine PI algebra of GK dimension 1 must be finite over its center. We therefore ask: Does there exist a prime affine PI Hopf algebra of GK dimension 1 that is not finitely generated as a left or right module over any of its normal commutative Hopf subalgebras?

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