

ON THE GAUSS MAP OF HYPERSURFACES WITH CONSTANT SCALAR CURVATURE IN SPHERES

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ABSTRACT. In this work we consider connected, complete and orientable hypersurfaces of the sphere \mathbb{S}^{n+1} with constant nonnegative r -mean curvature. We prove that under subsidiary conditions, if the Gauss image of M is contained in a closed hemisphere, then M is totally umbilic.

INTRODUCTION

One of the most celebrated theorems of minimal surfaces in \mathbb{R}^3 is Bernstein's theorem:

Theorem (Bernstein [4]). *Let $M \subset \mathbb{R}^3$ be a complete minimal surface in \mathbb{R}^3 that is given by an entire (defined over the whole \mathbb{R}^2) graph of a smooth function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$. Then M is a plane.*

The above result is also true under the weaker hypothesis that the image of the Gauss map of M lies in an open hemisphere of \mathbb{S}^{n+1} , as one can see in [3]. These results raise the following problem for the geometry of minimal surfaces in spheres: Does there exist a similar result for minimal hypersurfaces of the unit sphere? The answer to this question was obtained independently by E. De Giorgi ([6]) and J. Simons (see [13] - Theorem 5.2.1) as follows.

Theorem. *If the Gauss image (see the definition below) of a compact minimal hypersurface M^n in the Euclidean sphere lies in an open hemisphere of \mathbb{S}^{n+1} , then M must be a great hypersphere in \mathbb{S}^{n+1} .*

After that, K. Nomizu and Brian Smyth (see [9] - Theorem 2) were able to generalize this result to constant mean curvature hypersurfaces of \mathbb{S}^{n+1} , proving the following result:

Theorem (Nomizu-Smyth). *Let M be any compact connected orientable manifold of dimension $n \geq 2$ immersed in the sphere \mathbb{S}^{n+1} with constant mean curvature. If the Gauss image of M lies in a closed hemisphere of \mathbb{S}^{n+1} , then M is a hypersphere in \mathbb{S}^{n+1} .*

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The goal of this work is to extend these results to higher-order constant mean curvature hypersurfaces of the sphere. First let us fix some notation.

Let M^n be a compact orientable Riemannian manifold and let $x : M^n \rightarrow \mathbb{S}^{n+1}$ be an isometric immersion into the unit sphere $\mathbb{S}^{n+1} \subset \mathbb{R}^{n+2}$. Since M is orientable, we can choose a global unit normal field N . The Riemannian connections ∇ and $\tilde{\nabla}$ of M and \mathbb{S}^{n+1} , respectively, are related by

$$\tilde{\nabla}_X Y = \nabla_X Y + \langle A(X), Y \rangle N,$$

where A is the shape operator of the immersion, defined by

$$\tilde{\nabla}_X N = -A(X).$$

Let k_1, \dots, k_n be the eigenvalues of A . We define the r -mean curvature of the immersion at a point p by

$$H_r = \frac{1}{\binom{n}{r}} \sum_{i_1 < \dots < i_r} k_{i_1} \dots k_{i_r} = \frac{1}{\binom{n}{r}} S_r,$$

where S_r is the r -symmetric function of the k_1, \dots, k_n . In order to unify the notation, we will define $H_0 = 1$ and $H_r = 0$, for all $r \geq n + 1$. For $r = 1$, $H_1 = H$ is the mean curvature of the immersion, in the case $r = 2$, H_2 is the scalar curvature and for $r = n$, H_n is the Gauss-Kronecker curvature.

The Gauss map $\phi : M^n \rightarrow \mathbb{S}^{n+1}$ is defined by

$$\phi(P) = N(P) \in \mathbb{S}^{n+1}.$$

The set $\phi(M)$ is called the Gauss image of M . We observe that the Gauss image depends on the choice of the orientation of M , but the two possibilities are related by an antipodal mapping of \mathbb{S}^{n+1} . Thus the statement that the Gauss image of M is contained in a closed hemisphere of \mathbb{S}^{n+1} is independent of the orientation of M .

