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ENDOMORPHISMS OF STABLE CONTINUOUS-TRACE C^*-ALGEBRAS

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Abstract. We classify $C_0(X)$-endomorphisms of stable continuous-trace C^*-algebras up to inner automorphism by a surjective multiplicative invariant taking values in finite-dimensional vector bundles over the spectrum. Specializing to automorphisms, this gives a different approach to results of Lance, Smith, Phillips and Raeburn.

1. Introduction

The structure of automorphisms of continuous-trace C^*-algebras was studied by Lance ([L]) and Smith ([S]) (the case of $C(X)$-automorphisms of $C(X) \otimes \mathcal{K}$, or more precisely $C(X) \otimes \mathcal{B}(\mathcal{H})$), and subsequently by Phillips and Raeburn ([PhR]; see also [Ro], [RW]) for general continuous trace C^*-algebras. This paper is concerned with the question of generalizing some of those results to the situation of "unital" endomorphisms of stable continuous-trace C^*-algebras. We refer the reader to [RW] and [Di] for general references on the theory of continuous-trace C^*-algebras.

Throughout the paper, X will denote a locally compact Hausdorff space; $\mathcal{H}$ will denote a separable Hilbert space and $\mathcal{B}(\mathcal{H})$ will denote the bounded operators on $\mathcal{H}$; $\mathcal{M}(\mathcal{A})$ will denote the multiplier algebra of $\mathcal{A}$; $\mathcal{K}$ will denote the algebra of compact operators on a separable Hilbert space; automorphisms, endomorphisms and homomorphisms between C^*-algebras and C^*-bundles will always be $*$-homomorphisms; By a unital homomorphism between two nonunital C^*-algebras we mean a homomorphism which maps approximate units to approximate units.

We recall that every stable continuous-trace C^*-algebra for which all the irreducible representations act on a separable space is isomorphic to the algebra of sections of a locally trivial $\mathcal{K}$-bundle over a locally compact Hausdorff space. Let E be a locally trivial $\mathcal{K}$-bundle over X, and let $\mathcal{A} = \Gamma_0(E)$. By a $C_0(X)$-endomorphism we mean a unital endomorphism of $\mathcal{A}$ commutes with the multiplication action of $C_0(X)$ on $\mathcal{A}$.

Denote by $Aut_X(\mathcal{A})$ the $C_0(X)$-automorphisms of $\mathcal{A}$ and by $Inn(\mathcal{A})$ the group of inner automorphisms of $\mathcal{A}$ (the automorphisms of $\mathcal{A}$ which are given by $\alpha(A) = UAU^*$ for some $U \in \mathcal{M}(\mathcal{A})$). The results of Lance, Smith, Phillips and Raeburn, specialized to this case, can be summarized in the following exact sequence:

$$1 \to Inn(\mathcal{A}) \to Aut_X(\mathcal{A}) \to H^2(X; \mathbb{Z}) \to 1.$$

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This paper will be devoted to providing an adequate generalization to this sequence.

Recall ([Do]) that an open cover $\{U_\lambda\}$ of a topological space Y is said to be *numerable* if there is a locally finite partition of unity $\{f_\mu\}$ such that the open cover $\{f_\mu^{-1}((0,1])\}$ refines $\{U_\lambda\}$. A bundle ζ over Y is said to be *numerable* if there is a numerable open cover $\{U_\lambda\}$ of Y such that $\zeta|_{U_\lambda}$ is trivial for all λ. Note that any locally trivial bundle over a paracompact space is numerable.

Our main goal in this paper is to prove the following.

