

FALTINGS' THEOREM FOR THE ANNIHILATION
OF LOCAL COHOMOLOGY MODULES
OVER A GORENSTEIN RING

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ABSTRACT. In this paper we study the Annihilator Theorem and the Local-global Principle for the annihilation of local cohomology modules over a (not necessarily finite-dimensional) Noetherian Gorenstein ring.

1. INTRODUCTION

Let A be a commutative Noetherian ring, let \mathfrak{a} and \mathfrak{b} be ideals of A and let M be a finitely generated A -module. We first briefly recall the invariant $\lambda_{\mathfrak{a}}^{\mathfrak{b}}(M)$ defined in [3].

By the terminology of [3], the \mathfrak{b} -minimum \mathfrak{a} -adjusted depth $\lambda_{\mathfrak{a}}^{\mathfrak{b}}(M)$ of M is defined by

$$\lambda_{\mathfrak{a}}^{\mathfrak{b}}(M) = \inf\{\text{depth}M_{\mathfrak{p}} + \text{ht}(\mathfrak{a} + \mathfrak{p})/\mathfrak{p} : \mathfrak{p} \in \text{Spec}(A) \setminus V(\mathfrak{b})\}$$

where $V(\mathfrak{b})$ denotes the set of prime ideals containing \mathfrak{b} .

We shall say that the Annihilator Theorem (for local cohomology modules) holds over A if $\lambda_{\mathfrak{a}}^{\mathfrak{b}}(M) = f_{\mathfrak{a}}^{\mathfrak{b}}(M)$ for every choice of the finitely generated A -module M and for every choice of the ideals $\mathfrak{b}, \mathfrak{a}$ of A where $f_{\mathfrak{a}}^{\mathfrak{b}}(M)$ is the \mathfrak{b} -finiteness dimension of M relative to \mathfrak{a} . Also, we say that the Local-global Principle (for the annihilation of local cohomology modules) holds over A if

$$f_{\mathfrak{a}}^{\mathfrak{b}}(M) = \inf\{f_{\mathfrak{a}A_{\mathfrak{p}}}^{\mathfrak{b}A_{\mathfrak{p}}}(M_{\mathfrak{p}}) : \mathfrak{p} \in \text{Spec}(A)\}$$

for every choice of ideals $\mathfrak{a}, \mathfrak{b}$ of A and every choice of the finitely generated A -module M .

Faltings' Annihilator Theorem [5] states that if A is a homomorphic image of a regular ring or A has a dualizing complex, then the annihilator theorem (for local cohomology modules) holds over A . In [7], Raghavan deduced from Faltings' Annihilator Theorem [5] that if A is a homomorphic image of a regular ring, then the Local-global Principle (for the annihilation of local cohomology modules) holds over A . Recently, in [2], Brodmann, Rotthaus and Sharp showed, for every non-negative integer r , that if A is universally catenary and all formal fibres of all localizations of A satisfy Serre's Condition (S_r) , then the Annihilator Theorem (for

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local cohomology modules) holds at level r over A if and only if the Local-global Principle (for the annihilation of local cohomology modules) holds at level r over A . They also established the Local-global Principle (for the annihilation of local cohomology modules) over an arbitrary commutative Noetherian ring of dimension not exceeding 4.

In this paper, we show that the Local-global Principle and the Annihilator Theorem (for local cohomology modules) hold over a (homomorphic image of) a commutative Noetherian Gorenstein ring. The proof of the main theorem relies heavily on ideas in Brodmann's proof of [1, Satz 3.12] and in Raghavan's proof of [8, 3.1]. But our method is based on the theory of Gorenstein dimension or G-dimension. To the best of our knowledge, this is the first application of this theory in the study of Faltings' Annihilator Theorem.

Throughout the paper, A will denote a commutative Noetherian ring (with non-zero identity), \mathfrak{a} , and \mathfrak{b} will denote ideals of A , and M will denote a finitely generated A -module. We use \mathbb{N} and \mathbb{N}_0 to denote the set of positive and nonnegative integers respectively. For any unexplained notation and terminology concerning G-dimension we refer the reader to [4].

