

EVALUATION FIBRATIONS AND TOPOLOGY OF SYMPLECTOMORPHISMS

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ABSTRACT. There are two main results. The first states that isotropy subgroups of groups acting transitively on rationally hyperbolic spaces have infinitely generated rational cohomology algebra. Using this fact, we prove that the analogous statement holds for groups of symplectomorphisms of certain blowups.

1. INTRODUCTION

Let G be a topological group acting effectively and transitively on a topological space X . Denote by G_{pt} the isotropy subgroup for the point $pt \in X$. Let us consider the following **evaluation fibration** associated with the action

$$G_{pt} \rightarrow G \xrightarrow{ev} X.$$

Here $ev(\phi) = \phi(pt)$, where $pt \in X$ is a point. Our first aim is to prove the following.

Theorem 1.1. *If a topological group G acts transitively on a simply connected rationally hyperbolic space X , then the rational cohomology of the isotropy subgroup of a point $H^*(G_{pt}; \mathbb{Q})$ is infinitely generated as an algebra.*

The proof uses several known facts from rational homotopy theory, and here we sketch the argument. The first step is to show that the map induced by the evaluation on homotopy groups has a finite-dimensional image. This is achieved by the application of the Gottlieb theory (Theorem 2.4) under the assumption that the space X has finite Lusternik-Schnirelmann category. The second step is to use the existence of an exact sequence of, so-called, dual rational homotopy groups (Theorem 2.3) associated to the fibration. Then it follows from the first step that all but the finite-dimensional part of the rational homotopy of X comes from the rational homotopy of the isotropy subgroup. Since X is rationally hyperbolic, i.e., its rational homotopy is infinite dimensional, then so is the rational homotopy of the isotropy subgroup. Topological groups are H-spaces. Thus their dual rational homotopy generates the rational cohomology, and we obtain that the latter must be infinitely generated as an algebra.

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As an application of the above result we prove the following.

Theorem 1.2. *Let (M, ω) be a compact simply connected symplectic 4-manifold that is neither rational nor a ruled surface up to blowup. Suppose that $b_2 = \dim H^2(M) > 2$. Then the rational cohomology $H^*(\text{Symp}(\widetilde{M}_\varepsilon))$ of the symplectomorphism group of the symplectic blowup of (M, ω) is infinitely generated (as an algebra), for every sufficiently small $\varepsilon > 0$.*

In all known (very specific) examples, the rational cohomology rings of symplectomorphism groups are finitely generated [1, 2]. The above result shows that the topology of groups of symplectomorphisms is complicated in general.

2. RATIONAL HOMOTOPY METHODS

This section is devoted to recalling some facts and notions of rational homotopy theory. We refer to [3, 4, 5] for detailed expositions.

2.1. Minimal models. Given a connected topological space X , there is associated a free differential graded algebra $\mathcal{M}_X = (\Lambda V, d)$ such that

- (1) there exists a basis $\{v_\alpha\}_{\alpha \in J}$ of V for some well-ordered set J such that

$$d(v_\alpha) \in \Lambda V_{<\alpha} \quad \text{and} \quad \text{deg}(v_\alpha) < \text{deg}(v_\beta) \Rightarrow \alpha < \beta,$$

where $\Lambda V_{<\alpha}$ is a subalgebra in ΛV generated by all v_β for $\beta < \alpha$,

- (2) there exists a DGA-morphism $\varphi_X : \mathcal{M}_X \rightarrow A(X)$ that induces an isomorphism on cohomology. Here $A(X)$ denotes Sullivan’s rational forms. \mathcal{M}_X is called a **minimal model** of X .

The idea of minimal models can be extended to fibrations. Namely, one may try to write a minimal model of the total space of a fibration in terms of the minimal models of the fiber and the base. While this fails in general, however, it gives a certain model of the fibration as stated in the following result [3, Theorem (2.5.1)].

