PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 132, Number 12, Pages 3653–3659
S 0002-9939(04)07513-6
Article electronically published on July 26, 2004

A LOWER BOUND FOR THE STABILITY RADIUS OF TIME-VARYING SYSTEMS

ADINA LUMINIŢA SASU AND BOGDAN SASU

(Communicated by Joseph A. Ball)

Abstract. We introduce and characterize the stability radius of systems whose state evolution is described by linear skew-product semiflows. We obtain a lower bound for the stability radius in terms of the Perron operators associated to the linear skew-product semiflow. We generalize a result due to Hinrichsen and Pritchard.

1. Introduction

In the last few years, the theory of linear skew-product semiflows has proved to be a very useful tool in the study of evolution equations with unbounded coefficients (see [4]-[6], [10], [11], [14]-[20]). Significant questions concerning the asymptotic behaviour of linear skew-product semiflows have been answered. In this context, the theorems of Perron type or so-called "input-output" conditions of characterization of the asymptotic properties, have been obtained for uniform exponential stability (see [19]), for uniform exponential expansiveness (see [16]) and for pointwise and uniform exponential dichotomy, respectively, of discrete and continuous linear skew-product semiflows (see [4]-[6], [10], [11], [15], [17], [18], [20]).

Recently, theorems of Perron type for exponential dichotomy of different types of evolution families have been presented in [12] and in [13]. Significant results obtained with input-output techniques have been established in [1]-[3]. An important interpretation which justifies the term "frequency-response function" for time-varying finite-dimensional systems, was given in [3].

It is well known that the concept of stability radius introduced by Hinrichsen, Ilchmann and Pritchard (see [8], [9]) led to a systematic study of the stability of linear infinite-dimensional systems under structured time-varying perturbations. The idea was to estimate the size of the smallest disturbance operator under which the additively perturbed system loses exponential stability. An outstanding result expresses the lower bound of the stability radius of a systems associated to a mild evolution family in terms of the norm of the input-output operator (see [9], Theorem 3.2). This result has been recently proved by Clark, Latushkin, Montgomery-Smith and Randloph (see [7], Theorem 4.2), using an evolution semigroup technique.

Received by the editors April 7, 2003 and, in revised form, August 24, 2003.
2000 *Mathematics Subject Classification.* Primary 34D05.
Key words and phrases. Linear skew-product semiflows, stability radius.

Naturally, the question arises whether the concept of stability radius can be extended for the case of systems associated to linear skew-product semiflows, and how can we express the lower bound of this type of stability radius.

The purpose of this paper is to answer this question. We shall consider an abstract generalization of systems described by differential equations of the form

$$\begin{cases} \dot{x}(t) = A(\sigma(\theta,t))x(t) + B(\sigma(\theta,t))u(t), \\ y(t) = C(\sigma(\theta,t))x(t) \end{cases} \tag{1.1}$$

where σ is a semiflow on a locally compact metric space Θ. For every $\theta \in \Theta$, the operators $A(\theta)$ are generally unbounded operators on a Banach space X, while the operators $B(\theta) \in \mathcal{B}(U,X)$, $C(\theta) \in \mathcal{B}(X,Y)$, where U, Y are Banach spaces. Formally, in our investigations, the family $\{A(\theta)\}_{\theta\in\Theta}$ is subjected to additive structured perturbations, so that the perturbed system is

$$\dot{x}(t) = [A(\sigma(\theta,t)) + B(\sigma(\theta,t))\Delta(\sigma(\theta,t))C(\sigma(\theta,t))]x(t),$$

which can be interpreted as a system obtained by applying the feedback $u(t) = \Delta(\sigma(\theta,t))y(t)$ to the time-varying system (1.1).

It is well known that if X is a Banach space, Θ is a compact metric space, σ is a semiflow on Θ and $A : \Theta \to \mathcal{B}(X)$ is a continuous mapping, then $\Phi(\theta,t)$, the solution operator of the linear differential equation

$$\dot{x}(t) = A(\sigma(\theta,t))\, x(t), \quad t \geq 0,$$

is a cocycle and the pair $\pi = (\Phi,\sigma)$ is a linear skew-product semiflow. Therefore, in what follows, we consider an integral model for the above type of systems, defined in terms of linear skew-product semiflows. Our aim is to define and examine a stability radius for a general system associated to a linear skew-product semiflow, subjected to perturbations. The main tools in our study will be the theorems of Perron type, which connect the stability of a linear skew-product semiflow with the uniform boundedness of a family of Perron operators on $C_b(\mathbf{R}_+,X)$. We will express the lower bound of the stability radius in terms of the norms of input-output operators associated to the system. In this manner, we generalize a result proved by Hinrichsen and Pritchard for the case of mild evolution operators and by Clark, Latushkin, Montgomery-Smith and Randolph for strongly continuous evolution families.