For the case $H_r = 0$, we obtain that

Theorem A. *Let $M^n \rightarrow \mathbb{S}^{n+1}$ be a compact and connected hypersurface of \mathbb{S}^{n+1} with $H_r = 0$, for some $r = 1, \dots, n - 1$. Assume that the Gauss image of M is contained in a closed hemisphere and that H_{r-1} does not change sign in M . Then M is totally geodesic.*

If $H_r > 0$, we were able to prove that

Theorem B. *Let $M^n \rightarrow \mathbb{S}^{n+1}$ be a compact and connected hypersurface of \mathbb{S}^{n+1} with constant positive $(r + 1)$ -mean curvature H_{r+1} , for some $r = 0, \dots, n - 2$. Assume that the Gauss image of M is contained in a closed hemisphere, $H_r \geq 0$ and that the following inequality holds:*

$$H_1 H_r \geq H_{r+1}.$$

Then M is totally umbilic.

In the case of the scalar curvature, part of the hypothesis of the above theorems is trivially satisfied, and we obtain the following result.

Theorem C. *Let M^n be a compact orientable hypersurface of the sphere with constant scalar curvature $H_2 \geq 0$. In the case $H_2 = 0$, suppose also that H_1 does not change sign. If the Gauss image of M lies in a closed hemisphere of \mathbb{S}^{n+1} , then M is totally umbilic.*

The authors do not know if the hypotheses of Theorems A, B and C can be weakened.

Parts of these results were obtained by R. Reilly, [11], with the strong hypothesis that the Gauss image is contained in an open hemisphere.

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1. PRELIMINARIES

We introduce the r^{th} Newton tensors, $P_r : T_p M \rightarrow T_p M$, which are defined inductively by

$$\begin{aligned} P_0 &= I, \\ P_r &= S_r I - A P_{r-1}, \quad r > 1. \end{aligned}$$

It is easy to see that each P_r commutes with A , and if e_i is an eigenvector of A associated to the principal curvature k_i , then

$$P_1(e_i) = \mu_i e_i = (S_1 - k_i)e_i.$$

In [11], Reilly showed that the P_r 's satisfy the following.

Proposition 1.1 ([11], see also [2] - Lemma 2.1). *Let $x : M^n \rightarrow N^{n+1}$ be an isometric immersion between two Riemannian manifolds, and let A be its second fundamental form. The r^{th} Newton tensor P_r associated to A satisfies:*

- (1) $\text{trace}(P_r) = (n - r)S_r$,
- (2) $\text{trace}(A P_r) = (r + 1)S_{r+1}$,
- (3) $\text{trace}(A^2 P_r) = S_1 S_{r+1} - (r + 2)S_{r+2}$.

Associated to each Newton tensor P_r , we define a second-order differential operator

$$L_r(f) = \text{trace}(P_r \text{Hess } f).$$

We observe that for $r = 0$, L_0 is the Laplacian, which is always an elliptic operator. If N^{n+1} has constant sectional curvature, it follows from the Codazzi equation (see [12], p. 225) that L_r is

$$L_r(f) = \text{div}_M(P_r \nabla f).$$

Hence L_r is a self-adjoint operator. In general, for $r \geq 1$, L_r is not an elliptic operator. The following proposition give us a condition for L_r to be elliptic.

Proposition 1.2. *Let M^n be a connected, compact and orientable Riemannian manifold, and let $x : M^n \rightarrow \mathbb{S}^{n+1}$ be an isometric immersion with H_{r+1} constant. If M^n has one point where all principal curvatures are positive, then L_r is an elliptic operator.*

Proof. See the proof of Proposition 3.2 of [2].