2. RESULTS

Let us, firstly, introduce \mathfrak{b} -finiteness dimension of M relative to \mathfrak{a} (see [3, 9.1.5]):

$$f_{\mathfrak{a}}^{\mathfrak{b}}(M) = \inf\{i \in \mathbb{N}_0 : \mathfrak{b}^n H_{\mathfrak{a}}^i(M) \neq 0 \text{ for all } n \in \mathbb{N}\},$$

where, by convention, the infimum of the empty set of integers is interpreted by ∞ . Note that if $\mathfrak{a} = \mathfrak{b}$, then $f_{\mathfrak{b}}^{\mathfrak{b}}(M) = f_{\mathfrak{a}}(M)$ is the finiteness dimension of M relative to \mathfrak{a} .

We begin by the following remark.

Remark 2.1. It follows from [2, 3.7] that $f_{\mathfrak{a}}^{\mathfrak{b}}(M) = f_{\mathfrak{a}+\mathfrak{b}}^{\mathfrak{b}}(M)$. So one can reduce the study of the Local-global Principle (for the annihilation of local cohomology modules) for arbitrary ideals \mathfrak{b} and \mathfrak{a} to the study of this principle in the case that $\mathfrak{b} \subseteq \mathfrak{a}$.

We now prove some preliminary lemmas and a proposition which help us to conclude the main theorem.

Lemma 2.2. *Let \mathfrak{a} and \mathfrak{b} be ideals of A such that $\mathfrak{b} \subseteq \mathfrak{a}$ and $0 :_A M/\Gamma_{\mathfrak{b}}(M) \subseteq \mathfrak{a}$. Let s be a positive integer. Suppose that $f_{\mathfrak{a}A_{\mathfrak{p}}}^{\mathfrak{b}A_{\mathfrak{p}}}(M_{\mathfrak{p}}) > s$ for every prime ideal \mathfrak{p} of A . Then $s < \text{ht}_M \mathfrak{a}$.*

Proof. We suppose that $s \geq \text{ht}_M \mathfrak{a}$, and look for a contradiction. Firstly, we can use the exact sequence

$$0 \longrightarrow \Gamma_{\mathfrak{b}}(M) \longrightarrow M \longrightarrow M/\Gamma_{\mathfrak{b}}(M) \longrightarrow 0$$

to deduce the long exact sequence

$$\cdots \longrightarrow H_{\mathfrak{a}}^i(\Gamma_{\mathfrak{b}}(M)) \longrightarrow H_{\mathfrak{a}}^i(M) \longrightarrow H_{\mathfrak{a}}^i(M/\Gamma_{\mathfrak{b}}(M)) \longrightarrow \cdots.$$

Now, it follows from the assumption that $f_{\mathfrak{a}A_{\mathfrak{p}}}^{\mathfrak{b}A_{\mathfrak{p}}}(M/\Gamma_{\mathfrak{b}}(M))_{\mathfrak{p}} > s$ for all $\mathfrak{p} \in \text{Spec}(A)$. Since $\text{ht}_{M/\Gamma_{\mathfrak{b}}(M)} \mathfrak{a} \leq \text{ht}_M \mathfrak{a}$, we can thus replace M by $M/\Gamma_{\mathfrak{b}}(M)$ and assume, in our search for a contradiction, that there exists $x \in \mathfrak{b}$ that is a non-zero-divisor on M . Since $0 :_A M \subseteq \mathfrak{a}$, there exists a minimal prime ideal \mathfrak{p} over \mathfrak{a} such that

$\text{ht}_M \mathfrak{p} \leq s$. Set $t := \text{ht}_M \mathfrak{p}$. Then, by [3, 6.1.4], $H_{\mathfrak{a}A_{\mathfrak{p}}}^t(M_{\mathfrak{p}}) \neq 0$ and by hypothesis, $x^{n(\mathfrak{p})} H_{\mathfrak{a}A_{\mathfrak{p}}}^t(M_{\mathfrak{p}}) = 0$ for some $n(\mathfrak{p}) \in \mathbb{N}$. Now, using the exact sequence

$$0 \longrightarrow M \xrightarrow{x^{n(\mathfrak{p})}} M \longrightarrow \frac{M}{x^{n(\mathfrak{p})}M} \longrightarrow 0,$$

we obtain the exact sequence

$$H_{\mathfrak{a}A_{\mathfrak{p}}}^t(M_{\mathfrak{p}}) \xrightarrow{x^{n(\mathfrak{p})}} H_{\mathfrak{a}A_{\mathfrak{p}}}^t(M_{\mathfrak{p}}) \longrightarrow 0,$$

which, in turn, yields $H_{\mathfrak{a}A_{\mathfrak{p}}}^t(M_{\mathfrak{p}}) = 0$; this is the required contradiction. □