2. Preliminaries

Let X be a Banach space, let (Θ,d) be a locally compact metric space and let $\mathcal{E} = X\times\Theta$. We denote by $\mathcal{B}(X)$ the Banach algebra of all bounded linear operators from X into itself.

Definition 2.1. A continuous mapping $\sigma : \Theta\times\mathbf{R}_+ \to \Theta$ is called a *semiflow* on Θ, if $\sigma(\theta,0) = \theta$, for all $\theta\in\Theta$ and

$$\sigma(\theta,s+t) = \sigma(\sigma(\theta,s),t), \quad \forall(\theta,s,t) \in \Theta\times\mathbf{R}_+^2.$$

Definition 2.2. A pair $\pi = (\Phi,\sigma)$ is called a *linear skew-product semiflow* on $\mathcal{E} = X\times\Theta$ if σ is a semiflow on Θ and $\Phi : \Theta\times\mathbb{R}_+ \to \mathcal{B}(X)$ satisfies the following conditions:

(i) $\Phi(\theta,0) = I$, the identity operator on X, for all $\theta\in\Theta$;

(ii) $\Phi(\theta, t+s) = \Phi(\sigma(\theta, t), s)\Phi(\theta, t)$, for all $(\theta, t, s) \in \Theta \times \mathbf{R}_+^2$ (*the cocycle identity*);

(iii) $(\theta, t) \mapsto \Phi(\theta, t)x$ is continuous, for every $x \in X$;

(iv) there are $M \geq 1$ and $\omega > 0$ such that

$$||\Phi(\theta, t)|| \leq Me^{\omega t}, \quad \forall (\theta, t) \in \Theta \times \mathbf{R}_+.$$

The mapping Φ given by Definition 2.2 is called *the cocycle* associated to the linear skew-product semiflow $\pi = (\Phi, \sigma)$. Examples of linear skew-product semiflows can be found in [4]-[6], [10], [11], [14]-[20].

We denote by $\mathcal{C}_s(\Theta, \mathcal{B}(X))$ the space of all strongly continuous bounded mappings $H : \Theta \to \mathcal{B}(X)$.

Theorem 2.3. *Let $\pi = (\Phi, \sigma)$ be a linear skew-product semiflow on $\mathcal{E} = X \times \Theta$. If $P \in \mathcal{C}_s(\Theta, \mathcal{B}(X))$ there is a unique linear skew product semiflow $\pi_P = (\Phi_P, \sigma)$ on $X \times \Theta$ such that*

$$\Phi_P(\theta, t)x = \Phi(\theta, t)x + \int_0^t \Phi(\sigma(\theta, s), t-s)\, P(\sigma(\theta, s))\, \Phi_P(\theta, s)x\, ds \tag{2.1}$$

for all $(x, \theta, t) \in X \times \Theta \times \mathbf{R}_+$.

Proof. See [14], Theorem 2.1. □

Let $C(\mathbf{R}_+, X)$ be the linear space of all continuous functions $u : \mathbf{R}_+ \to X$ and $C_b(\mathbf{R}_+, X) = \{u \in C(\mathbf{R}_+, X) : \sup_{t \geq 0} ||u(t)|| < \infty\}$. $C_b(\mathbf{R}_+, X)$ is a Banach space with respect to the norm $|||u||| := \sup_{t \geq 0} ||u(t)||$.

Let $\pi = (\Phi, \sigma)$ be a linear skew-product semiflow on $\mathcal{E} = X \times \Theta$. For every $\theta \in \Theta$ we define the Perron operator

$$P_\theta : C(\mathbf{R}_+, X) \to C(\mathbf{R}_+, X), \quad (P_\theta u)(t) = \int_0^t \Phi(\sigma(\theta, s), t-s)u(s)ds. \tag{2.2}$$

Definition 2.4. A linear skew-product semiflow $\pi = (\Phi, \sigma)$ on $\mathcal{E} = X \times \Theta$ is called *uniformly exponentially stable* if there are $N \geq 1$ and $\nu > 0$ such that

$$||\Phi(\theta, t)|| \leq Ne^{-\nu t}, \quad \forall (\theta, t) \in \Theta \times \mathbf{R}_+.$$

Definition 2.5. A linear skew-product semiflow $\pi = (\Phi, \sigma)$ on $\mathcal{E} = X \times \Theta$ is said to be uniformly $(C_b(\mathbf{R}_+, X), C_b(\mathbf{R}_+, X))$-*stable* if

(i) $P_\theta u \in C_b(\mathbf{R}_+, X)$, for all $(u, \theta) \in C_b(\mathbf{R}_+, X) \times \Theta$;

(ii) there is $K > 0$ such that $|||P_\theta u||| \leq K\, |||u|||$, for all $(u, \theta) \in C_b(\mathbf{R}_+, X) \times \Theta$.