Theorem 1. *Let $\mathcal{A} = \Gamma_0(E)$ as above. To each $C_0(X)$-endomorphism α of $\mathcal{A}$ we associate a numerable Hermitian vector bundle $vect(\alpha)$, such that the following hold:*

1. *$vect(\alpha) \cong vect(\beta)$ if and only if $\alpha = AdU \circ \beta$ for some unitary $U \in \mathcal{M}(\mathcal{A})$.*
2. *$vect(\alpha \circ \beta) \cong vect(\alpha) \otimes vect(\beta)$.*
3. *$vect(\alpha)$ is a trivial bundle if and only if α is inner, in the sense that there exist isometries $S_1, ..., S_n \in \mathcal{M}(\mathcal{A})$ with $S_i^* S_j = 0$ for $i \neq j$, $\sum S_i S_i^* = 1$, such that $\alpha(A) = \sum S_i A S_i^*$.*
4. *For any numerable Hermitian vector bundle v over X there exists a $C_0(X)$-endomorphism α of $\mathcal{A}$ such that $vect(\alpha) \cong v$.*

α will be an automorphism if and only if $vect(\alpha)$ is a line bundle. So, if we restrict our attention to automorphisms, we get a map from the group of $C_0(X)$-automorphisms onto the group of Hermitian line bundles, whose kernel is the inner automorphisms. The group of line bundles is isomorphic to $H^2(X;\mathbb{Z})$ (when X is paracompact; see for example [K], I.3), thus we recover the exact sequence above.

We first examine the topology of the space of endomorphisms of $\mathcal{K}$. We then proceed to prove the main theorem.

Remark 2. For concreteness, we use ordinary (complex) C^*-algebras; however, it is worth noticing that the analogous results of this paper along with their proofs hold for endomorphisms of the real stable continuous trace C^*-algebras (replacing complex bundles by real bundles, unitaries by orthogonals and so on).

2. Endomorphisms of $\mathcal{K}$

Let α be a unital endomorphism of $\mathcal{K}$. Let

$$V_\alpha = \{T \in \mathcal{M}(\mathcal{K}) \cong \mathcal{B}(\mathcal{H}) \mid TA = \alpha(A)T \ \forall A \in \mathcal{K}\};$$

V_α is clearly a vector space. If $T, S \in V_\alpha$, then $T^*SA = AT^*S$ for all $A \in \mathcal{K}$, and therefore T^*S is a scalar multiple of the identity, and indeed $\langle S, T\rangle = T^*S$ is an inner product. Note, in particular, that any element of V_α of norm 1 is an isometry.

Lemma 3. *V_α is finite dimensional and nonzero. Any orthonormal basis $S_1, ..., S_n$ for V_α satisfies $\sum_{i=1}^n S_i S_i^* = 1$ and $\alpha(A) = \sum_{i=1}^n S_i A S_i^*$ for any $A \in \mathcal{K}$.*

Proof. Letting $\mathcal{K}$ act irreducibly on some space $\mathcal{H}$, we observe that α gives a nondegenerate representation of $\mathcal{K}$ on $\mathcal{H}$. This representation decomposes into a finite direct sum of irreducible representations on pairwise orthogonal subspaces $\mathcal{H}_1 \oplus ... \oplus \mathcal{H}_n = \mathcal{H}$ ($n \leq \infty$), and there are isometries S_i from $\mathcal{H}$ onto $\mathcal{H}_i$, $i = 1, ..., n$, which implement $\alpha|_{\mathcal{H}_i}$, i.e., $S_i A S_i^* \xi_i = \alpha(A)\xi_i$ for all $\xi_i \in \mathcal{H}_i$. One easily verifies that $S_i \in V_\alpha$ for all i, and therefore $V_\alpha \neq 0$, and that $\sum_{i=1}^n S_i S_i^* = 1$. The S_i's are orthonormal as elements of V_α, and if $T \in V_\alpha$, then $T = (\sum_{i=1}^n S_i S_i^*)T = \sum_{i=1}^n \langle T, S_i\rangle S_i$, and therefore $S_1, ..., S_n$ forms a basis for V_α. Checking that if

$T_1, ..., T_n$ is another orthonormal basis for V_α then $\alpha(A) = \sum_{i=1}^n T_i A T_i^*$ for all $A \in \mathcal{K}$ is straightforward. □

Denote by $End(\mathcal{K})$ the space of unital endomorphisms of $\mathcal{K}$, endowed with the point-norm topology, and denote

$$End^n(\mathcal{K}) = \{\alpha \in End(\mathcal{K}) \mid dim(V_\alpha) = n\}.$$

Notice that $End(\mathcal{K})$ (and hence all the $End^n(\mathcal{K})$'s) is metrizable, hence paracompact (if $A_1, A_2, ...$ is a dense sequence in the unit sphere of $\mathcal{K}$, then $d(\alpha, \beta) = \sum_k \|\alpha(A_k) - \beta(A_k)\|/2^k$ is a metric inducing the point-norm topology).