Theorem 2.1 (Grivel-Halperin-Thomas). *Let $\pi : E \rightarrow B$ be a Serre fibration of path connected spaces, and let $F = \pi^{-1}(b)$ be the fiber over the base point b . Suppose that:*

- (1) F is path connected,
- (2) $\pi_1(B)$ acts nilpotently on $H^k(F)$ for all $k \geq 1$,
- (3) either B or F has finite \mathbb{K} -type.¹

Then there exists a KS-model² of the fibration

$$\begin{array}{ccccc} A(B) & \xrightarrow{A(\pi)} & A(E) & \xrightarrow{A(i)} & A(F) \\ \uparrow \varphi_B & & \uparrow \varphi & & \uparrow \varphi_F \\ \mathcal{M}_B & \longrightarrow & \mathcal{M}_B \otimes \mathcal{M}_F & \longrightarrow & \mathcal{M}_F \end{array}$$

in which $\varphi_B : \mathcal{M}_B \rightarrow A(B)$ is a minimal model of B , $\varphi_F : \mathcal{M}_F \rightarrow A(F)$ is a minimal model of F and $\varphi : \mathcal{M}_B \otimes \mathcal{M}_F \rightarrow A(E)$ induces an isomorphism on cohomology. □

¹Recall that a path connected X has finite \mathbb{K} -type if $H^k(X; \mathbb{K})$ is finite dimensional for $k \geq 1$.
²KS stands for Koszul and Sullivan.

2.2. Dual rational homotopy groups. Let $\mathcal{M}_X = (\Lambda V, d)$ be a minimal model of the space X . Recall that $V = \bigoplus_{k>0} V^k$ is the graded rational vector space. The subspaces V^k are called the **dual rational homotopy groups** of X and denoted by $\Pi^k(X)$. Notice that Π^* is a contravariant functor from the category of path connected topological spaces to the category of graded rational vector spaces (see [3] for details). The next two theorems are also taken from the book [3] (Theorem (2.3.7) and Corollary (2.5.2)).

Theorem 2.2. *Let X be a nilpotent space of finite \mathbb{Q} -type³. Then for each $k \geq 2$, there is a natural isomorphism*

$$\Pi^k(X) \cong \text{Hom}_{\mathbb{Z}}(\pi_k(X), \mathbb{Q}).$$

Furthermore, this holds for $k = 1$, provided that $\pi_1(X)$ is abelian. \square

Theorem 2.3. *Under the conditions of Theorem 2.1, there is a long exact sequence of dual homotopy groups*

$$\begin{aligned} 0 \rightarrow \Pi^1(B) \rightarrow \Pi^1(E) \rightarrow \Pi^1(F) \rightarrow \Pi^2(B) \rightarrow \dots \\ \rightarrow \Pi^k(B) \rightarrow \Pi^k(E) \rightarrow \Pi^k(F) \rightarrow \Pi^{k+1}(B) \rightarrow \dots \end{aligned}$$

\square

2.3. Gottlieb theory. Let X be a path connected topological space, and let $HE(X)$ be the space of self-homotopy equivalences of X . Then we have the evaluation map $ev : HE(X) \rightarrow X$. The images of the homomorphisms induced by the evaluation on homotopy groups were first studied by Gottlieb [7, 8], now called the Gottlieb subgroups. We denote them by $G_k(X) := im(\pi_k(ev)) \subset \pi_k(X)$.

To formulate the next result we need the notion of the Lusternik-Schnirelmann category of a space. The **LS category** of X , denoted $cat(X)$, is the least integer m (or ∞), such that X is the union of $m + 1$ open subsets contractible in X . The following theorem is an easy consequence of Proposition 28.8 in [5].

Theorem 2.4. *Suppose that X is a simply connected topological space of finite category. Then*

- (1) $G_{2k} \otimes \mathbb{Q} = 0$ and
- (2) $\dim G_* \otimes \mathbb{Q} \leq cat(X)$.

\square

3. THE RATIONAL DICHOTOMY AND PROOF OF THEOREM 1.1

Suppose that X is simply connected and has the homotopy type of a finite CW-complex. Let G be a connected group acting transitively on X with the isotropy group of a point $pt \in X$ denoted by G_{pt} .

