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ON LOCAL REPRESENTATIONS OF VON NEUMANN ALGEBRAS

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ABSTRACT. We prove that every bounded, linear, 2-local Hilbert space representation of a von Neumann algebra is a representation. In contrast, 1-local representations may fail to be multiplicative, even at the 2 by 2 matrix algebra level.

Cohomology and representations have long been central to the understanding of the fine structure of von Neumann algebras. A derivation of a von Neumann algebra M into a dual, normal M-bimodule X is a linear map $\delta : M \to X$ satisfying $\delta(xy) = \delta(x)y + x\delta(y)$, for all $x, y \in M$. Kadison [7] and Sakai [16] proved that every derivation $\delta : M \to M$ is inner, that is, $\delta(x) = ax - xa$ for some $a \in M$. The situation is quite different if $X = B(H)$. These derivations are known to be implemented by operators $a \in B(H)$ for all von Neumann algebras except the finite ones, for which, despite significant progress, the problem remains open. Remarkably, Kirchberg [10] showed that the derivation (into $B(H)$) problem is equivalent to the celebrated similarity problem: is every bounded representation of a C^*-algebra similar to a *-representation?

Kadison [8] and Larson and Sourour [11] introduced local derivations. These are linear maps $\theta : M \to X$ such that, for every $a \in M$, there exists a derivation $\delta_a : M \to X$ satisfying $\theta(a) = \delta_a(a)$. Kadison [8] proved that every local derivation is a derivation. Building upon Larson and Sourour's results, Brešar and Šemrl [2] proved that, for separable Hilbert spaces H, every linear local automorphism of $B(H)$ is an automorphism. Šemrl [17] removed the linearity assumptions and was led to introduce 2-local derivations and 2-local automorphisms. He proved that, in the separable Hilbert space case, 2-local automorphisms (derivations) of $B(H)$ are automorphisms (derivations), no assumption on linearity, surjectivity or continuity being required. For other related results we refer to [1], [2], [3], [4], [6], [11], [12], [13], [17] and the references therein.

This paper originated in our attempt to investigate whether local representations of von Neumann algebras are representations. This was a natural thing to ask, in view of Kadison's and Kirchberg's results. While our focus was on linear maps, 2-local representations (so far relevant only in the absence of linearity) were to play a surprisingly important role. Our main result is that every bounded, linear 2-local representation of a von Neumann algebra into $B(H)$ is a representation. In contrast to this situation, 1-local representations are multiplicative only on abelian algebras.

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In section 2 we present an example of a local representation of the algebra of 2 by 2 matrices that fails to be a representation. We conclude with an application to 2-local automorphisms.

1. Local representations

In this section we begin the investigation of local representations, with the goal of going as far as possible within the 1-local property.

1.1. **Definition.** Let M be a von Neumann algebra. A bounded, linear map $\phi : M \to B(H)$ is called a local (or 1-local) representation if, for every $x \in M$, there exists a bounded representation $\pi_x : M \to B(H)$ satisfying $\phi(x) = \pi_x(x)$. (It is not assumed that M and $B(H)$ have the same identity.)

1.2. *Remark.* Let $e \in M$ be an idempotent ($e^2 = e$). Then $\phi(e) = \pi_e(e) = \pi_e(e^2) = \pi_e(e)^2 = \phi(e)^2$, which shows that $\phi(e)$ is an idempotent as well.

1.3. **Lemma.** *Let e and f be idempotents in $B(H)$ such that $ef + fe = 0$. Then $ef = fe = 0$.*

Proof. We have $0 = e(ef + fe) = ef + efe$ and $0 = (ef + fe)e = efe + fe$; hence $ef = fe$, which implies $ef = fe = 0$. $\square$

1.4. **Proposition.** *If M is an abelian von Neumann algebra, then every local representation $\phi : M \to B(H)$ is multiplicative, therefore is a representation.*

Proof. Let p and q be mutually orthogonal (self-adjoint) projections in M. Then $p + q$ is a projection and, by 1.2, we have $\phi(p) + \phi(q) = \phi(p + q) = \phi(p + q)^2 = (\phi(p) + \phi(q))^2 = \phi(p)^2 + \phi(q)^2 + \phi(p)\phi(q) + \phi(q)\phi(p)$, which shows that $\phi(p)\phi(q) + \phi(q)\phi(p) = 0$. Now 1.3 implies that $\phi(p)\phi(q) = 0$.