Let P be a minimal projection in $\mathcal{K}$. Observe that $dim(V_\alpha) = trace(\alpha(P))$. Suppose $\alpha_k \to \alpha$. For sufficiently large k, we have $\|\alpha_k(P) - \alpha(P)\| < 1$. Therefore, $\alpha_k(P)$ and $\alpha(P)$ are Murray–von Neumann equivalent, and, in particular, they have the same trace. Therefore, the $End^n(\mathcal{K})$ are closed and open in $End(\mathcal{K})$ (cf. [Pr] Proposition 2.3).

Let $\mathcal{C}_n = \{(S_1, ..., S_n) \in \mathcal{B}(\mathcal{H})^n \mid S_i^* S_j = \delta_{ij} 1 \, \forall i, j \ , \ \sum S_i S_i^* = 1\}$ equipped with the strong operator topology. Note that $\mathcal{C}_n$ is metrizable (since the unit sphere of $\mathcal{B}(\mathcal{H})$ is metrizable in the SOT), hence paracompact. We have a surjective map $\pi : \mathcal{C}_n \to End^n(\mathcal{K})$, given by $\pi(S_1, ..., S_n)(A) = \sum_{i=1}^n S_i x S_i^*$. It is easy to see that π is continuous.

We have a Hermitian vector bundle

$$\begin{tikzcd} \mathcal{I}_n \arrow[d] \\ End^n(\mathcal{K}), \end{tikzcd}$$

namely, the sub-bundle of $End^n(\mathcal{K}) \times \mathcal{B}(\mathcal{H})$ whose fiber over α is V_α (where the topology on $\mathcal{B}(\mathcal{H})$ is the strong operator topology).

$$\begin{tikzcd} \mathcal{C}_n \arrow[d, "\pi"'] \\ End^n(\mathcal{K}) \end{tikzcd}$$

is the associated principal $\mathcal{U}_n$-bundle (the bundle of orthonormal frames).

Lemma 4. $\pi : \mathcal{C}_n \to End^n(\mathcal{K})$ *has local cross sections, i.e., the bundles above are locally trivial (and since the base space is paracompact, they are numerable).*

Proof (*cf.* [RW], *Proposition* 1.6). We think of $\mathcal{K}$ as acting irreducibly on $\mathcal{H}$. Fix $\alpha \in End^n(\mathcal{K})$, and let $P = \nu \otimes \bar{\nu} \in \mathcal{K}$ be a fixed minimal projection. Let $\omega_1, ..., \omega_n$ be an orthonormal basis for $\alpha(P)\mathcal{H}$, and define $S_i \in \mathcal{B}(\mathcal{H})$, $i = 1, \ldots, n$, by $S_i \xi = \alpha(\xi \otimes \bar{\nu})\omega_i$.

For any $i, j \in 1, ..., n$ and $\xi, \eta \in \mathcal{H}$, we have

$$\begin{aligned} \langle S_i \xi, S_j \eta \rangle &= \langle \alpha(\xi \otimes \bar{\nu})\omega_i, \alpha(\eta \otimes \bar{\nu})\omega_j \rangle = \langle \alpha(\eta \otimes \bar{\nu})^* \alpha(\xi \otimes \bar{\nu})\omega_i, \omega_j \rangle \\ &= \langle \alpha((\eta \otimes \bar{\nu})^* \xi \otimes \bar{\nu})\omega_i, \omega_j \rangle = \langle \xi, \eta \rangle \langle \alpha(\nu \otimes \bar{\nu})\omega_i, \omega_j \rangle = \langle \xi, \eta \rangle \langle \omega_i, \omega_j \rangle = \langle \xi, \eta \rangle \delta_{ij} \end{aligned}$$

so $S_1, ..., S_n$ are isometries with orthogonal ranges.

