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CORRIGENDUM TO "EXISTENCE THEORY FOR FIRST ORDER DISCONTINUOUS FUNCTIONAL DIFFERENTIAL EQUATIONS"

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The main result in [3], Theorem 3.3, is incorrect. We will present a counterexample and a repair.

To construct a counterexample, we use a result of Davis [1]: a necessary and sufficient condition for a lattice to be complete is that every nondecreasing function from the lattice into itself has a fixed point.

For $r > 0$, let $\mathcal{L}^\infty([-r,0])$ be the set of all bounded Lebesgue-measurable real functions on $[-r,0]$, and let $\bar{\alpha}, \bar{\beta} \in \mathcal{L}^\infty([-r,0])$ be given by $\bar{\alpha} \equiv 0$ and $\bar{\beta} \equiv 1$. The functional interval $[\bar{\alpha}, \bar{\beta}] = \{\gamma \in \mathcal{L}^\infty([-r,0]) : \bar{\alpha} \le \gamma \le \bar{\beta}\}$ is an incomplete lattice. Thus, by Davis' result, there exists a nondecreasing mapping $A : [\bar{\alpha}, \bar{\beta}] \to [\bar{\alpha}, \bar{\beta}]$ with no fixed point.

The problem

$$(0.1) \qquad \begin{cases} x'(t) = 0, & \text{for a.a. } t \in [0,1], \\ x(\theta) = Ax_{|[-r,0]}(\theta), & \text{for all } \theta \in [-r,0], \end{cases}$$

fits the assumptions required in Theorem 3.3 with the lower and upper solutions $\alpha \equiv 0$ and $\beta \equiv 1$ on $[-r,1]$, and the mapping

$$B : \xi \in \mathcal{S}_r^1 \mapsto B\xi := A(\sup\{\bar{\alpha}, \inf\{\xi_{|[-r,0]}, \bar{\beta}\}\}) \in \mathcal{L}^\infty([-r,0]).$$

Notice that $B\xi = A\xi_{|[-r,0]}$ whenever $\alpha \le \xi \le \beta$. But, contrary to the statement of Theorem 3.3, (0.1) has no solution between α and β in the sense of Definition 3.2, i.e., a solution $x : [-r,1] \to \mathbb{R}$ with $x_{|[-r,0]} \in \mathcal{L}^\infty([-r,0])$, because the restriction of such a solution to $[-r,0]$ would be a fixed point for A, a contradiction.

The analogue of Theorem 3.3 with $\mathcal{L}^\infty([-r,0])$ replaced by $\mathcal{B}([-r,0])$, the set of all bounded real functions on $[-r,0]$, is true.

The proof remains the same as in [3] except that Claim 2 must be replaced by

Claim $2'$. *If C is a chain in $[\alpha,\beta]$, then* $\sup G[C]$ *and* $\inf G[C]$ *exist in* $[\alpha,\beta]$. In particular, $\sup G[C]$ and $\inf G[\bar{C}]$ exist in $\mathcal{S}_r^T$ when C is the well-ordered chain of G-iterations of α, and $\bar{C}$ is the inversely well-ordered chain of G-iterations of β.

Claim $2'$ follows from Theorem 1.2.3 in [2], included as reference [7] in [3].

The corrected version of Theorem 3.3, which is obtained by replacing $\mathcal{L}^\infty([-r,0])$ with $\mathcal{B}([-r,0])$, does not change the conclusions in Examples 4.1 and 4.2 in [3].

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