Theorem 2.6. *Let $\pi = (\Phi, \sigma)$ be a linear skew-product semiflow on $\mathcal{E} = X \times \Theta$. Then π is uniformly exponentially stable if and only if it is uniformly $(C_b(\mathbf{R}_+, X), C_b(\mathbf{R}_+, X))$-stable.*

Proof. See [19], Theorem 5.1. □

3. Stability radius

If U, Y are Banach spaces we denote by $\mathcal{B}(U, Y)$ the space of all bounded linear operators from U into Y. If Θ is a locally compact metric space, we denote by

$\mathcal{C}_s(\Theta, \mathcal{B}(U,Y))$ the space of all strongly continuous bounded mappings $H : \Theta \to \mathcal{B}(U,Y)$, which is a Banach space with respect to the norm

$$||H|| := \sup_{\theta \in \Theta} ||H(\theta)||.$$

Let X, Y, U be Banach spaces, and let Θ be a locally compact metric space. Let $B \in \mathcal{C}_s(\Theta, \mathcal{B}(U,X))$ and $C \in \mathcal{C}_s(\Theta, \mathcal{B}(X,Y))$. Let $\pi = (\Phi, \sigma)$ be a linear skew-product semiflow on $\mathcal{E} = X \times \Theta$.

Consider the system $S = (\pi, B, C)$ described by the following integral model:

$$\begin{cases} x(\theta, t, x_0, u) &= \Phi(\theta, t)x_0 + \int_0^t \Phi(\sigma(\theta, s), t-s)B(\sigma(\theta, s))u(s)\, ds, \\ \\ y(\theta, t, x_0, u) &= C(\sigma(\theta, t))x(\theta, t, x_0, u) \end{cases}$$

where $t \geq 0, (x_0, \theta) \in \mathcal{E}$ and $u \in L^1_{loc}(\mathbf{R}_+, U)$.

Definition 3.1. Suppose that $\pi = (\Phi, \sigma)$ is uniformly exponentially stable. The *stability radius* of π with respect to the perturbation structure (B, C) is defined by

$$r_{stab}(\pi, B, C) = \sup\,\{r \geq 0 : \forall \Delta \in \mathcal{C}_s(\Theta, \mathcal{B}(Y,U)) \text{ with}$$
$$||\Delta|| \leq r \Longrightarrow \pi_{B\Delta C} = (\Phi_{B\Delta C}, \sigma) \text{ is uniformly exponentially stable}\}.$$

In what follows we shall obtain a lower bound for the stability radius of π.

For every $\theta \in \Theta$ we define the bounded linear operators

$$B_\theta : C_b(\mathbf{R}_+, U) \to C_b(\mathbf{R}_+, X), \quad (B_\theta u)(t) = B(\sigma(\theta, t))u(t),$$
$$C_\theta : C_b(\mathbf{R}_+, X) \to C_b(\mathbf{R}_+, Y), \quad (C_\theta u)(t) = C(\sigma(\theta, t))u(t).$$

Let $\{P_\theta\}_{\theta \in \Theta}$ be the family of operators associated to π according to relation (2.2). Since π is uniformly exponentially stable, using Theorem 2.6, it follows that for every $\theta \in \Theta$ and every $u \in C_b(\mathbf{R}_+, U)$, $C_\theta P_\theta B_\theta u \in C_b(\mathbf{R}_+, Y)$. Then it makes sense to consider the family of *input-output operators* $\{L_\theta\}_{\theta \in \Theta}$, where

$$L_\theta : C_b(\mathbf{R}_+, U) \to C_b(\mathbf{R}_+, Y), \quad L_\theta = C_\theta P_\theta B_\theta.$$

Using Theorem 2.6, we obtain that

$$\alpha(\pi) := \sup_{\theta \in \Theta} ||L_\theta|| < \infty.$$

In what follows we suppose that the mapping C is *uniformly positive*, which means that there is $m > 0$ such that

$$(3.1) \qquad ||C(\theta)x||_Y \geq m||x||_X, \quad \forall (\theta, x) \in \Theta \times X.$$