Let p and q be commuting projections in M. Then $p = p_1 + r$ and $q = q_1 + r$, where p_1, q_1, and r are mutually orthogonal projections. From the first part of the proof we have $\phi(p)\phi(q) = (\phi(p_1) + \phi(r))(\phi(q_1) + \phi(r)) = \phi(r)^2 = \phi(r) = \phi(pq)$. Since every von Neumann algebra is equal to the norm-closed linear span of its projections, the conclusion follows. $\square$

We now turn our attention to the non-commutative case. Let M be the 2 by 2 matrix algebra, generated by two mutually orthogonal, equivalent projections e and f and partial isometries v and v^* satisfying

$$ef = 0,\ v^*v = e,\ vv^* = f,\ v^2 = 0,\ (v^*)^2 = 0,\ ve = v,\ vf = 0,$$
$$fv = v,\ ev = 0,\ ev^* = v^*,\ fv^* = 0,\ v^*f = v^*,\ v^*e = 0.$$

1.5. **Lemma.** *Let $\phi : M \to B(H)$ be a local representation. Then the operators $\phi(e), \phi(f), \phi(v)$, and $\phi(v^*)$ satisfy the relations*

$$\phi(v)^2 = \phi(v^*)^2 = 0,$$
$$\phi(v) = \phi(e)\phi(v) + \phi(v)\phi(e) = \phi(f)\phi(v) + \phi(v)\phi(f),$$
$$\phi(v^*) = \phi(e)\phi(v^*) + \phi(v^*)\phi(e) = \phi(f)\phi(v^*) + \phi(v^*)\phi(f),$$
$$\phi(v)\phi(e) = \phi(f)\phi(v);\ \phi(v)\phi(f) = \phi(e)\phi(v),$$
$$\phi(v^*)\phi(e) = \phi(f)\phi(v^*);\ \phi(v^*)\phi(f) = \phi(e)\phi(v^*),$$
$$\phi(v)\phi(v^*) + \phi(v^*)\phi(v) = \phi(e) + \phi(f).$$

Proof. Let π_v be the representation satisfying $\pi_v(v) = \phi(v)$. Then $\phi(v)^2 = \pi_v(v)^2 = \pi_v(v^2) = \pi_v(0) = 0$. Similarly, $\phi(v^*)^2 = 0$. We apply 1.2 to the idempotents $e + v, f + v, e + v^*$, and $f + v^*$:

$$\begin{aligned}\phi(e) + \phi(v) = \phi(e + v) = \phi(e + v)^2 &= (\phi(e) + \phi(v))^2\\ &= \phi(e)^2 + \phi(v)^2 + \phi(e)\phi(v) + \phi(v)\phi(e),\end{aligned}$$

which implies $\phi(v) = \phi(e)\phi(v) + \phi(v)\phi(e)$. Similar computations show that $\phi(v) = \phi(f)\phi(v) + \phi(v)\phi(f)$, $\phi(v^*) = \phi(e)\phi(v^*) + \phi(v^*)\phi(e)$, and $\phi(v^*) = \phi(f)\phi(v^*) + \phi(v^*)\phi(f)$.

From this it follows that

$$\begin{aligned}\phi(v)\phi(e) &= (\phi(f)\phi(v) + \phi(v)\phi(f))\phi(e) = \phi(f)\phi(v)\phi(e),\\ \phi(f)\phi(v) &= \phi(f)(\phi(e)\phi(v) + \phi(v)\phi(e)) = \phi(f)\phi(v)\phi(e);\end{aligned}$$

hence $\phi(v)\phi(e) = \phi(f)\phi(v)$. Similarly, $\phi(v)\phi(f) = \phi(e)\phi(v)$, $\phi(v^*)\phi(e) = \phi(f)\phi(v^*)$, $\phi(v^*)\phi(f) = \phi(e)\phi(v^*)$.