Let $A = \xi \otimes \bar{\eta}$, and let $\zeta \in \mathcal{H}$. Then

$$\begin{aligned} S_i A \zeta &= (S_i \xi \otimes \bar{\eta})\zeta = \langle \zeta, \eta \rangle S_i \xi = \langle \zeta, \eta \rangle \alpha(\xi \otimes \bar{\nu})\omega_i \\ &= \alpha((\xi \otimes \bar{\eta})(\zeta \otimes \bar{\nu}))\omega_i = \alpha(A)\alpha(\zeta \otimes \bar{\nu})\omega_i = \alpha(A) S_i \zeta; \end{aligned}$$

so $S_i \in V_\alpha$.

Therefore, $\pi(S_1, ..., S_n) = \alpha$. Now, define $f_1, ..., f_n : End^n(\mathcal{K}) \to \mathcal{H}$ by

$$f_i(\beta) = \beta(P)\omega_i.$$

The f_i are all clearly continuous, and $f_i(\alpha) = \omega_i$. Then there exists an open neighborhood N of α in which $f_1, ..., f_n$ are linearly independent. Let $g_1, ..., g_n : N \to \mathcal{H}$ be obtained from $f_1, ..., f_n$ via the Gram-Schmidt orthonormalization process. Then the g_i's are continuous as well. Now, let $s_1, ..., s_n : N \to \mathcal{B}(\mathcal{H})$ be given by

$$s_i(\beta)\xi = \beta(\xi \otimes \bar{\nu})g_i(\beta) \ , \ \xi \in \mathcal{H}.$$

Then the s_i's are clearly continuous (when $\mathcal{B}(\mathcal{H})$ is given the strong operator topology), and from the above, we have $(s_1(\beta), ..., s_n(\beta)) \in \mathcal{C}_n$ for all β, and $\pi(s_1(\beta), ..., s_n(\beta)) = \beta$; so we have a local cross section, as required. □

Before stating our next theorem, we recall some results concerning numerable bundles (see [Do]):

- Any pull back of a numerable bundle is numerable.
- A numerable principal $\mathcal{U}_n$-bundle (resp. Hermitian vector bundle) ζ over a space B is said to be *universal* if for any numerable principal $\mathcal{U}_n$-bundle (resp. Hermitian vector bundle) ξ over a space Y there exists a continuous map $f : Y \to B$, unique up to homotopy, such that $f^*\zeta = \xi$.
- A Hermitian vector bundle is universal if and only if its associated principal $\mathcal{U}_n$-bundle is universal.
- *Dold's Theorem* ([Do], Theorem 7.5): A numerable principal $\mathcal{U}_n$-bundle is universal if and only if the total space is contractible.

Theorem 5. *The two bundles above are universal.*

Proof. By Dold's theorem, it suffices to show that $\mathcal{C}_n$ is contractible. Fix $(S_1, ..., S_n) \in \mathcal{C}_n$. Recall that $\mathcal{U}(\mathcal{H})$ (the unitary group of $\mathcal{H}$) is contractible in the strong operator topology (see [RW], Lemma 4.72). Define a map $\mathcal{U}(\mathcal{H}) \to \mathcal{C}_n$ by $U \mapsto (US_1, ..., US_n)$. This map can readily be seen to be a homeomorphism. □

3. PROOF OF THE MAIN THEOREM

Let E be a locally trivial bundle over X with fiber $\mathcal{K}$. Note that $\mathcal{M}(\Gamma_0(E))$ can be identified with the sections of a bundle with fiber $\mathcal{B}(\mathcal{H})$ and the same Dixmier-Douady class (see [PhR], Proposition 2.15 and the following remark). We refer to this bundle henceforth as the *multiplier bundle of* E. Denote by $\mathcal{B}_x$ the fiber of the multiplier bundle over a point $x \in X$. For a $C_0(X)$-endomorphism α, we denote by α_x the action of α on the fiber over x (coming from the induced action of α on the quotient algebra corresponding to the primitive ideal represented by x).

