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# DETECTING THE INDEX OF A SUBGROUP IN THE SUBGROUP LATTICE

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ABSTRACT. A theorem by Zacher and Rips states that the finiteness of the index of a subgroup can be described in terms of purely lattice-theoretic concepts. On the other hand, it is clear that if $G$ is a group and $H$ is a subgroup of finite index of $G$, the index $|G : H|$ cannot be recognized in the lattice $\mathfrak{L}(G)$ of all subgroups of $G$, as for instance all groups of prime order have isomorphic subgroup lattices. The aim of this paper is to give a lattice-theoretic characterization of the number of prime factors (with multiplicity) of $|G : H|$.

## 1. Introduction

For every group $G$, we shall denote by $\mathfrak{L}(G)$ the lattice of all subgroups of $G$. If $G$ and $\bar{G}$ are groups, an isomorphism from the lattice $\mathfrak{L}(G)$ onto the lattice $\mathfrak{L}(\bar{G})$ is also called a *projectivity* from $G$ onto $\bar{G}$; one of the main problems in the theory of subgroup lattices is to find group properties that are invariant under projectivities. In 1980, Zacher [5] and Rips proved independently that any projectivity from a group $G$ onto a group $\bar{G}$ maps each subgroup of finite index of $G$ to a subgroup of finite index of $\bar{G}$. In addition, Zacher gave a lattice-theoretic characterization of the finiteness of the index of a subgroup in a group; other characterizations were given by Schmidt [3]. On the other hand, it is clear that if $G$ is a group and $H$ is a subgroup of finite index of $G$, the index $|G : H|$ cannot be recognized in the subgroup lattice $\mathfrak{L}(G)$, as for instance all groups of prime order have the same lattice of subgroups.

The aim of this paper is to find an arithmetic invariant related to the index of a subgroup and preserved under projectivities. In fact, if $H$ is a subgroup of finite index of any group $G$, we will give a lattice-theoretic characterization of the number of prime factors (with multiplicity) of $|G : H|$, so that this number can be detected in the lattice $\mathfrak{L}(G)$.

Most of our notation is standard and can be found in [2]; for definitions and properties concerning lattices and subgroup lattices we refer to the monograph [4]. In particular, if $\mathfrak{L}$ is any complete lattice, the smallest and the largest element of $\mathfrak{L}$ will be denoted by 0 and $I$, respectively; moreover, for each pair $(a, b)$ of elements of $\mathfrak{L}$ such that $a \leq b$, we put $[b/a] = \{x \in \mathfrak{L} \mid a \leq x \leq b\}$. If $a$ is any non-zero

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element of the finite lattice $\mathfrak{L}$, we put

$$\phi(a) = \inf\{x \in \mathfrak{L} \,|\, x <\!\cdot\, a\}$$

(where the symbol $x <\!\cdot\, a$ means that $x$ is a maximal (proper) element of the lattice $[a/0]$). Finally, for each positive integer $n$ we will denote by $M_n$ the lattice of length 2 with $n$ atoms.

## 2. The weight of a finite lattice

A finite lattice $\mathfrak{L}$ is called *perfect* if it has no maximal elements that are modular.

**Lemma 2.1.** *Let $\mathfrak{L}$ be a finite lattice, and let $x$ and $y$ be elements of $\mathfrak{L}$ such that the intervals $[x/0]$ and $[y/0]$ are perfect lattices. Then also the lattice $[x \vee y/0]$ is perfect.*

*Proof.* Assume for a contradiction that $[x \vee y/0]$ contains a maximal element $z$ that is modular. Since $x \wedge z$ is modular in the perfect lattice $[x/0]$ and the lattices $[x \vee z/z]$ and $[x/x \wedge z]$ are isomorphic, it follows that $x \wedge z = x$ and hence $x \leq z$. We obtain similarly that $y \leq z$ and so $z = x \vee y$, a contradiction. Therefore the lattice $[x \vee y/0]$ is perfect. □

Let $\mathfrak{L}$ be a finite lattice. It follows from Lemma 2.1 that $\mathfrak{L}$ contains a largest element $r$ such that the interval $[r/0]$ is a perfect lattice; such an element $r$ will be called the *perfect radical* of $\mathfrak{L}$ and denoted by $r(\mathfrak{L})$. Clearly, the lattice $\mathfrak{L}$ is perfect if and only if $r(\mathfrak{L}) = I$.