We now apply 1.2 to the projection $p = \frac{1}{2}(e + f + v + v^*)$ and use the results obtained so far. $\phi(p) = \phi(p)^2$ implies

$$\begin{gathered}2(\phi(e) + \phi(f) + \phi(v) + \phi(v^*)) =\\ \phi(e)^2 + \phi(f)^2 + \phi(v)^2 + \phi(v^*)^2 + \phi(e)\phi(f) + \phi(e)\phi(v)+\\ \phi(e)\phi(v^*) + \phi(f)\phi(e) + \phi(f)\phi(v) + \phi(f)\phi(v^*) + \phi(v)\phi(e)+\\ \phi(v)\phi(f) + \phi(v)\phi(v^*) + \phi(v^*)\phi(e) + \phi(v^*)\phi(f) + \phi(v^*)\phi(v),\\ 2(\phi(e) + \phi(f) + \phi(v) + \phi(v^*)) = \phi(e) + \phi(f) + 2\phi(v) + 2\phi(v^*)+\\ \phi(v)\phi(v^*) + \phi(v^*)\phi(v),\\ \phi(v)\phi(v^*) + \phi(v^*)\phi(v) = \phi(e) + \phi(f).\end{gathered}$$

□

1.6. **Lemma.** *If $S, T \in M$ and $ST = TS$, then $\phi(ST) = \phi(S)\phi(T)$.*

Proof. Let $S = a_1e + b_1v^* + c_1v + d_1f$ and $T = a_2e + b_2v^* + c_2v + d_2f$. Then $ST = (a_1a_2 + b_1c_2)e + (a_1b_2 + b_1d_2)v^* + (c_1a_2 + d_1c_2)v + (c_1b_2 + d_1d_2)f$ and $TS = (a_2a_1 + b_2c_1)e + (a_2b_1 + b_2d_1)v^* + (c_2a_1 + d_2c_1)v + (c_2b_1 + d_2d_1)f$.

$$\begin{gathered}\phi(S)\phi(T) = a_1a_2\phi(e) + a_1b_2\phi(e)\phi(v^*) + a_1c_2\phi(e)\phi(v)+\\ b_1a_2\phi(v^*)\phi(e) + b_1c_2\phi(v^*)\phi(v) + b_1d_2\phi(v^*)\phi(f) + c_1a_2\phi(v)\phi(e) + c_1b_2\phi(v)\phi(v^*)+\\ c_1d_2\phi(v)\phi(f) + d_1b_2\phi(f)\phi(v^*) + d_1c_2\phi(f)\phi(v) + d_1d_2\phi(f).\end{gathered}$$

$ST = TS$ implies $b_1c_2 = b_2c_1, a_1b_2 + b_1d_2 = a_2b_1 + b_2d_1$, and $a_2c_1 + c_2d_1 = a_1c_2 + c_1d_2$. Hence, by using the relations 1.5, we obtain

$$\begin{gathered}\phi(S)\phi(T) = (a_1a_2 + b_1c_2)\phi(e) + (b_2c_1 + d_1d_2)\phi(f) + (a_1b_2 + b_1d_2)\phi(v^*)+\\ (a_2c_1 + c_2d_1)\phi(v) = \phi(ST).\end{gathered}$$

□

2. A COUNTEREXAMPLE

The computations in section 1 and the obstructions therein suggest the following example of a local representation of the algebra of 2 by 2 matrices, which fails to be a representation. Define $\phi : \mathbf{M}_2(\mathbf{C}) \to B(\mathbf{C}^4)$ by

$$\phi\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) = \begin{pmatrix} a & 0 & b & 0 \\ 0 & a & 0 & c \\ c & 0 & d & 0 \\ 0 & b & 0 & d \end{pmatrix}.$$

For $X = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $Y = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$, it is easy to check that $\phi(XY) \neq \phi(X)\phi(Y)$, so ϕ is not a representation.