Theorem 3.1. *With the above notation, there is the following estimate:*

$$\dim \pi_k(X) \otimes \mathbb{Q} \leq \dim \pi_k(G_{pt}) \otimes \mathbb{Q},$$

for k large enough.

³A path connected space X is said to be nilpotent if $\pi_1(X)$ is a nilpotent group and acts nilpotently on higher homotopy groups. It has finite \mathbb{Q} -type if $H^n(X; \mathbb{Q})$ are finite dimensional for all $n \geq 1$.

Proof. The assumptions are made in order to apply Theorems 2.1 and 2.3 to the evaluation fibration

$$G_{pt} \rightarrow G \rightarrow X.$$

Thus Theorem 2.3 implies that there exists the following exact sequence:

$$\cdots \rightarrow \Pi^{k-1}(G_{pt}) \rightarrow \Pi^k(X) \rightarrow \Pi^k(G) \rightarrow \Pi^k(G_{pt}) \rightarrow \cdots.$$

Since X is up to homotopy a finite CW-complex, then it has finite LS-category. It follows from Theorem 2.4 that $\dim G_*(X) \oplus \mathbb{Q}$ is finite; hence $ev^* : \Pi^k(X) \rightarrow \Pi^k(G)$ is trivial for k large enough. Now the statement follows from the exactness of the sequence of dual homotopy groups. \square

The rational dichotomy discovered by Félix [4, 5] states that if X is an n -dimensional simply connected space with the rational cohomology of finite type and finite category, then either its rational homotopy is finite dimensional or the dimensions of rational homotopy groups grow exponentially. In the first case, the space is called **rationally elliptic** and in the second **rationally hyperbolic**. As in other branches of mathematics, hyperbolicity is in a sense a “generic” feature. This is illustrated in the following.

Proposition 3.2. *If X is rationally elliptic, then*

- (1) $\dim \pi_{\text{even}}(X) \otimes \mathbb{Q} \leq \dim \pi_{\text{odd}}(X) \otimes \mathbb{Q} \leq \text{cat}(X)$,
- (2) $\chi(X) \geq 0$, where χ denotes the Euler characteristic. \square

Proof of Theorem 1.1. Since G_{pt} is a topological group, then its rational cohomology is freely generated by its dual rational homotopy. If X is rationally hyperbolic, then Theorem 3.1 implies that the dual rational homotopy is of infinite dimension. \square

Example 3.3. Let X be a 4-dimensional simply connected finite CW-complex. Then it follows from the basic properties of the Lusternik-Schirelmann category that $\text{cat}(X) \leq 2$. Now the above proposition implies that if $\pi_2(X) \otimes \mathbb{Q} \geq 3$, then X is rationally hyperbolic. Thus every simply connected 4-manifold X whose $b^2(X) := \dim H^2(X; \mathbb{Q}) \geq 3$ is rationally hyperbolic. By Theorem 1.1, we get that if a topological group acts transitively on X , then the isotropy subgroup has infinitely generated rational cohomology.

4. THE COHOMOLOGY OF A SYMPLECTOMORPHISM GROUP IS INFINITELY GENERATED

4.1. The argument. In this section we prove Theorem 1.2. We need to introduce some additional notions. Let B_ε be the standard symplectic ball of capacity ε and $\text{Symp}(M, \omega)$ denote the component of the identity in the group of symplectomorphisms of (M, ω) . Notice that since M is simply connected, then $\text{Symp}(M, \omega) = \text{Ham}(M, \omega)$, the group of Hamiltonian symplectomorphisms. We also need the following:

- $\text{Symp}^{U(2)}(M, B_\varepsilon)$ is the subgroup of $\text{Symp}(M, \omega)$ consisting of elements acting $U(2)$ -linearly on B_ε .
- $\text{Emb}^{U(2)}(B_\varepsilon, M)$ is the set of symplectic embeddings of the ε -ball into (M, ω) modulo the $U(2)$ reparametrizations of the source.