Theorem 3.2. *Let $\Delta \in \mathcal{C}_s(\Theta, \mathcal{B}(Y,U))$ with*

$$||\Delta|| < \frac{1}{\alpha(\pi)}.$$

Then there is $M \geq 1$ such that

$$||\Phi_{B\Delta C}(\theta, t)|| \leq M, \quad \forall (\theta, t) \in \Theta \times \mathbf{R}_+.$$

Proof. For every $(x, \theta, t) \in \mathcal{E} \times \mathbf{R}_+$, let $M(x, \theta, t) = \sup_{s \in [0,t]} ||C(\sigma(\theta, s))\Phi_{B\Delta C}(\theta, s)x||$.

Let $(x, \theta, t_0) \in \mathcal{E} \times \mathbf{R}_+^*$. Let $\varepsilon > 0$. We consider a continuous function $\alpha_\varepsilon : \mathbf{R}_+ \to [0, 1]$ such that

$$\alpha_\varepsilon(t) = 1, \quad \forall t \in [0, t_0] \quad \text{and} \quad \alpha_\varepsilon(t) = 0, \quad \forall t \geq t_0 + \varepsilon.$$

We define the function

$$u_\varepsilon : \mathbf{R}_+ \to X, \quad u_\varepsilon(t) = \alpha_\varepsilon(t)\ \Phi_{B\Delta C}(\theta, t)x.$$

Then $u_\varepsilon \in C_b(\mathbf{R}_+, X)$. Using (2.1) for every $t \in [0, t_0]$, we have

$$C(\sigma(\theta, t))\Phi_{B\Delta C}(\theta, t)x = C(\sigma(\theta, t))\Phi(\theta, t)x + (L_\theta \Delta_\theta C_\theta u_\varepsilon)(t).$$

If $N \geq 1$ is given by Definition 2.4, from the above relation we obtain

$$||C(\sigma(\theta, t))\Phi_{B\Delta C}(\theta, t)x|| \leq ||C||\ N\ ||x|| + ||L_\theta||\ ||\Delta||\ |||C_\theta u_\varepsilon|||, \quad \forall t \in [0, t_0].$$

We set $\delta = \alpha(\pi)\ ||\Delta||$, and we obtain

$$M(x, \theta, t_0) \leq ||C||\ N\ ||x|| + \delta\ M(x, \theta, t_0 + \varepsilon), \quad \forall \varepsilon > 0. \tag{3.2}$$

Using the continuity of $\pi_{B\Delta C}$ we obtain that $M(x, \theta, t_0 + \varepsilon) \longrightarrow M(x, \theta, t_0)$, as $\varepsilon \to 0$. Then, for $\varepsilon \to 0$ in (3.2) we deduce that

$$M(x, \theta, t_0) \leq \frac{N\ ||C||}{1 - \delta}\ ||x||. \tag{3.3}$$

Let $m > 0$ be given by (3.1). Then, from (3.3) we have that

$$m||\Phi_{B\Delta C}(\theta, t)x|| \leq \frac{N\ ||C||}{1 - \delta}\ ||x||, \quad \forall t \in [0, t_0].$$

Since x, θ, t_0 were arbitrary, we obtain

$$||\Phi_{B\Delta C}(\theta, t)|| \leq \frac{N\ ||C||}{m(1 - \delta)}, \quad \forall (\theta, t) \in \Theta \times \mathbf{R}_+.$$

□

Theorem 3.3. *For the system $S = (\pi, B, C)$ we have*

$$r_{stab}(\pi, B, C) \geq \frac{1}{\alpha(\pi)}.$$

Proof. Let $\Delta \in \mathcal{C}_s(\Theta, \mathcal{B}(Y, U))$ with $||\Delta|| < 1/\alpha(\pi)$. We set $\delta = \alpha(\pi)\ ||\Delta||$. By Theorem 3.2, there is $M \geq 1$ such that $||\Phi_{B\Delta C}(\theta, t)|| \leq M$, for all $(\theta, t) \in \Theta \times \mathbf{R}_+$.