Recall that an element $c$ of a finite lattice $\mathfrak{L}$ is called *cyclic* if the interval $[c/0]$ is a distributive lattice. Moreover, an element $a$ of $\mathfrak{L}$ is said to be *modularly embedded* in $\mathfrak{L}$ if the interval $[a \vee c/0]$ is a modular lattice for each cyclic element $c$ of $\mathfrak{L}$; a *modular chain* in $\mathfrak{L}$ is a chain of elements of $\mathfrak{L}$ of the form

$$0 = a_0 < a_1 < \ldots < a_t = I$$

such that $a_{i+1}$ is modularly embedded in $[I/a_i]$ for each non-negative integer $i < t$.

For our purposes, we will consider the subset $P(\mathfrak{L})$ of $\mathfrak{L}$ consisting of all elements $a$ satisfying the following conditions:

- the lattice $[a/0]$ has a modular chain;
- every interval of $[a/0]$ is directly indecomposable;
- if $x <\!\cdot\, y \leq a$ and $[x/0]$ is a chain of length 2, then either $[y/0]$ is a modular lattice or it is isomorphic to the subgroup lattice $\mathfrak{L}(D_8)$ of the dihedral group of order 8.

In particular, $P(\mathfrak{L})$ contains any element $a$ of $\mathfrak{L}$ such that $[a/0]$ is a modular lattice whose intervals are directly indecomposable. Note also that, in the special case of the subgroup lattice of a finite group $G$, it turns out that the elements of $P(\mathfrak{L}(G))$ are precisely the primary subgroups and the $P$-subgroups of $G$ (see [4], Theorem 7.4.10). Here a group is called a *$P$-group* if it is the semidirect product of an abelian normal subgroup $A$ of prime exponent by a group $\langle x \rangle$ of prime order such that $x$ induces on $A$ a power automorphism; in particular, all abelian groups of prime exponent are $P$-groups.

For a finite lattice $\mathfrak{L}$, we let $A_{\mathfrak{L}}$ be the set of all atoms of $\mathfrak{L}$. For every prime number $p$, we define two subsets of $A_{\mathfrak{L}}$, namely

$$R_{\mathfrak{L}}(p) = \{a \in A_{\mathfrak{L}} \mid \exists b \in P(\mathfrak{L}) \text{ such that } a \leq b \text{ and } [b/\phi(b)] \simeq M_{p+1}\}$$

and

$$\begin{aligned} S_{\mathfrak{L}}(p) &= \{a \in A_{\mathfrak{L}} \mid \exists b \in \mathfrak{L} \ \text{ such that } \ [b/0] \text{ is a chain}, \phi(b) \neq 0, \\ &\quad [a \vee \phi(b)/0] \text{ is distributive and } \ [a \vee b/\phi(b)] \simeq M_{p+1}\}; \end{aligned}$$

furthermore, we let $T_{\mathfrak{L}}(p) = R_{\mathfrak{L}}(p) \cup S_{\mathfrak{L}}(p)$ and

$$T_{\mathfrak{L}} = \bigcup_{p \in \mathbb{P}} T_{\mathfrak{L}}(p),$$

where $\mathbb{P}$ is the set of all prime numbers. Then, clearly, $T_{\mathfrak{L}} \subseteq A_{\mathfrak{L}}$ and in general $T_{\mathfrak{L}} \neq A_{\mathfrak{L}}$, for instance if $\mathfrak{L}$ is a non-trivial chain. Finally, for every atom $a$ of $\mathfrak{L}$, we define

$$\omega_{\mathfrak{L}}(a) = \text{Min}\{p \in \mathbb{P} \mid a \in T_{\mathfrak{L}}(p)\}$$

if $a \in T_{\mathfrak{L}}$ and $\omega_{\mathfrak{L}}(a) = 0$ if $a \in A_{\mathfrak{L}} \setminus T_{\mathfrak{L}}$. Then

$$\omega_{\mathfrak{L}} : A_{\mathfrak{L}} \longrightarrow \mathbb{P} \cup \{0\}$$

is a well-defined map described entirely in the (finite) lattice $\mathfrak{L}$.