For hypersurfaces of \mathbb{R}^{n+1} with $H_r = 0$, Hounie and Leite, [8], were able to give a geometric condition that is equivalent to L_r being elliptic. In fact, their proof can be generalized to hypersurfaces of the sphere, and we have the following result.

Proposition 1.3 ([8] - Proposition 1.5). *Let M be a hypersurface in \mathbb{R}^{n+1} or S^{n+1} with $H_r = 0$, $2 \leq r < n$. Then the operator $L_{r-1}(f) = \text{div}(P_{r-1} \nabla f)$ is elliptic at $p \in M$ if and only if $H_{r+1}(p) \neq 0$.*

Since the r -mean curvatures of M^n are symmetric means of the n -uple of principal curvatures of M , they are related by the following algebraic inequalities (see [7], p. 52, and [5], p. 285):

$$(1.1) \quad H_{i-1}H_{i+1} \leq H_i^2, \quad \forall i, 1 \leq i < n.$$

Also, provided that the H_r 's are nonnegative, $r = 1, \dots, i$,

$$(1.2) \quad H_1 \geq H_2^{1/2} \geq H_3^{1/3} \geq \dots \geq H_i^{1/i}.$$

Furthermore, the equality in (1.1) and (1.2) holds only if $k_1 = k_2 = \dots = k_n$.

2. INTEGRAL FORMULA

Consider the functions $f, g : M \rightarrow \mathbb{R}$, given by

$$f(P) = \langle N(P), \alpha \rangle$$

and

$$g(P) = \langle x(P), \alpha \rangle,$$

where α is a fixed vector of \mathbb{R}^{n+2} . These functions satisfy (see [2], Lemma 5.2)

$$(2.1) \quad L_r(g) = -(r+1)S_{r+1}f - (n-r)S_r g,$$

$$(2.2) \quad L_r(f) = -(S_1S_{r+1} - (r+2)S_{r+2})f - (r+1)S_{r+1}g,$$

where, in the last equation, we use the fact that S_{r+1} is constant. In particular, for $r = 0$, we get

$$(2.3) \quad \Delta(g) = -S_1f - ng,$$

$$(2.4) \quad \Delta(f) = -(S_1^2 - 2S_2)f - S_1g = -\|A\|^2f - S_1g.$$

The following integral formula will be needed.

Proposition 2.1. *Let $M^n \rightarrow \mathbb{S}^{n+1}$ be a compact orientable hypersurface isometrically immersed in \mathbb{S}^{n+1} , with H_{r+1} constant, for some r with $0 \leq r < n-2$. Then,*

$$(2.5) \quad \int_M [(n-r-1)S_1S_{r+1} - n(r+2)S_{r+2}]f \, dM = 0.$$

Proof. Observe that, since S_{r+1} is constant, by (2.2) and (2.3), we obtain that

$$\begin{aligned} L_r f - \frac{(r+1)}{n} S_{r+1} \Delta g &= -(S_1 S_{r+1} - (r+2) S_{r+2}) f \\ &\quad - (r+1) S_{r+1} g + \frac{(r+1)}{n} S_{r+1} S_1 f + \frac{(r+1)}{n} S_{r+1} n g \\ &= -S_1 S_{r+1} f + (r+2) S_{r+2} f + \frac{(r+1)}{n} S_{r+1} S_1 f \\ &= \frac{1}{n} [-n S_1 S_{r+1} f + n(r+2) S_{r+2} f + (r+1) S_{r+1} S_1 f] \\ &= \frac{1}{n} [(-n+r+1) S_1 S_{r+1} f + n(r+2) S_{r+2} f] \\ &= \frac{-1}{n} [(n-r-1) S_1 S_{r+1} - n(r+2) S_{r+2}] f. \end{aligned}$$

Integrating this last expression and applying Stokes' Theorem, one has that

$$\begin{aligned} & \int_M [(n - r - 1)S_1S_{r+1} - n(r + 2)S_{r+2}]f \, dM \\ &= \int_{\partial M} \langle P_r \nabla f - \frac{(r + 1)}{n} S_{r+1} \nabla g, \nu \rangle \, dS = 0, \end{aligned}$$

where the last equality follows from the fact that $\partial M = \emptyset$. □

3. THE CASE $H_r = 0$

In this section we consider hypersurfaces of the sphere with $H_r = 0$. We have the following result.