Lemma 2.3. *Let \mathfrak{a} and \mathfrak{b} be ideals of A , and let M be a finitely generated A -module. Let \mathfrak{c} be an ideal of A such that $\mathfrak{c} \subseteq 0 :_A M/\Gamma_{\mathfrak{b}}(M)$. Then $f_{\mathfrak{a}+\mathfrak{c}}^{\mathfrak{b}}(M) = f_{\mathfrak{a}}^{\mathfrak{b}}(M)$.*

Proof. Let n be a positive integer such that $\Gamma_{\mathfrak{b}}(M) = 0 :_M \mathfrak{b}^n$. Hence, it follows from our assumption that $\mathfrak{b}^n r H_{\mathfrak{a}}^i(M) = 0$, for all $r \in \mathfrak{c}$ and $i \geq 0$, and so $\mathfrak{b}^n (H_{\mathfrak{a}}^i(M))_r = 0$. Now, the claim follows from the exact sequence

$$\dots \longrightarrow H_{\mathfrak{a}+A_r}^i(M) \longrightarrow H_{\mathfrak{a}}^i(M) \longrightarrow (H_{\mathfrak{a}}^i(M))_r \longrightarrow H_{\mathfrak{a}+A_r}^{i+1}(M) \longrightarrow \dots \quad \square$$

For an A -module M the biduality map is the canonical map

$$\delta_M : M \longrightarrow \text{Hom}_A(\text{Hom}_A(M, A), A),$$

defined by $\delta_M(x)(\psi) = \psi(x)$ for $\psi \in \text{Hom}_A(M, A)$ and $x \in M$. It is a homomorphism of A -modules.

Definition 2.4. [4, 1.1.2] A finitely generated A -module M belongs to the G-class $G(A)$ if and only if

- (1) $\text{Ext}_A^t(M, A) = 0$ for $t > 0$;
- (2) $\text{Ext}_A^t(\text{Hom}_A(M, A), A) = 0$ for $t > 0$; and
- (3) the biduality map $\delta_M : M \longrightarrow \text{Hom}_A(\text{Hom}_A(M, A), A)$ is an isomorphism.

Definition 2.5. [4, 1.2.1] A G-resolution of a finitely generated A -module M is a sequence of modules in $G(A)$,

$$\dots \longrightarrow G_l \longrightarrow G_{l-1} \longrightarrow \dots \longrightarrow G_1 \longrightarrow G_0 \longrightarrow 0,$$

which is exact at G_l for $l > 0$ and has $G_0/\text{Im}(G_1 \longrightarrow G_0) \cong M$. That is, there is an exact sequence

$$\dots \longrightarrow G_l \longrightarrow G_{l-1} \longrightarrow \dots \longrightarrow G_1 \longrightarrow G_0 \longrightarrow M \longrightarrow 0.$$

The resolution is said to be of length n if $G_n \neq 0$ and $G_l = 0$ for $l > n$.

Note that every finitely generated A -module has a resolution by finitely generated free modules and, thereby, a G-resolution.

Definition 2.6. [4, 1.2.3] A finitely generated A -module M is said to have finite G-dimension if it has a G-resolution of finite length. We set $G - \dim_A 0 = -\infty$, and for $M \neq 0$ we define the G-dimension of M as follows: For any positive integer n we say that M has G-dimension at most n , and write $G - \dim_A M \leq n$ for short, if and only if M has a G-resolution of length n . If M has no G-resolution of finite length, then we say that it has infinite G-dimension and write $G - \dim_A M = \infty$.

Remark 2.7. By [4, 1.4.9], a commutative Noetherian local ring A is Gorenstein if and only if every finite A -module has a finite G-resolution.

