ERRATUM TO “SUPERCYCLIC AND CHAOTIC TRANSLATION SEMIGROUPS”

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(Communicated by Joseph A. Ball)

In [1], the proof of Theorem 3 in Section 4 is incorrect, as pointed out to us by Alfredo Peris. In proving that “(2) implies (1)” , \( v_p \) has not been confirmed to belong to the space \( C_{0,\rho}(I) \). As for the characterizations of chaoticity of the semigroup in the space \( C_{0,\rho}(I) \), we give the following Theorem 3’ instead of Theorems 3 and 4 in [1]. As the result, Example 2 in [1] does not give a chaotic translation semigroup.

**Theorem 3’.** Let \( I \) be the interval \((-\infty, \infty)\) (resp. \( I = [0, \infty) \)), and let \( X \) be \( C_{0,\rho}(I) \). For the translation semigroup \( \{T(t)\} \) on \( X \), the following assertions are equivalent:

1. \( \lim_{\tau \to \pm \infty} \rho(\tau) = 0 \) (resp. \( \tau \to \infty \));
2. \( \{T(t)\} \) is chaotic.

**Proof.** (1) implies (2): It is clear that \( \{T(t)\} \) is hypercyclic by Theorem A in [1]. So we only have to show that the set \( X_{\text{per}} \) of periodic points is dense in \( X \). Since the set \( X_{0,0} \) of all the functions with compact support is dense in \( X \), we shall show that for any \( z \in X_{0,0} \) with \( \text{supp}(z) \subset [-l, l] \) and \( \varepsilon > 0 \), there exists \( v \in X_{\text{per}} \) such that \( \|z - v_p\| < \varepsilon \).

From Lemma B in [1], there exists \( M_{2l} \geq 1 \) satisfying

\[
\frac{1}{M_{2l}} \rho(\sigma - l) \leq \rho(\tau) \leq M_{2l} \rho(\sigma + l)
\]

for any \( \sigma \in I \) and any \( \tau \in [\sigma - l, \sigma + l] \).

Take \( \varepsilon' \) such as \( 0 < \varepsilon' < \frac{\rho(l)}{M_{2l} \|z\|} \). By (1), there exists \( \tau_0 \) such that \( \rho(\tau) < \varepsilon' \) for any \( |\tau| > \tau_0 \). Let \( P \) satisfy \( P > \max\{\tau_0 + l, 2l\} \). We shall put \( v_p(\tau) = \sum_{n \in \mathbb{Z}} z(\tau - nP) \). Then clearly \( T(P)v_p = v_p \) and \( \rho(l + nP) < \varepsilon' \) for \( n \in \mathbb{Z} \setminus \{0\} \). We calculate \( \|z - v_p\|\):

\[
\|z - v_p\| = \left\| \sum_{n \in \mathbb{Z} \setminus \{0\}} z(\tau - nP) \right\| \leq \sup_{n \in \mathbb{Z} \setminus \{0\}} \sup_{\tau \in [-l + nP, l + nP]} \rho(\tau) \cdot \|z(\tau - nP)\| \leq \sup_{n \in \mathbb{Z} \setminus \{0\}} \frac{M_{2l}^2 \rho(l + nP)}{\rho(l)} \cdot \|z\| < \varepsilon.
\]
(In the case when $I = [0, \infty)$, replace $-l$, $2l$, $Z$ and $Z \setminus \{0\}$ with $0$, $l$, $Z_+$ and $N$.) Since $v_p \in C_{0,\rho}(I)$ follows from (1) and $v_p$ belongs to $X_{\text{per}}$, the set $X_{\text{per}}$ is dense in $X$. Therefore $\{T(t)\}$ is chaotic.

(2) implies (1): We shall consider the case $I = [0, \infty)$. Let $z_2 \in X$ satisfy $z(0) = 1$ and $z(t) = 0$ for $t > 1$.

Since $\{T(t)\}$ is chaotic, there exists $v \in X_{\text{per}}$ such that $\|z - v\| < \rho(0)/2$ and $P > 0$ such that $v(\tau) = v(\tau + nP)$ for any $n \in \mathbb{N}$. Then

$$v(0)\rho(0) \geq z(0)\rho(0) - \rho(0)/2 \geq \rho(0) - \rho(0)/2 = \rho(0)/2.$$ 

Therefore $v(0) \geq \frac{1}{2}$.

By Lemma B in [1], there exists $M_P$ satisfying

$$(*) \quad \frac{1}{M_P}\rho(nP) \leq \rho(\tau) \leq M_P\rho((n + 1)P)$$

for any $n \in \mathbb{N}$ and any $\tau \in [nP, (n + 1)P]$.

Since $v$ belongs to $X$, $\lim_{\tau \to \infty} v(\tau)\rho(\tau) = 0$ holds. So for any $\varepsilon > 0$, putting $\varepsilon_1 = \frac{\varepsilon}{2M_P}$, there exists $\tau_0$ such that $|v(\tau)\rho(\tau)| < \varepsilon_1$ for $\tau > \tau_0$. If $|nP| > \tau_0$, then $|v(nP)\rho(nP)| < \varepsilon_1$ holds. By the relation $v(0) = v(nP)$, we have

$$\rho(nP) < \frac{\varepsilon_1}{|v(0)|} < 2\varepsilon_1.$$ 

For $\tau > \tau_0$, there exists $n \in \mathbb{Z}$ such that $nP \leq \tau < (n + 1)P$. By (*), we have

$$\rho(\tau) < M_P\rho((n + 1)P) < M_P \cdot 2\varepsilon_1 = \varepsilon,$$

which implies that $\lim_{\tau \to \infty} \rho(\tau) = 0$. In the case when $I = (-\infty, \infty)$, we can prove the theorem in a similar way.

Acknowledgement

The authors would like to thank Alfredo Peris for pointing out the mistake in [1].

References