Theorem 3.1 (Theorem A of the Introduction). *Let $M^n \rightarrow \mathbb{S}^{n+1}$ be a compact and connected hypersurface of \mathbb{S}^{n+1} with $H_r = 0$, for some $r = 1, \dots, n - 1$. Assume that the Gauss image of M is contained in a closed hemisphere and that H_{r-1} does not change sign in M . Then M is totally geodesic.*

Proof. By (1.1) and the fact that $H_r = 0$, it follows that

$$H_{r+1}H_{r-1} \leq 0.$$

Thus, since H_{r-1} does not change sign in M , H_{r+1} also does not change sign on M .

On the other hand, our hypothesis on the Gauss image implies that there exists a vector $\alpha \in \mathbb{R}^{n+2}$ such that

$$f(P) = \langle N(P), \alpha \rangle$$

is nonnegative along M . Hence, $f(P)S_{r+1}(P)$ does not change sign along M . The equation (2.5), in our case, reads

$$\int_M f(P)S_{r+1}(P)dM = 0.$$

Thus,

$$(3.1) \quad f(P)S_{r+1}(P) = 0, \quad \forall P \in M.$$

Let $\mathcal{A} \subset M$ be the set of all points of M where $S_{r+1}(P) > 0$. In \mathcal{A} , by equation (3.1), $f \equiv 0$. By continuity, f is zero along $\overline{\mathcal{A}}$, where $\overline{\mathcal{A}}$ is the closure of \mathcal{A} . On the other hand, the set $M/\overline{\mathcal{A}}$ is an open set of M where

$$H_r = H_{r+1} = 0.$$

Hence equality holds in (1.1), for all $P \in M/\overline{\mathcal{A}}$. This means that all points in $M/\overline{\mathcal{A}}$ are umbilic. That is, for all $P \in M/\overline{\mathcal{A}}$,

$$k_1(P) = \dots = k_n(P) = a(P).$$

Thus,

$$0 = S_r(P) = a^r(P).$$

This implies that all points of $M/\overline{\mathcal{A}}$ are totally geodesic, and hence f is constant along each connected component of $M/\overline{\mathcal{A}}$. Since along the boundary of those sets, $f = 0$, we conclude that f is identically zero on M , that is, M is totally geodesic (see Theorem 1 of [9]). □

Remark. For the case $r = 1$, we observe that $S_{r-1} = S_0 = 1$ does not change sign. Hence, the theorem is a generalization of Theorem 2 in [9], in the minimal case.

4. THE CASE $H_{r+1} > 0$

Let us consider the case $H_{r+1} > 0$. We have the following result:

Theorem 4.1 (Theorem B of the Introduction). *Let $M^n \rightarrow \mathbb{S}^{n+1}$ be a compact and connected hypersurface of \mathbb{S}^{n+1} with constant positive $(r+1)$ -mean curvature H_{r+1} , for some $r = 0, \dots, n-2$. Assume that the Gauss image of M is contained in a closed hemisphere, $H_r \geq 0$ and that the following inequality holds:*

$$(4.1) \quad H_1 H_r \geq H_{r+1}.$$

Then M is totally umbilic.