Proposition 2.8. *Let M be a finitely generated A -module and let $G - \dim_A M = 0$. Then $\text{grade}(\mathfrak{a}, M) \geq \text{grade}(\mathfrak{a}, A)$, for every ideal \mathfrak{a} of A .*

Proof. If $\mathfrak{a}M = M$, then $\text{grade}(\mathfrak{a}, M) = \infty$ and the result is clear. We therefore suppose that $\mathfrak{a}M \neq M$. Set $g := \text{grade}(\mathfrak{a}, A)$. We argue by induction on g . When $g = 0$ the result is clear. We therefore suppose inductively that $g > 0$ and that the result has been proved for smaller values of g . Since $G - \dim_A M = 0$, by [4, 1.1.8], we have that $\text{Ass}M \subseteq \text{Ass}A$. Hence there exists $x \in \mathfrak{a}$ such that x is an M - and A -sequence. Also, by [4, 1.3.6], $G - \dim_{A/xA} M/xM = 0$. Note that by our assumption $xM \neq M$. Therefore, by the inductive hypotheses, $\text{grade}(\mathfrak{a}, M/xM) \geq \text{grade}(\mathfrak{a}, A/xA)$ and so $\text{grade}(\mathfrak{a}, M) \geq \text{grade}(\mathfrak{a}, A)$. \square

Lemma 2.9. *Let M be a finitely generated A -module, and let $\mathfrak{p} \in \text{Spec}(A)$ be such that $M_{\mathfrak{p}} \in G(A_{\mathfrak{p}})$. Then there exists $s \in A \setminus \mathfrak{p}$ such that $sH_{\mathfrak{a}}^i(M) = 0$ for all $i < \text{grade}(\mathfrak{a}, A)$.*

Proof. We use an inductive argument on i . Let M be a finitely generated A -module, and let $\mathfrak{p} \in \text{Spec}(A)$ be such that $M_{\mathfrak{p}} \in G(A_{\mathfrak{p}})$. If $i = 0$, since $H_{\mathfrak{a}}^0(M)$ is a finitely generated A -module and, in view of 2.8, $\mathfrak{p} \notin \text{Supp}H_{\mathfrak{a}}^0(M)$, there exists $s \in A \setminus \mathfrak{p}$ such that $sH_{\mathfrak{a}}^0(M) = 0$. Now suppose that $1 \leq i < \text{grade}(\mathfrak{a}, A)$ and that the result holds for smaller values of i . Consider the exact sequence

$$0 \longrightarrow K \longrightarrow M \xrightarrow{\delta_M} M^{**} \longrightarrow L \longrightarrow 0,$$

where M^{**} is the bidual of M . Since the A -modules K and L are finitely generated and $\mathfrak{p} \notin \text{Supp}(K) \cup \text{Supp}(L)$, there exists $t \in A \setminus \mathfrak{p}$ such that $tH_{\mathfrak{a}}^i(K) = 0 = tH_{\mathfrak{a}}^i(L)$ for all $i \geq 0$. Hence, by breaking the above exact sequence into short exact sequences, we need only to prove the result for M^{**} . Note that, by [4, 1.1.7], $M_{\mathfrak{p}}^{**} \in G(A_{\mathfrak{p}})$. Now, let

$$0 \longrightarrow W \longrightarrow F \longrightarrow M^* \longrightarrow 0$$

be a finite presentation of M^* . By applying the duality functor $-^* = \text{Hom}_A(-, A)$, we obtain the exact sequence

$$0 \longrightarrow M^{**} \longrightarrow F \longrightarrow W^* \longrightarrow \text{Ext}_A^1(M^*, A) \longrightarrow 0,$$

which breaks up into two short exact sequences

$$0 \longrightarrow M^{**} \longrightarrow F \longrightarrow D \longrightarrow 0 \quad \text{and} \quad 0 \longrightarrow D \longrightarrow W^* \longrightarrow \text{Ext}_A^1(M^*, A) \longrightarrow 0.$$

Since $\text{grade}(\mathfrak{a}, A) > i$, we get from the induced long exact sequence of $H_{\mathfrak{a}}^i$,

- (1) $H_{\mathfrak{a}}^i(M^{**}) \cong H_{\mathfrak{a}}^{i-1}(D)$, and
- (2) $H_{\mathfrak{a}}^{i-2}(\text{Ext}_A^1(M^*, A)) \longrightarrow H_{\mathfrak{a}}^{i-1}(D) \longrightarrow H_{\mathfrak{a}}^{i-1}(W^*)$ is exact.

