PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 132, Number 12, Pages 3751–3752
S 0002-9939(04)07608-7
Article electronically published on July 14, 2004

ERRATUM TO "SUPERCYCLIC AND CHAOTIC TRANSLATION SEMIGROUPS"

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(Communicated by Joseph A. Ball)

In [1], the proof of Theorem 3 in Section 4 is incorrect, as pointed out to us by Alfredo Peris. In proving that "(2) implies (1)", v_p has not been confirmed to belong to the space $C_{0,\rho}(I)$. As for the characterizations of chaoticity of the semigroup in the space $C_{0,\rho}(I)$, we give the following Theorem 3′ instead of Theorems 3 and 4 in [1]. As the result, Example 2 in [1] does not give a chaotic translation semigroup.

Theorem 3′. *Let I be the interval $(-\infty,\infty)$ (resp. $I=[0,\infty)$), and let X be $C_{0,\rho}(I)$. For the translation semigroup $\{T(t)\}$ on X, the following assertions are equivalent*:

(1) $\lim_{\tau\to\pm\infty}\rho(\tau)=0$ (*resp.* $\tau\to\infty$);
(2) $\{T(t)\}$ *is chaotic.*

Proof. (1) implies (2): It is clear that $\{T(t)\}$ is hypercyclic by Theorem A in [1]. So we only have to show that the set X_{per} of periodic points is dense in X. Since the set $X_{0,0}$ of all the functions with compact support is dense in X, we shall show that for any $z\in X_{0,0}$ with $\mathrm{supp}(z)\subset[-l,l]$ and $\varepsilon>0$, there exists $v\in X_{per}$ such that $\|z-v_p\|<\varepsilon$.

From Lemma B in [1], there exists $M_{2l}\geq 1$ satisfying

$$\frac{1}{M_{2l}}\rho(\sigma-l)\leq\rho(\tau)\leq M_{2l}\rho(\sigma+l)$$

for any $\sigma\in I$ and any $\tau\in[\sigma-l,\sigma+l]$.

Take ε' such as $0<\varepsilon'<\frac{\rho(-l)}{M_{2l}^2\|z\|}\varepsilon$. By (1), there exists τ_0 such that $\rho(\tau)<\varepsilon'$ for any $|\tau|>\tau_0$. Let P satisfy $P>\max\{\tau_0+l,2l\}$. We shall put $v_p(\tau)=\sum_{n\in\mathbf{Z}}z(\tau-nP)$. Then clearly $T(P)v_p=v_p$ and $\rho(l+nP)<\varepsilon'$ for $n\in\mathbf{Z}\setminus\{0\}$. We calculate $\|z-v_p\|$:

$$\begin{aligned}\|z-v_p\|=\left\|\sum_{n\in\mathbf{Z}\setminus\{0\}}z(\tau-nP)\right\| &\leq \sup_{n\in\mathbf{Z}\setminus\{0\}}\ \sup_{\tau\in[-l+nP,\ l+nP]}\rho(\tau)\cdot|z(\tau-nP)|\\ &\leq \sup_{n\in\mathbf{Z}\setminus\{0\}}\frac{M_{2l}^2\rho(l+nP)}{\rho(-l)}\cdot\|z\|<\varepsilon.\end{aligned}$$

Received by the editors December 3, 2003.
2000 *Mathematics Subject Classification.* Primary 47D60; Secondary 47D06.
Key words and phrases. Admissible weight function, supercyclic semigroups, chaotic semigroups.

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(In the case when $I = [0, \infty)$, replace $-l$, $2l$, $\mathbf{Z}$ and $\mathbf{Z} \setminus \{0\}$ with 0, l, $\mathbf{Z}_+$ and $\mathbf{N}$.) Since $v_p \in C_{0,\rho}(I)$ follows from (1) and v_p belongs to X_{per}, the set X_{per} is dense in X. Therefore $\{T(t)\}$ is chaotic.

(2) implies (1): We shall consider the case $I = [0, \infty)$. Let $z \in X$ satisfy $z(0) = 1$ and $z(\tau) = 0$ for $\tau \geq 1$.

Since $\{T(t)\}$ is chaotic, there exists $v \in X_{per}$ such that $\|z - v\| < \rho(0)/2$ and $P > 0$ such that $v(\tau) = v(\tau + nP)$ for any $n \in \mathbf{N}$. Then

$$v(0)\rho(0) \geq z(0)\rho(0) - \rho(0)/2 \geq \rho(0) - \rho(0)/2 = \rho(0)/2.$$

Therefore $v(0) \geq \frac{1}{2}$.

By Lemma B in [1], there exists M_P satisfying

$$(*) \qquad \frac{1}{M_P}\rho(nP) \leq \rho(\tau) \leq M_P \rho((n+1)P)$$

for any $n \in \mathbf{N}$ and any $\tau \in [nP, (n+1)P]$.

Since v belongs to X, $\lim_{\tau \to \infty} v(\tau)\rho(\tau) = 0$ holds. So for any $\varepsilon > 0$, putting $\varepsilon_1 = \frac{\varepsilon}{2M_P}$, there exists τ_0 such that $|v(\tau)\rho(\tau)| < \varepsilon_1$ for $\tau > \tau_0$. If $|nP| > \tau_0$, then $|v(nP)\rho(nP)| < \varepsilon_1$ holds. By the relation $v(0) = v(nP)$, we have

$$\rho(nP) < \frac{\varepsilon_1}{|v(0)|} < 2\varepsilon_1.$$

For $\tau > \tau_0$, there exists $n \in \mathbf{Z}$ such that $nP \leq \tau < (n+1)P$. By (*), we have

$$\rho(\tau) < M_P \rho((n+1)P) < M_P \cdot 2\varepsilon_1 = \varepsilon,$$

which implies that $\lim_{\tau \to \infty} \rho(\tau) = 0$. In the case when $I = (-\infty, \infty)$, we can prove the theorem in a similar way. □

Acknowledgement

The authors would like to thank Alfredo Peris for pointing out the mistake in [1].

References

1. Mai Matsui, Mino Yamada and Fukiko Takeo, *Supercyclic and chaotic translation semigroups*, Proc. Amer. Math. Soc. **131** (2003), 3535–3546. MR 2004c:47018

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