Proof. By Proposition 2.1, we have that for a fixed $\alpha \in \mathbb{R}^{n+2}$, the function $f = \langle N(P), \alpha \rangle$ satisfies

$$(4.2) \quad \int_M [(n-r-1)S_1 S_{r+1} - n(r+2)S_{r+2}] f \, dM = 0.$$

We are going to prove that the integrand has a fixed sign, for some $\alpha \in \mathbb{R}^{n+2}$. Since the Gauss image of M lies in a closed hemisphere, there exists a vector $\alpha \in \mathbb{R}^{n+2}$ such that

$$(4.3) \quad f(P) = \langle N(P), \alpha \rangle \geq 0, \quad \forall P \in M.$$

On the other hand, the relation $H_1 H_r \geq H_{r+1}$ implies that $H_1 H_{r+1} \geq H_{r+2}$. In fact, by using equation (1.1), one has that

$$(4.4) \quad H_r H_{r+2} \leq H_{r+1}^2 \leq H_r H_1 H_{r+1}.$$

Observe that $H_r \neq 0$; otherwise, the last inequality implies that H_r and H_{r+1} are equal to zero, which is a contradiction. Hence, $H_r > 0$ and we can divide (4.4) by

$$(4.5) \quad H_1 H_{r+1} \geq H_{r+2}.$$

Since

$$H_i = \frac{S_i}{\binom{n}{i}},$$

by (4.5), one has

$$\frac{S_1}{n} \frac{S_{r+1}}{\binom{n}{r+1}} \geq \frac{S_{r+2}}{\binom{n}{r+2}}.$$

This implies that

$$(4.6) \quad (n-r-1)S_1 S_{r+1} - n(r+2)S_{r+2} \geq 0.$$

The inequalities (4.3) and (4.6) imply that

$$[(n-r-1)S_1 S_{r+1} - n(r+2)S_{r+2}] f \geq 0.$$

Thus, by (4.2), we have that

$$[(n-r-1)S_1 S_{r+1} - n(r+2)S_{r+2}] f = 0.$$

Observe that the function f is not identically zero, since in this case, M has to be totally geodesic (see Theorem 1 of [9]) and hence $H_r = 0$, which is a contradiction. Let $\mathcal{B} \subset M$ be the open and nonempty set where $f > 0$. Along \mathcal{B} , we have

$$(n-r-1)S_1 S_{r+1} - n(r+2)S_{r+2} = 0,$$

that is, equality holds in (4.6). This means that equality also holds in (1.1), since this inequality was used to obtain (4.6). Hence, all points of \mathcal{B} are umbilic. In this

case, M has an elliptic point and $S_r = \text{constant} > 0$. Thus, by Proposition 1.2, the operator L_r is an elliptic operator. By the principle of analytic continuation, since M is totally umbilic in an open set, it has to be totally umbilic. \square

Observe that in the case $r = 2$, part of the hypotheses of Theorems 3.1 and 4.1 is trivially satisfied, and we have the following result.

Corollary 4.1 (Theorem C of the Introduction). *Let M^n be a compact orientable hypersurface of the sphere with constant scalar curvature $H_2 \geq 0$. In the case $H_2 = 0$, suppose also that H_1 does not change sign. If the Gauss image of M lies in a closed hemisphere of \mathbb{S}^{n+1} , then M is totally umbilic.*

Proof. The case $H_2 = 0$ is the statement of Theorem 3.1. For the case $H_2 > 0$, the hypothesis (4.1) in Theorem 4.1 reads

$$H_1^2 \geq H_2,$$

which is always true by equation (1.1). The above equation also says that H_1 is different from zero on M . Hence we can choose the orientation of M so that $H_1 > 0$. The sign of H_2 does not depend on the orientation; thus the result follows directly from Theorem 4.1.