Consider the following commutative diagram of fibrations:

$$\begin{array}{ccc}
 \text{Sym}^{U(2)}(M, B_\varepsilon) & \longrightarrow & \text{Sym}(M, pt) \\
 \downarrow & & \downarrow \\
 \text{Sym}(M, \omega) & \xlongequal{\quad} & \text{Sym}(M, \omega) \\
 \downarrow & & \downarrow \text{ev} \\
 \text{Emb}^{U(2)}(B_\varepsilon, M) & \xrightarrow{f} & M
 \end{array}$$

The argument is split into several lemmas, which we shall prove in the next section.

Lemma 4.1. *Let (M, ω) be a compact symplectic manifold that is neither a rational nor ruled surface up to blowup. Let $(\widetilde{M}_\varepsilon, \omega_\varepsilon)$ be the symplectic blowup of (M, ω) . Then for any almost complex structure J compatible with ω_ε there exists a unique J -holomorphic exceptional sphere that is embedded.*

Lemma 4.2. *If for any almost complex structure J compatible with ω_ε there exists unique J -holomorphic exceptional sphere that is embedded, then $\text{Sym}(\widetilde{M}_\varepsilon)$ is weak homotopy equivalent to $\text{Sym}^{U(2)}(M, B_\varepsilon)$.*

Lemma 4.3. *In the above diagram, the map f induces a surjection on rational homotopy groups, except π_2 on which it depends on the first Chern class of (M, ω) .*

Proof of Theorem 1.2. Consider the long exact sequences of homotopy groups:

$$\begin{array}{ccc}
 \pi_{k+1}(\text{Emb}^{U(2)}(B_\varepsilon, M)) & \xrightarrow{f_*} & \pi_{k+1}(M) \\
 \downarrow & & \downarrow \\
 \pi_k(\text{Sym}^{U(2)}(M, B_\varepsilon)) & \longrightarrow & \pi_k(\text{Sym}(M, pt)) \\
 \downarrow & & \downarrow \\
 \pi_k(\text{Sym}(M, \omega)) & \xlongequal{\quad} & \pi_k(\text{Sym}(M, \omega)) \\
 \downarrow & & \downarrow \text{ev}_* \\
 \pi_k(\text{Emb}^{U(2)}(B_\varepsilon, M)) & \xrightarrow{f_*} & \pi_k(M)
 \end{array}$$

Theorem 2.4 implies that the rank of the image of the connecting homomorphism $\pi_{*+1}(M) \rightarrow \pi_*(\text{Sym}(M, pt))$ is infinite. Thus, according to Lemma 4.3, the same is true for $\pi_{*+1}(\text{Emb}^{U(2)}(B_\varepsilon, M)) \rightarrow \pi_*(\text{Sym}^{U(2)}(M, B_\varepsilon))$. In particular, $\dim \pi_*(\text{Sym}^{U(2)}(M, B_\varepsilon)) \otimes \mathbb{Q} = \infty$. The statement of the theorem follows immediately from Lemma 4.2 and the proof of Theorem 1.1. \square

4.2. Proofs of the lemmas.

Proof of Lemma 4.1. This is essentially Theorem 4.39 in [10]. First, it is clear that there exists an exceptional sphere that is embedded for any generic J . According to the criterion of Hofer, Lizan and Sikorav [6] we see that it is regular. It is also unique due to positivity of intersections.

Suppose that for some J the class E of the exceptional sphere is represented by some cusp-curve $\sum C_i$. Then $1 = C_1(E) = \sum c_1(C_i)$, which implies that for some i ,

$c_1(C_i) \geq 0$. Now Theorem 4.39 in [10] says that (M, ω) is a blowup of a rational or ruled surface, which is excluded by the hypothesis (cf. Proposition 2.6 in [9]). \square

Proof of Lemma 4.2. This was proved by Lalonde and Pinsonnault in [9], Lemmas 2.3 and 2.4. \square