On the other hand, since $\text{Ext}_A^1(M^*, A)$ is a finitely generated A -module and $\mathfrak{p} \notin \text{Supp}(\text{Ext}_A^1(M^*, A))$, there exists $s \in A \setminus \mathfrak{p}$ such that $sH_{\mathfrak{a}}^i(\text{Ext}_A^1(M^*, A)) = 0$ for all $i \in \mathbb{N}_0$. Also, by [4, 1.1.10(a)], we have that $W_{\mathfrak{p}}^* \in G(A_{\mathfrak{p}})$. Now the conclusion follows by applying the inductive hypotheses on W^* in conjunction with (1) and (2). \square

Theorem 2.10. *The Local-global Principle (for the annihilation of local cohomology modules) holds over a commutative Noetherian Gorenstein ring.*

Remark 2.11. Whenever A is the homomorphic image of a Gorenstein ring and M a finitely generated A -module then, by [2, 3.4(i)] and Faltings' Annihilator Theorem

(in the local case), for every choice of the ideals $\mathfrak{b}, \mathfrak{a}$ of A , $\lambda_{\mathfrak{a}}^{\mathfrak{b}}(M) = \inf\{f_{\mathfrak{a}_{\mathfrak{p}}}^{\mathfrak{b}}(M_{\mathfrak{p}}) \mid \mathfrak{p} \in \text{Supp}(M)\}$.

Proof of 2.10. We suppose, for ideals \mathfrak{a} and \mathfrak{b} of a Gorenstein ring A and a finitely generated A -module M , that $f_{\mathfrak{a}}^{\mathfrak{b}}(M) < f_{\mathfrak{a}_{\mathfrak{p}}}^{\mathfrak{b}}(M_{\mathfrak{p}})$ for all $\mathfrak{p} \in \text{Spec}(A)$ and look for a contradiction. We can (and do) assume that \mathfrak{a} is a maximal member of the set $\{\mathfrak{c} : \mathfrak{c} \text{ is an ideal of } A \text{ such that, for some finitely generated } A\text{-module } M', f_{\mathfrak{c}}^{\mathfrak{b}}(M') < f_{\mathfrak{c}_{\mathfrak{p}}}^{\mathfrak{b}}(M'_{\mathfrak{p}}) \text{ for all } \mathfrak{p} \in \text{Spec}(A)\}$. By [2, 3.2], it follows that \mathfrak{a} is a prime ideal of A .

By 2.1 and 2.3, we can assume, in our search for a contradiction, that $\mathfrak{b} \subseteq \mathfrak{a}$, and $0 : M/\Gamma_{\mathfrak{b}}(M) \subseteq \mathfrak{a}$. Set $s := \inf\{f_{\mathfrak{a}_{\mathfrak{p}}}^{\mathfrak{b}}(M_{\mathfrak{p}}) : \mathfrak{p} \in \text{Supp}(M)\}$. By 2.2, $s \leq \text{ht } \mathfrak{a}$. There are two cases to consider.

Case 1. Suppose that $\text{ht } \mathfrak{a} = s$. Let \mathfrak{p} be a prime ideal of A such that $G - \dim_{A_{\mathfrak{p}}} M_{\mathfrak{p}} = 0$. Then, by 2.9, there exists $s_{\mathfrak{p}} \in A \setminus \mathfrak{p}$ such that $s_{\mathfrak{p}} H_{\mathfrak{a}}^i(M) = 0$ for all $i < s$. Set

$$S := \{\mathfrak{q} \in \text{Spec}(A) : M_{\mathfrak{q}} \in G(A_{\mathfrak{q}})\},$$

and let g be the ideal generated by all $s_{\mathfrak{p}}$'s, where \mathfrak{p} ranges in S . In case $S = \emptyset$, we set $g := 0$. Observe that if $g \subseteq \mathfrak{p}$, then $\mathfrak{p} \notin S$. Consequently, $G - \dim_{A_{\mathfrak{p}}} M_{\mathfrak{p}} > 0$. Also note that $g H_{\mathfrak{a}}^i(M) = 0$ for all $i < s$. If $\mathfrak{b} \subseteq \text{Rad}(g)$, then we have nothing to do any more. So we may assume that $\mathfrak{b} \not\subseteq \text{Rad}(g)$. Let $\mathfrak{p}_1, \dots, \mathfrak{p}_t$ be the minimal