To show that ϕ is a local representation, it is enough to prove that $I_2 \otimes X$ and $\phi(X)$ are similar for all $X \in \mathbf{M}_2(\mathbf{C})$. This is a routine exercise in linear algebra: two square matrices A and B are similar if and only if they have the same canonical Jordan form. Equivalently, they have the same characteristic polynomial and, for every common eigenvalue λ, the matrices $\lambda I - A$ and $\lambda I - B$ have the same rank. If $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $\begin{pmatrix} a & 0 & b & 0 \\ 0 & a & 0 & b \\ c & 0 & d & 0 \\ 0 & c & 0 & d \end{pmatrix}$ and $\begin{pmatrix} a & 0 & b & 0 \\ 0 & a & 0 & c \\ c & 0 & d & 0 \\ 0 & b & 0 & d \end{pmatrix}$ have the same characteristic polynomial $P(\lambda) = (\lambda^2 - (a+d)\lambda + ad - bc)^2$. Note that, for $t \in \mathbf{C}$, X and Y are similar if and only if $X + tI$ and $Y + tI$ are similar. We distinguish several cases:

1. If $a = d$, subtract aI. Then the operators
$$\begin{pmatrix} 0 & 0 & b & 0 \\ 0 & 0 & 0 & b \\ c & 0 & 0 & 0 \\ 0 & c & 0 & 0 \end{pmatrix} \text{ and } \begin{pmatrix} 0 & 0 & b & 0 \\ 0 & 0 & 0 & c \\ c & 0 & 0 & 0 \\ 0 & b & 0 & 0 \end{pmatrix}$$
are unitary equivalent via the unitary
$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$$
2. If $a \neq d$, we may assume (by subtracting dI and dividing by $a - d$) that $a = 1$ and $d = 0$, and thus reduce the problem to showing that
$$A = \begin{pmatrix} 1 & 0 & b & 0 \\ 0 & 1 & 0 & b \\ c & 0 & 0 & 0 \\ 0 & c & 0 & 0 \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 0 & b & 0 \\ 0 & 1 & 0 & c \\ c & 0 & 0 & 0 \\ 0 & b & 0 & 0 \end{pmatrix} \text{ are similar.}$$
3. If $b = c = 0$, there is nothing to prove.
4. If one of b and c is 0, say $c = 0, b \neq 0$, then both matrices have characteristic polynomial $P(\lambda) = (\lambda(\lambda-1))^2$. Similarity follows once we prove that $\lambda I - A$ and $\lambda I - B$ have the same rank, $\lambda = 0, 1$.

If $\lambda = 0$, both $\begin{pmatrix} 1 & 0 & b & 0 \\ 0 & 1 & 0 & b \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ and $\begin{pmatrix} 1 & 0 & b & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & b & 0 & 0 \end{pmatrix}$ have rank 2. If $\lambda = 1$, both $\begin{pmatrix} 0 & 0 & -b & 0 \\ 0 & 0 & 0 & -b \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ and $\begin{pmatrix} 0 & 0 & -b & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -b & 0 & 1 \end{pmatrix}$ have rank 2.

(5) If $b \neq 0, c \neq 0$, then $I_2 \otimes X$ and $\phi(X)$ have the same characteristic polynomial $P(\lambda) = (\lambda(\lambda - 1) - bc)^2$. If $P(\lambda) = 0$, then $\lambda \neq 0, 1$, and elementary row operations show that $\begin{pmatrix} \lambda - 1 & 0 & -b & 0 \\ 0 & \lambda - 1 & 0 & -b \\ -c & 0 & \lambda & 0 \\ 0 & -c & 0 & \lambda \end{pmatrix}$ has the same rank as $\begin{pmatrix} \lambda(\lambda - 1) & 0 & -\lambda b & 0 \\ 0 & \lambda(\lambda - 1) & 0 & -\lambda b \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$, which is 2. In the same fashion, $\begin{pmatrix} \lambda - 1 & 0 & -b & 0 \\ 0 & \lambda - 1 & 0 & -c \\ -c & 0 & \lambda & 0 \\ 0 & -b & 0 & \lambda \end{pmatrix}$ and $\begin{pmatrix} \lambda(\lambda - 1) & 0 & -\lambda b & 0 \\ 0 & \lambda(\lambda - 1) & 0 & -\lambda c \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ have rank 2. This completes the proof.