We need some preparation to prove Lemma 4.3. Let J be an almost complex structure on M compatible with ω . Let $B_\varepsilon \rightarrow TM_\varepsilon \rightarrow M$ be the ε -ball subbundle of the tangent bundle (with respect to the Riemannian metric given by J). We need the following parametrized version of the Darboux theorem:

Proposition 4.4 (Universal thickening property). *There exists a map $\phi : TM_\varepsilon \rightarrow M$ such that for any $p \in M$ the composition $\phi \circ i_m : B_\varepsilon \rightarrow M$ is a symplectic embedding. Here $i_m : B_\varepsilon \rightarrow TM_\varepsilon$ is an inclusion of the fiber over the point $m \in M$.*

Proof. We start with the exponential map associated to the Riemannian metric,

$$\exp : TM_\varepsilon \rightarrow M.$$

Then $\exp^* \omega \in \Omega^2(TM_\varepsilon)$, and we have a map

$$V : \Omega^2(TM_\varepsilon) \rightarrow \Omega_{vert}^2(TM_\varepsilon),$$

where the latter space consists of 2-forms evaluated only on the vectors tangent to the fibers of TM_ε . In other words, this is a space of sections of the vector bundle over M whose fibers are 2-forms on the fibers of TM_ε ; that is,

$$\Omega_{vert}^2(TM_\varepsilon) = \Gamma(M, P \times_{U(n)} \Omega^2(B_\varepsilon)),$$

where $P \rightarrow M$ is a $U(n)$ -principal frame bundle of M . The differential d_B on B_ε defines a morphism of vector bundles

$$d : P \times_{U(n)} \Omega^1(B_\varepsilon) \rightarrow P \times_{U(n)} \Omega^2(B_\varepsilon)$$

by $d[p, \alpha] = [p, d_B(\alpha)]$. It is well defined because $U(n)$ acts linearly on B_ε . Since the ball B_ε is contractible, then the map d is surjective.

The bundle $P \times_{U(n)} \Omega^2(B_\varepsilon)$ has an obvious section $\widehat{\omega}_0 : M \ni m \mapsto [p, \omega_0]$, where ω_0 is the standard symplectic form on B_ε . Let us consider the section

$$V(\exp^* \omega) - \widehat{\omega}_0 \in \Gamma(M, P \times_{U(n)} \Omega^2(B_\varepsilon)).$$

We can assume (after possibly composing \exp with a linear automorphism of TM_ε) that the above section is equal to zero when restricted to the zero section of TM_ε .

Because the map d is surjective we can find a section σ of $P \times_{U(n)} \Omega^1(B_\varepsilon)$ such that $d \circ \sigma = V(\exp^* \omega) - \widehat{\omega}_0$. Moreover, σ can be chosen so that it vanishes along the zero section of TM_ε . Indeed, if σ' is any such section, then we take $\sigma := \sigma' - \sigma'_0$, where σ'_0 over $m \in M$ is the constant 1-form equal to $\sigma'(m)$ at the origin. It is clearly closed; so $d \circ \sigma = d \circ (\sigma' - \sigma'_0) = V(\exp^* \omega) - \widehat{\omega}_0 = a'$.

Now, in the presence of a symplectic form in each fiber (the constant section ω_0) the section σ gives rise to the section of

$$P \times_{U(n)} \mathfrak{X}(B_\varepsilon),$$

where $\mathfrak{X}(B_\varepsilon)$ denotes vector fields on B_ε . In other words, we get a vector field on TM_ε tangent to the fibers. Moreover, this vector field is trivial along the zero section. Take the flow ψ^t of this vector field (and smaller ε if necessary). We obtain that

$$V(\exp \circ \psi^1)^* \omega = \widehat{\omega}_0.$$

Notice that when restricted to a fiber of TM_ε the above argument reduces to the standard proof of the Darboux theorem [10, Theorem 3.15]. Finally, we define $\phi := \exp \circ \psi^1$. Then for an inclusion of the fiber $i_m : B_\varepsilon \rightarrow TM_\varepsilon$ we get that $\exp \circ \psi^1 \circ i_m : B_\varepsilon \rightarrow M$ is a Darboux chart, that is, a symplectic embedding. \square