primes over g not containing \mathfrak{b} . It is easy to see that $\bigcap_{i=1}^t \mathfrak{p}_i \not\subseteq \mathfrak{a}$. (If $\mathfrak{p}_i \subset \mathfrak{a}$, by the same argument as in [3, 9.4.8], one can deduce that $G - \dim M_{\mathfrak{p}_i} > 0$ and so, by the Auslander-Bridger formula (cf. [4, 1.4.8]), $\text{depth} R_{\mathfrak{p}_i} - \text{depth} M_{\mathfrak{p}_i} > 0$. This implies that $\lambda_{\mathfrak{a}}^{\mathfrak{b}}(M) \leq \text{depth} M_{\mathfrak{p}_i} + \text{ht} \frac{\mathfrak{a} + \mathfrak{p}_i}{\mathfrak{p}_i} < \text{depth} R_{\mathfrak{p}_i} + \text{ht} \frac{\mathfrak{a}}{\mathfrak{p}_i} \leq \text{ht } \mathfrak{a}$, which is a

contradiction.) Let $x \in \bigcap_{i=1}^t \mathfrak{p}_i \setminus \mathfrak{a}$. Note that $\mathfrak{b}_x \subseteq (\text{Rad}(g))_x$. Now it follows from the exact sequence

$$\dots \longrightarrow H_{\mathfrak{a} + Ax}^i(M) \longrightarrow H_{\mathfrak{a}}^i(M) \longrightarrow H_{\mathfrak{a}_x}^i(M_x) \longrightarrow \dots$$

that $f_{(\mathfrak{a} + Ax)_{A_{\mathfrak{p}}}}^{\mathfrak{b}}(M_{\mathfrak{p}}) \geq s$. Thus by the "maximality" assumption about \mathfrak{a} , we have that $f_{\mathfrak{a} + Ax}^{\mathfrak{b}}(M) \geq s$. Now, we deduce from the above exact sequence that $f_{\mathfrak{a}}^{\mathfrak{b}}(M) \geq s$.

Case 2. Suppose that $\text{ht } \mathfrak{a} > s$. Consider the finite presentation

$$0 \longrightarrow K \longrightarrow F \longrightarrow M \longrightarrow 0$$

of M to deduce the long exact sequence

$$(*) \quad \dots \longrightarrow H_{\mathfrak{a}}^i(M) \longrightarrow H_{\mathfrak{a}}^{i+1}(K) \longrightarrow H_{\mathfrak{a}}^{i+1}(F) \longrightarrow \dots$$

Therefore, $f_{\mathfrak{a}_{\mathfrak{p}}}^{\mathfrak{b}}(K_{\mathfrak{p}}) > s$ for all $\mathfrak{p} \in \text{Spec}(A)$. Hence, by case 1, $f_{\mathfrak{a}}^{\mathfrak{b}}(K) \geq s + 1$. Now, by using the exact sequence (*), we can deduce that $f_{\mathfrak{a}}^{\mathfrak{b}}(M) \geq s$. \square

Corollary 2.12. *The Annihilator Theorem (for local cohomology modules) holds over a commutative Noetherian Gorenstein ring.*

Proof. Let A be a Gorenstein ring, let \mathfrak{a} and \mathfrak{b} be ideals of A , and let M be a finitely generated A -module. We must show that $f_{\mathfrak{a}}^{\mathfrak{b}}(M) = \lambda_{\mathfrak{a}}^{\mathfrak{b}}(M)$. By [2, 3.7], we may assume that $\mathfrak{b} \subseteq \mathfrak{a}$. Hence, by [3, 9.3.5], the only nontrivial point is the

proof that $\lambda_{\mathfrak{a}}^{\mathfrak{b}}(M) \leq f_{\mathfrak{a}}^{\mathfrak{b}}(M)$. By 2.10, there exists a prime ideal \mathfrak{p} of A such that $f_{\mathfrak{a}}^{\mathfrak{b}}(M) = f_{\mathfrak{a}A_{\mathfrak{p}}}^{\mathfrak{b}A_{\mathfrak{p}}}(M_{\mathfrak{p}})$. Also, by [5], $f_{\mathfrak{a}A_{\mathfrak{p}}}^{\mathfrak{b}A_{\mathfrak{p}}}(M_{\mathfrak{p}}) = \lambda_{\mathfrak{a}A_{\mathfrak{p}}}^{\mathfrak{b}A_{\mathfrak{p}}}(M_{\mathfrak{p}})$. We now use [3, 9.2.5] to complete the proof. \square

Corollary 2.13. *The Local-global Principle and the Annihilator Theorem (for local cohomology modules) hold over a homomorphic image of a commutative Noetherian Gorenstein ring.*

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