3. 2-LOCAL REPRESENTATIONS

Having obtained as much as possible from 1-local representations, we now address the question of whether the 2-local property yields any additional information.

3.1. **Definition.** A bounded, linear map $\phi : M \to B(H)$ is called a 2-local representation if, for every $x, y \in M$, there exists a bounded representation $\pi_{x,y} : M \to B(H)$ such that $\phi(x) = \pi_{x,y}(x)$ and $\phi(y) = \pi_{x,y}(y)$.

3.2. *Remark.* Let $\phi : M \to B(H)$ be a 2-local representation and $x, y \in M$. If $xy = 0$, then $\phi(x)\phi(y) = 0$.

Proof. $\phi(x)\phi(y) = \pi_{x,y}(x)\pi_{x,y}(y) = \pi_{x,y}(xy) = 0$. □

3.3. The above property allows us to complete relations 1.5. If $M = \mathbf{M}_2(\mathbf{C})$ and if $\phi : M \to B(H)$ is a 2-local representation, then the relations immediately preceding Lemma 1.5 show that

$$\phi(v)\phi(f) = \phi(e)\phi(v) = \phi(f)\phi(v^*) = \phi(v^*)\phi(e) = 0.$$

Relations 1.5 become

$$\begin{gathered} \phi(v) = \phi(v)\phi(e) = \phi(f)\phi(v), \\ \phi(v^*) = \phi(e)\phi(v^*) = \phi(v^*)\phi(f), \\ \phi(v^*)\phi(v) = \phi(e), \ \phi(v)\phi(v^*) = \phi(f). \end{gathered}$$

We need to prove only the last two. From the first two and 1.5 we have

$$\begin{aligned}\phi(v^*)\phi(v) &= \phi(e)\phi(v^*)\phi(v)\phi(e)\\ &= \phi(e)(\phi(e)+\phi(f)-\phi(v)\phi(v^*))\phi(e)\\ &= \phi(e)+\phi(e)\phi(f)\phi(e)-\phi(e)\phi(v)\phi(v^*)\phi(e)=\phi(e).\end{aligned}$$

A similar argument establishes $\phi(v)\phi(v^*)=\phi(f)$.

3.4. **Proposition.** *Every 2-local representation $\phi : \mathbf{M}_2(\mathbf{C}) \to B(H)$ is a representation.*

Proof. We use the notation in 1.6. Recall that $S = a_1e + b_1v^* + c_1v + d_1f,\ T = a_2e + b_2v^* + c_2v + d_2f$ and $ST = (a_1a_2 + b_1c_2)e + (a_1b_2 + b_1d_2)v^* + (c_1a_2 + d_1c_2)v + (c_1b_2 + d_1d_2)f$. Relations 3.3, applied to 1.6, lead to

$$\begin{gathered}\phi(S)\phi(T) = a_1a_2\phi(e) + a_1b_2\phi(e)\phi(v^*) + a_1c_2\phi(e)\phi(v)+\\ b_1a_2\phi(v^*)\phi(e) + b_1c_2\phi(v^*)\phi(v) + b_1d_2\phi(v^*)\phi(f) + c_1a_2\phi(v)\phi(e) + c_1b_2\phi(v)\phi(v^*)+\\ c_1d_2\phi(v)\phi(f) + d_1b_2\phi(f)\phi(v^*) + d_1c_2\phi(f)\phi(v) + d_1d_2\phi(f)\\ = a_1a_2\phi(e) + a_1b_2\phi(v^*) + b_1c_2\phi(e) + b_1d_2\phi(v^*) + c_1a_2\phi(v)+\\ c_1b_2\phi(f) + d_1c_2\phi(v) + d_1d_2\phi(f)\\ = (a_1a_2 + b_1c_2)\phi(e) + (a_1b_2 + b_1d_2)\phi(v^*) + (c_1a_2 + d_1c_2)\phi(v)+\\ (c_1b_2 + d_1d_2)\phi(f) = \phi(ST).\end{gathered}$$

