CORRIGENDUM

"EXISTENCE THEORY FOR FIRST ORDER DISCONTINUOUS
FUNCTIONAL DIFFERENTIAL EQUATIONS"

JOSÉ ÁNGEL CID, EDUARDO LIZ, AND RODRIGO L. POUSO

(Communicated by Carmen C. Chicone)

The main result in [3], Theorem 3.3, is incorrect. We will present a counterexample and a repair.

To construct a counterexample, we use a result of Davis [1]: a necessary and sufficient condition for a lattice to be complete is that every nondecreasing function from the lattice into itself has a fixed point.

For $r > 0$, let $L^1([-r,0])$ be the set of all bounded Lebesgue-measurable real functions on $[-r,0]$, and let $\alpha, \beta \in L^\infty([-r,0])$ be given by $\alpha \equiv 0$ and $\beta \equiv 1$. The functional interval $[\alpha, \beta] = \{\gamma \in L^\infty([-r,0]) : \alpha \leq \gamma \leq \beta\}$ is an incomplete lattice. Thus, by Davis’ result, there exists a nondecreasing mapping $A : [\alpha, \beta] \to [\alpha, \beta]$ with no fixed point.

The problem

$$\begin{cases} x'(t) = 0, & \text{for a.a. } t \in [0,1], \\ x(\theta) = Ax_{[-r,0]}(\theta), & \text{for all } \theta \in [-r,0], \end{cases}$$

fits the assumptions required in Theorem 3.3 with the lower and upper solutions $\alpha \equiv 0$ and $\beta \equiv 1$ on $[-r,1]$, and the mapping

$$B : \xi \in \mathcal{S}^1 \mapsto B\xi := A(\sup\{\alpha, \inf\{\xi_{[-r,0]}(\theta)\}\}) \in L^\infty([-r,0]).$$

Notice that $B\xi = A\xi_{[-r,0]}$ whenever $\alpha \leq \xi \leq \beta$. But, contrary to the statement of Theorem 3.3, (0.1) has no solution between $\alpha$ and $\beta$ in the sense of Definition 3.2, i.e., a solution $x : [-r,1] \to \mathbb{R}$ with $x_{[-r,0]} \in L^\infty([-r,0])$, because the restriction of such a solution to $[-r,0]$ would be a fixed point for $A$, a contradiction.

The analogue of Theorem 3.3 with $L^\infty([-r,0])$ replaced by $B([-r,0])$, the set of all bounded real functions on $[-r,0]$, is true.

The proof remains the same as in [3] except that Claim 2 must be replaced by

Claim 2’. If $C$ is a chain in $[\alpha, \beta]$, then $\sup G[C]$ and $\inf G[C]$ exist in $[\alpha, \beta]$. In particular, $\sup G[C]$ and $\inf G[C]$ exist in $\mathcal{S}^T_r$ when $C$ is the well-ordered chain of $G$-iterations of $\alpha$, and $\tilde{C}$ is the inversely well-ordered chain of $G$-iterations of $\beta$.

Claim 2’ follows from Theorem 1.2.3 in [2], included as reference [7] in [3].

The corrected version of Theorem 3.3, which is obtained by replacing $L^\infty([-r,0])$ with $B([-r,0])$, does not change the conclusions in Examples 4.1 and 4.2 in [3].
References


Departamento de Análise Matemática, Facultade de Matemáticas, Campus Sur, Universidade de Santiago de Compostela, 15782 Santiago de Compostela, Spain

E-mail address: angelcid@usc.es

Departamento de Matemática Aplicada, E.T.S.I. Telecomunicación, Universidad de Vigo, Campus Marcosende, 36280 Vigo, Spain

E-mail address: eliz@dma.uvigo.es

Departamento de Análise Matemática, Facultade de Matemáticas, Campus Sur, Universidade de Santiago de Compostela, 15782 Santiago de Compostela, Spain

E-mail address: rodrigolp@usc.es