Let $(u, \theta) \in C_b(\mathbf{R}_+, X) \times \Theta$ and

$$Q_{u,\theta} : \mathbf{R}_+ \to X, \quad Q_{u,\theta}(t) = \int_0^t \Phi_{B\Delta C}(\sigma(\theta, s), t - s)u(s)ds.$$

For every $n \in \mathbb{N}^*$ we consider a continuous function $\alpha_n : \mathbf{R}_+ \to [0, 1]$ with $\alpha_n(t) = 1$, for all $t \in [0, n]$ and $\alpha_n(t) = 0$, for all $t \geq n + 1$. For every $n \in \mathbb{N}^*$ we define $u_n := \alpha_n u$ and we consider

$$Q_{u_n,\theta} : \mathbf{R}_+ \to X, \quad Q_{u_n,\theta}(t) = \int_0^t \Phi_{B\Delta C}(\sigma(\theta, s), t - s)u_n(s)ds.$$

For every $t \geq n + 1$, using the cocycle identity, we obtain

$$Q_{u_n,\theta}(t) = \Phi_{B\Delta C}(\sigma(\theta, n + 1), t - n - 1) \int_0^{n+1} \Phi_{B\Delta C}(\sigma(\theta, s), n + 1 - s)u_n(s)ds.$$

It follows that

$$||Q_{u_n,\theta}(t)|| \leq M||Q_{u_n,\theta}(n + 1)||, \quad \forall t \geq n + 1;$$

so $Q_{u_n,\theta} \in C_b(\mathbf{R}_+, X)$. Using the relation (2.1), we obtain that

$$(C_\theta Q_{u_n,\theta})(t) = (C_\theta P_\theta u_n)(t) \ + \ C(\sigma(\theta,t)) \int_0^t \int_0^{t-\tau} \Phi(\sigma(\theta,\tau+s), t-\tau-s)$$
$$B(\sigma(\theta,\tau+s))\Delta(\sigma(\theta,\tau+s))C(\sigma(\theta,\tau+s))\Phi_{B\Delta C}(\sigma(\theta,\tau), s)u_n(\tau)ds\ d\tau$$
$$= (C_\theta P_\theta u_n)(t) \ + \ C(\sigma(\theta,t)) \int_0^t \int_\tau^t \Phi(\sigma(\theta,\xi), t-\xi)$$
$$B(\sigma(\theta,\xi))\Delta(\sigma(\theta,\xi))C(\sigma(\theta,\xi))\Phi_{B\Delta C}(\sigma(\theta,\tau), \xi-\tau)u_n(\tau)d\xi\ d\tau.$$

Using Fubini's theorem it follows that

$$(C_\theta Q_{u_n,\theta})(t) = (C_\theta P_\theta u_n)(t) \ + \ C(\sigma(\theta,t)) \int_0^t \int_0^\xi \Phi(\sigma(\theta,\xi), t-\xi)$$
$$B(\sigma(\theta,\xi))\Delta(\sigma(\theta,\xi))C(\sigma(\theta,\xi))\Phi_{B\Delta C}(\sigma(\theta,\tau), \xi-\tau)u_n(\tau)d\tau\ d\xi$$
$$= (C_\theta P_\theta u_n)(t) + (C_\theta P_\theta B_\theta \Delta_\theta C_\theta Q_{u_n,\theta})(t), \quad \forall t \geq 0.$$

It follows that

$$||C_\theta Q_{u_n,\theta}(t)|| \leq ||C||\ K\ |||u_n||| + \delta\ |||C_\theta Q_{u_n,\theta}|||, \quad \forall t \geq 0$$

where $K = \sup_{\theta\in\Theta} ||P_\theta||$. Thus, we deduce that

$$|||C_\theta Q_{u_n,\theta}||| \leq \frac{K||C||}{1-\delta}\ |||u|||, \quad \forall n \in \mathbb{N}^*.$$

If $m > 0$ is given by (3.1), then we obtain that

$$m\ |||Q_{u_n,\theta}||| \leq \frac{K||C||}{1-\delta}\ |||u|||, \quad \forall n \in \mathbb{N}^*.$$

Since for $t > 0$ and $n = [t] + 1$ we have that $Q_{u_n,\theta}(t) = Q_{u,\theta}(t)$, using the above inequality, we obtain that

$$|||Q_{u,\theta}||| \leq \frac{K||C||}{m(1-\delta)}\ |||u|||, \quad \forall (u,\theta) \in C_b(\mathbf{R}_+, X) \times \Theta.$$

Then by Theorem 2.6, we conclude that $\pi_{B\Delta C}$ is uniformly exponentially stable, which ends the proof. □