We define

$$vect(\alpha) = \{(x, T) \mid T \in \mathcal{B}_x \ , \ TA = \alpha_x(A)T \ \ \forall A \in E_x\}.$$

If E_1,E_2 are two locally trivial $\mathcal{K}$-bundles over X, then $E_1 \otimes E_2$ is a locally trivial $\mathcal{K}$-bundle over X, and if α_1,α_2 are $C_0(X)$-endomorphisms of $\Gamma(E_1)$, $\Gamma(E_2)$ respectively, then we have a $C_0(X)$-endomorphism $\alpha_1 \otimes \alpha_2$ of $\Gamma(E_1 \otimes E_2)$.

Lemma 6. *Let E_1,E_2 be two locally trivial $\mathcal{K}$-bundles over X, and let α_1,α_2 be $C_0(X)$-endomorphisms of $\Gamma(E_1)$, $\Gamma(E_2)$, respectively.*

(1) $vect(\alpha \otimes \beta) \cong vect(\alpha) \otimes vect(\beta)$.

(2) *If* $\varphi : \Gamma(E_1) \to \Gamma(E_2)$ *is an isomorphism commuting with the multiplication action of* $C_0(X)$*, then* $vect(\varphi\alpha\varphi^{-1}) \cong vect(\alpha)$*.*

(Cf. [RW]*, Lemma 6.6.)*

Proof. Let M_1, M_2 denote the multiplier bundles of E_1, E_2 respectively. We denote by $M_1 \otimes M_2$ the fiberwise tensor product bundle, where the tensor product is taken to be the spatial tensor product, and we observe that $M_1 \otimes M_2$ is canonically the multiplier bundle of $E_1 \otimes E_2$. $vect(\alpha \otimes \beta)$ is a sub-bundle of $M_1 \otimes M_2$, as is $vect(\alpha) \otimes vect(\beta)$, and it is immediate to verify that they are, in fact, the same. That proves (1). (2) is immediate. □

Proof of the main theorem. If E is trivial, then the multiplier bundle is trivial, in which case $x \mapsto \alpha_x$ can be thought of as a continuous map $X \to End^n(\mathcal{K})$. $vect(\alpha)$ is the pull-back of the bundle $\mathcal{I}_n \to End^n(\mathcal{K})$ via that map, and therefore is numerable. Any continuous map $X \to End^n(\mathcal{K})$ gives rise to a $C_0(X)$-endomorphism of $\mathcal{A}$ (in the trivial case); so by Theorem 5, any numerable bundle arises as $vect(\alpha)$ for some $C_0(X)$-endomorphism α. This proves part (4) when E is trivial. For the general case, we use a trick similar to the one used in [RW], p. 160.

Suppose v is a given numerable vector bundle. Let E be our given $\mathcal{K}$-bundle, and let E_0 be the trivial bundle. We proved that there exists a $C_0(X)$-endomorphism α_0 of $\Gamma(E_0) \cong C_0(X) \otimes \mathcal{K}$ with $vect(\alpha_0) \cong v$. Note that $vect(id_{\Gamma_0(E)})$ is the trivial line bundle ϵ_0 over X. Define a $C_0(X)$-endomorphism α of $\Gamma_0(E)$ by $\alpha = \alpha_0 \otimes id_{\Gamma_0(E)}$. Then

$$vect(\alpha) \cong vect(\alpha_0) \otimes vect(id_{\Gamma_0(E)}) \cong v \otimes \epsilon_0 \cong v,$$

establishing part (4) in general.

To show that $vect(\alpha)$ is numerable, let E^{op} be the opposite bundle, and let $\alpha_0 = \alpha \otimes id_{E^{op}}$. Then α_0 is an endomorphism of $E_0 \cong E \otimes E^{op}$, and as above, $vect(\alpha) \cong vect(\alpha_0)$. We know that $vect(\alpha_0)$ is numerable, and therefore $vect(\alpha)$ is numerable as well.