□

3.5. **Corollary.** *Let M be the direct sum of an abelian von Neumann algebra, and a type* I_2 *factor. Then every 2-local representation $\phi : M \to B(H)$ is multiplicative.*

Proof. Let e be the central projection in M such that $M(I-e)$ is isomorphic to $\mathbf{M}_2(\mathbf{C})$ and Me is abelian. If $x, y \in M$, then $x = a_1 + x_1$ and $y = a_2 + x_2$, where $a_i \in Me, x_i \in M(I-e),\ i = 1,2$. Since $\phi(a_1a_2) = \phi(a_1)\phi(a_2),\ \phi(x_1x_2) = \phi(x_1)\phi(x_2),\ \phi(a_i) = \phi(e)\phi(a_i)\phi(e),\ \phi(x_i) = \phi(f)\phi(x_i)\phi(f),\ i = 1,2$, we get $\phi(x)\phi(y) = \phi(xy)$. □

3.6. **Corollary.** *Let M be a von Neumann algebra, and let $\phi : M \to B(H)$ be a 2-local representation. Then ϕ is a representation.*

Proof. The von Neumann algebra generated by any two projections e and f in M is either abelian, of type I_2, or the direct sum of an abelian and a type I_2 ([9], 12.4.11). From 1.4, 3.4 and 3.5 we infer that $\phi(ef) = \phi(e)\phi(f)$. Since M equals the norm-closed linear span of its projections, ϕ is multiplicative on M. □

We conclude with some remarks on 2-local automorphisms of von Neumann algebras. From [11], [2], [17] we know that every 2-local automorphism of $B(H)$ (H separable) is an automorphism. In fact, no linearity, surjectivity or continuity are assumed. We generalize this result, in the context of linear maps.

3.7. **Definition.** Let M be a von Neumann algebra. A bounded, linear map $\phi : M \to M$ is called a 2-local automorphism of M if, for every $x, y \in M$, there exists a bounded, linear (not necessarily self-adjoint) automorphism $\pi_{x,y}$ of M such that $\phi(x) = \pi_{x,y}(x)$ and $\phi(y) = \pi_{x,y}(y)$.

Recall that a von Neumann algebra M is singly generated if there is an operator $S \in M$ such that M equals the ultraweak closure of the algebra generated by I, S, and S^*.

3.8. **Proposition.** *Let M be a singly generated von Neumann algebra. Then every ultraweakly continuous 2-local automorphism of M is an automorphism.*

Proof. Let $\phi : M \to M$ be an ultraweakly continuous 2-local automorphism. Then, by 3.6, ϕ is a representation. Let π_I be the automorphism of M such that $\pi_I(I) = \phi(I)$. Then $\pi_I(I) = I$; hence $\phi(I) = I$. If $\phi(x) = 0$, then $\phi(x) = \pi_x(x)$ for some automorphism π_x. This shows that $x = 0$, so ϕ is injective. If S is the generator of M, there exists an automorphism $\pi = \pi_S$ of M such that $\phi(S) = \pi(S)$ and $\phi(S^*) = \pi(S^*)$. Then ϕ and π are equal on the algebra generated by I, S, and S^*. It is well known that π is ultraweakly continuous. In addition, ultraweak continuity of ϕ shows that the range of ϕ must be the entire M, which concludes the proof. □

3.9. The class of singly generated von Neumann algebras contains all type I algebras [14] and all properly infinite ones [18]. While it is still unknown whether every finite von Neumann algebra is singly generated, the answer is affirmative for at least three important classes: finite algebras containing Cartan subalgebras [15], type II_1 factors with property Γ and tensor products of type II_1 factors [5].

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