We now give conditions that imply condition (4.1). First of all, if H_i is nonnegative for $i = 1, \dots, r - 1$, then (4.1) holds. This fact was stated in [12], p. 232, and we are including its proof here for the sake of completeness. Let (x_1, \dots, x_n) be an n -uple of real numbers, and let S_r be the r -symmetric function of the x_1, \dots, x_n . Let H_r be defined by

$$H_r = \frac{1}{\binom{n}{r}} S_r = \frac{1}{\binom{n}{r}} \sum_{i_1 < \dots < i_r} x_{i_1} \dots x_{i_r}.$$

Proposition 4.1. *With the above notation, if $H_i \geq 0$ for all $i = 1, \dots, r - 1$, then*

$$(4.7) \quad H_1 H_{i+1} \geq H_{i+2}, \quad \forall i = 1, \dots, r - 1.$$

Moreover,

$$(4.8) \quad (n - i - 1) S_1 S_{i+1} - n(i + 2) S_{i+2} \geq 0, \quad \forall i = 1, \dots, r - 1.$$

Proof. By using (1.1), we have that

$$H_r H_{r-2} \geq H_{r-1}^2 \geq 0$$

and

$$H_{r+1} H_{r-1} \geq H_r^2 \geq 0.$$

Since H_{r-2} and H_{r-1} are nonnegative, it follows that $H_r \geq 0$ and $H_{r+1} \geq 0$. Let us prove (4.7). We will argue by induction on i . By using (1.1), with $i = 1$, and the fact that $H_0 = 1$, we obtain

$$H_1^2 \geq H_0 H_2 = H_2.$$

Hence (4.7) holds for $i = 0$. By induction, let us suppose that

$$(4.9) \quad H_1 H_i \geq H_{i+1}.$$

This implies, using equation (1.1), that

$$(4.10) \quad H_i H_{i+2} \leq H_{i+1}^2 \leq H_{i+1} H_1 H_i.$$

If $H_i = 0$, then (4.9) implies that $H_{i+1} \leq 0$. Since $H_{i+1} \geq 0$, it follows that $H_{i+1} = 0$. Thus we have equality in (1.2), which implies that $x_k = 0$, $\forall k = 1, \dots, n$. Hence (4.7) holds in this case.

Let us suppose $H_i > 0$. In this case, we can divide (4.10) by H_i and obtain

$$(4.11) \quad H_1 H_{i+1} \geq H_{i+2},$$

and we finish the proof of (4.7). In order to obtain (4.8), just observe that

$$H_i = \frac{S_i}{\binom{n}{i}}.$$

Then, by (4.11), one has

$$\frac{S_1}{n} \frac{S_{i+1}}{\binom{n}{i+1}} \geq \frac{S_{i+2}}{\binom{n}{i+2}}.$$

This implies that

$$(n - i - 1)S_1 S_{i+1} - n(i + 2)S_{i+2} \geq 0, \quad \forall i = 1, \dots, r - 2. \quad \square$$

Thus, we have the following result.

Corollary 4.2. *Let $M^n \rightarrow \mathbb{S}^{n+1}$ be a compact and connected hypersurface of \mathbb{S}^{n+1} with constant positive r -mean curvature H_r , for some $r = 1, \dots, n - 1$. Assume that the Gauss image of M is contained in a closed hemisphere and that $H_i \geq 0$ for all $i = 1, \dots, r - 1$. Then M is totally umbilic.*

In the following proposition (see Proposition 2.3 in [2]) we have another geometric condition that gives $H_i \geq 0$ for all $i = 1, \dots, r - 1$.

Proposition 4.2. *Let M^n be a connected compact Riemannian manifold, and let $x : M^n \rightarrow \mathbb{S}^{n+1}$ be an isometric immersion. If $H_r > 0$ and $x(M)$ is contained in an open hemisphere of \mathbb{S}^{n+1} , then $H_i > 0$ for all $i = 1, \dots, r - 1$.*

This and Corollary 4.2 imply

Corollary 4.3. *Let $x : M^n \rightarrow \mathbb{S}^{n+1}$ be an isometric immersion of a compact and connected hypersurface of \mathbb{S}^{n+1} with constant positive r -mean curvature H_r , for some $r = 1, \dots, n - 1$. Assume that the Gauss image of M is contained in a closed hemisphere and that $x(M)$ is contained in an open hemisphere of \mathbb{S}^{n+1} . Then M is totally umbilic.*

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