$vect(\alpha)$ determines α: if $N \subseteq X$ is such that $vect(\alpha)|_N$,$E|_N$ are trivial, then choose orthonormal sections $S_1, ..., S_n$ of the restricted bundle, and then $\alpha|_N(A) = \sum S_i A S_i^*$ for $A \in \Gamma(E|_N)$; those restrictions determine α.

If there is a unitary $U \in \mathcal{M}(\Gamma_0(E))$ such that $\alpha(A) = U^*\beta(A)U$ for all $A \in \Gamma_0(E)$, then $TA_x = \alpha_x(A_x)T$ if and only if $(U_xT)A_x = \beta_x(A_x)(U_xT)$; so left multiplication by U gives a vector bundle isomorphism $vect(\alpha) \cong vect(\beta)$.

Conversely, suppose $vect(\alpha) \cong vect(\beta)$. Note that $Hom(vect(\alpha), vect(\beta))$ is isomorphic to the sub-bundle of the multiplier bundle whose fiber over x is

$$span\{TS^* \mid T \in vect(\beta)\ ,\ S \in vect(\alpha)\}.$$

$vect(\alpha) \cong vect(\beta)$ means that there is a unitary element in $Hom(vect(\alpha), vect(\beta))$, i.e., that there is a unitary section U of the above sub-bundle (and, in particular, $U \in \mathcal{M}(\Gamma_0(E))$). Any such U satisfies $U\alpha(A)U^* = \beta(A)$ for all $A \in \Gamma_0(E)$. This proves part (1) of the theorem.

Part (3) is immediate. For part (2), Define a map

$$vect(\alpha) \otimes vect(\beta) \to vect(\alpha \circ \beta)$$

by

$$(x, T \otimes S) \mapsto (x, TS).$$

Verifying that this map is a well-defined bundle isomorphism is immediate. □

Corollary 7. *If α, β are $C_0(X)$-endomorphisms of $\Gamma_0(E)$, then $\alpha \circ \beta$ and $\beta \circ \alpha$ differ by an inner automorphism (cf.* [RW]*, Corollary 5.43).*

Remark 8. Composition of $C_0(X)$-endomorphisms maps under *vect* to tensor products. There is also a direct sum operation (defined up to homotopy, or equivalently inner automorphism), which is mapped under *vect* to the direct (Whitney) sum: if α, β are $C_0(X)$-endomorphisms of $\mathcal{A}$, then we have a $C_0(X)$-map $\mathcal{A} \to M_2(\mathcal{A})$ given by $A \mapsto \begin{pmatrix} \alpha(A) & 0 \\ 0 & \beta(A) \end{pmatrix}$. Composing this map with a $C_0(X)$-isomorphism $M_2(\mathcal{A}) \to \mathcal{A}$ gives us a $C_0(X)$-endomorphism of $\mathcal{A}$, and it is easy to check that its corresponding bundle will be isomorphic to $vect(\alpha) \oplus vect(\beta)$. The choice of isomorphism is unique up to homotopy.

We can do so explicitly: choose two isometries $s_1, s_2 \in \mathcal{M}(\mathcal{A})$ such that $s_1 s_1^* + s_2 s_2^* = 1$, and define $(\alpha \oplus_{(s_1,s_2)} \beta)(A) = s_1 \alpha(A) s_1^* + s_2 \beta(A) s_2^*$. The map $\Psi : vect(\alpha) \oplus vect(\beta) \to vect(\alpha \oplus_{(s_1,s_2)} \beta)$ given by $\Psi(x, T_1 \oplus T_2) = (x, s_1(x)T_1 + s_2(x)T_2)$ (where here s_1, s_2 are thought of as sections) gives a bundle isomorphism. The choice of the pair s_1, s_2 is unique up to homotopy (since $\mathcal{C}_2$ is contractible).

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