References

1. J. A. Ball, I. Gohberg, and M. A. Kaashoek, *Two-sided Nudelman interpolation for input-output operators of discrete time-varying systems*, Integral Equations Operator Theory **21** (1995), 174–211. MR 95m:47018
2. J. A. Ball, I. Gohberg, and M. A. Kaashoek, *Input-output operators of J-unitary time-varying continuous time systems*, Oper. Theory Adv. Appl. **75** (1995), 57–94. MR 96c:47109
3. J. A. Ball, I. Gohberg, and M. A. Kaashoek, *A frequency response function for linear, time-varying systems*, Math. Control Signals Systems **8** (1995), 334–351. MR 97h:93014
4. C. Chicone and Y. Latushkin, *Evolution Semigroups in Dynamical Systems and Differential Equations*, Mathematical Surveys and Monographs, vol. 70, Amer. Math. Soc., Providence, RI, 1999. MR 2001e:47068
5. S.-N. Chow and H. Leiva, *Existence and roughness of the exponential dichotomy for linear skew-product semiflows in Banach space*, J. Differential Equations **120** (1995), 429–477. MR 97a:34121
6. S.-N. Chow and H. Leiva, *Unbounded perturbation of the exponential dichotomy for evolution equations*, J. Differential Equations **129** (1996), 509–531. MR 98d:34090

7. S. Clark, Y. Latushkin, S. Montgomery-Smith, and T. Randolph, *Stability radius and internal versus external stability in Banach spaces: an evolution semigroup approach*, SIAM J. Control Optimization **38** (2000), 1757–1793. MR 2001k:93085
8. D. Hinrichsen, A. Ilchmann, and A. J. Pritchard, *Robustness of stability of time-varying linear systems*, J. Differential Equations **82** (1989), 219–250. MR 91b:93122
9. D. Hinrichsen and A. J. Pritchard, *Robust stability of linear operators on Banach spaces*, SIAM J. Control Optim. **32** (1994), 1503–1541. MR 95i:93109
10. Y. Latushkin, S. Montgomery-Smith, and T. Randolph, *Evolutionary semigroups and dichotomy of linear skew-product flows on spaces with Banach fibers*, J. Differential Equations **125** (1996), 73–116. MR 97a:47056
11. Y. Latushkin and R. Schnaubelt, *Evolution semigroups, translation algebras and exponential dichotomy of cocycles*, J. Differential Equations **159** (1999), 321–369. MR 2000k:47054
12. M. Megan, B. Sasu, and A. L. Sasu, *On nonuniform exponential dichotomy of evolution operators in Banach spaces*, Integral Equations Operator Theory **44** (2002), 71–78. MR 2003d:47058
13. M. Megan, A. L. Sasu, and B. Sasu, *Discrete admissibility and exponential dichotomy for evolution families*, Discrete Contin. Dynam. Systems **9** (2003), 383–397. MR 2003k:47054
14. M. Megan, A. L. Sasu, and B. Sasu, *Stabilizability and controllability of systems associated to linear skew-product semiflows*, Rev. Mat. Complutense **15** (2002), 599–618. MR 2004b:93114
15. M. Megan, A. L. Sasu, and B. Sasu, *Theorems of Perron type for uniform exponential dichotomy of linear skew-product semiflows*, Bull. Belg. Math. Soc. Simon Stevin **10** (2003), 1–21.
16. M. Megan, A. L. Sasu, and B. Sasu, *Perron conditions for uniform exponential expansiveness of linear skew-product flows*, Monatsh. Math. **138** (2003), 145–157. MR 2003k:47055
17. M. Megan, A. L. Sasu, and B. Sasu, *Perron conditions for pointwise and global exponential dichotomy of linear skew-product flows*, accepted for publication in Integral Equations Operator Theory.
18. M. Megan, A. L. Sasu, and B. Sasu, *Uniform exponential dichotomy and admissibility for linear skew-product semiflows*, Oper. Theory Adv. Appl. (2003).
19. M. Megan, A. L. Sasu, and B. Sasu, *Theorems of Perron type for uniform exponential stability of linear skew-product semiflows*, accepted for publication in Dynam. Contin. Discrete Impuls. Systems.
20. R. J. Sacker and G. R. Sell, *Dichotomies for linear evolutionary equations in Banach spaces*, J. Differential Equations **113** (1994), 17–67. MR 96k:34136

Department of Mathematics, Faculty of Mathematics and Computer Science, West University of Timişoara, Romania

E-mail address: `sasu@math.uvt.ro`

Department of Mathematics, Faculty of Mathematics and Computer Science, West University of Timişoara, Romania

E-mail address: `lbsasu@yahoo.com`