In general we define a **twisted family of embeddings of F into M parametrized by B** to be a map $\phi : E \rightarrow M$, where $F \rightarrow E \rightarrow B$ is a bundle and ϕ restricted to every fiber is an embedding. This is in contrast to the usual notion of a family of maps where the domain is always a product. Thus the map $\phi : TM_\varepsilon \rightarrow M$ in the above proposition is an example of a twisted family of *symplectic* embeddings. This notion is proved useful below in the proof of Lemma 4.3. The idea is that after restriction of the parameter space the twisted family becomes trivial, i.e., a usual family.

Corollary 4.5 (Thickening property). *Let $f : X \rightarrow (M, \omega)$ be a continuous (or smooth) map. Then there exists a continuous (or smooth) twisted family of symplectic embeddings of small balls B_ε parametrized by X .*

Proof. Take the pull-back bundle

$$\begin{array}{ccc} f^*TM_\varepsilon & \xrightarrow{\hat{f}} & TM_\varepsilon \\ \downarrow & & \downarrow \\ X & \xrightarrow{f} & M. \end{array}$$

The twisted family of symplectic embeddings is now defined by $\phi \circ \hat{f} : f^*TM_\varepsilon \rightarrow M$. \square

Remark 4.6. We call the above facts the *thickening property* because they say that given any map we can always “thicken” it to get a twisted family of embeddings of symplectic balls.

Proof of Lemma 4.3. Let $s \in \pi_k(M)$. According to Corollary 4.5, we have a map $\phi : s^*TM_\varepsilon \rightarrow M$ such that its restriction to each fiber is a symplectic embedding. If the bundle s^*TM_ε is trivial, then it defines an element of $\pi_k(\text{Emb}(B_\varepsilon, M))$. Notice that the rational homotopy of the structure group of s^*TM_ε (which is $Sp(4, \mathbb{R})$) is equal to the exterior algebra $\Lambda(c_1, c_2)$, where c_i is of degree $2i - 1$. Thus in degrees different from 2 and 4 we get that $k s^*(TM_\varepsilon)$ is trivial, where $k \in \mathbb{Z}$. This means that

$$f_* : \pi_k(\text{Emb}(B_\varepsilon, M)) \otimes \mathbb{Q} \rightarrow \pi_k(M) \otimes \mathbb{Q}$$

is surjective for $k \neq 2, 4$. In degree 4 this is also true because there is no nonzero degree map $S^4 \rightarrow M$, where M is a symplectic 4-dimensional manifold. In degree 2, we have an obvious dependence on the corresponding Chern classes. \square

4.3. Final remarks. (1) It is very likely that the assumption in Theorem 1.2 saying that the manifold is not a blowup of either a rational or a ruled surface can be weakened. Then one has to do more work to show that for every compatible almost complex structure there exists an embedded exceptional curve (cf. Proposition 2.6 in [9]).

(2) Theorem 1.2 can be proved in a slightly different way. Namely, it is possible to prove that the map $\pi_k(\text{Sym}^{U(2)}(M, B_\varepsilon)) \rightarrow \pi_k(\text{Sym}(M, pt))$ is a surjection.

This together with Lemma 4.2 proves the theorem. I learned this argument from François Lalonde.

(3) Notice that the infinite-dimensional part of the rational homotopy of the symplectomorphism group detected by Theorem 1.2 is robust. That is, it survives when we embed symplectomorphisms into homeomorphisms or even into homotopy equivalences.

(4) Although the rational cohomology ring of the group of symplectomorphisms is infinitely generated, it does not mean that the topology of this group cannot be understood. The hope is that there is more structure. Namely, there is the Pontryagin product on homology and with respect to it the homology may be finitely generated. Thus the more appropriate structure to investigate in the topology of symplectomorphism groups is their homology with the Pontryagin product.

(5) Similarly, the cohomology ring of the classifying space need not be infinitely generated in this case. If such is the case, then this means that there are a lot of nontrivial fibrations with trivial characteristic classes.

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