An element $x \in \mathfrak{L}$ is called a *p-element* of $\mathfrak{L}$ if $\omega_{\mathfrak{L}}(a) = p$ for every atom $a$ of $[x/0]$. As usual, the length $l(\mathfrak{L})$ of $\mathfrak{L}$ is the largest length of a chain in $\mathfrak{L}$, and we denote the largest length of a chain consisting of $p$-elements in $\mathfrak{L}$ by $\ell_p(\mathfrak{L})$. The *weight* $||\mathfrak{L}||$ of $\mathfrak{L}$ is now defined by

$$||\mathfrak{L}|| = \ell([I/r(\mathfrak{L})]) + \sum_{p \in \mathbb{P}} \ell_p([r(\mathfrak{L})/0]),$$

where $r(\mathfrak{L})$ is the perfect radical of $\mathfrak{L}$ defined above.

## 3. The order of a finite group

It is well known that a finite group is perfect if and only if its subgroup lattice is perfect (see [4], Theorem 5.3.3). It follows that for any finite group $G$, the perfect radical of the lattice $\mathfrak{L}(G)$ is the largest perfect subgroup of $G$ (and so it coincides with the soluble residual of $G$).

**Lemma 3.1.** *Let $H$ be a minimal subgroup of a finite group $G$, and let $p$ be a prime number. If $H \in S_{\mathfrak{L}(G)}(p)$, then $|H| = p$.*

*Proof.* Since $H \in S_{\mathfrak{L}(G)}(p)$, there exists a cyclic subgroup $K$ of prime power order such that $\phi(K) \neq \{1\}$, $\langle H, \phi(K)\rangle$ is cyclic and

$$[\langle H, K\rangle/\phi(K)] \simeq M_{p+1}.$$

Thus $H$ is not contained in $K$, so that $\langle H, \phi(K)\rangle = H \times \phi(K)$, and in particular $H$ and $K$ have coprime orders. So $\langle H, K\rangle/\phi(K)$ cannot be a $p$-group, and hence it is non-abelian of order $pq$ where $p > q \in \mathbb{P}$. Thus $[H, K] \neq \{1\}$ and $[H, \phi(K)] = \{1\}$. Therefore $K$ cannot be normal in $\langle H, K\rangle$, and it follows that $|K/\phi(K)| = q$ and $|H| = p$. □

**Lemma 3.2.** *Let $G$ be a finite group having no normal Sylow complement. Then $\omega_{\mathfrak{L}(G)}(H) = |H|$ for every minimal subgroup $H$ of $G$.*

*Proof.* Let $|H| = p$. We claim that it suffices to show that $H$ belongs to $T_{\mathfrak{L}(G)}(p)$.

Indeed, this clearly implies that $0 < \omega_{\mathfrak{L}(G)}(H) \leq p$. If $H$ were contained in $T_{\mathfrak{L}(G)}(q)$ for some prime $q < p$, then by Lemma 3.1, $H \in R_{\mathfrak{L}(G)}(q)$ and so there would exist $Q \in P(\mathfrak{L}(G))$ such that $H \leq Q$ and $[Q/\phi(Q)] \simeq M_{q+1}$. In this case, $Q$ would be a $q$-group or a $P$-group of order $qr$ with $q \geq r \in \mathbb{P}$ (see [4], Theorem 7.4.10). This would contradict the fact that $H \leq Q$ and $|H| = p > q$. Thus $\omega_{\mathfrak{L}(G)}(H) = p = |H|$.

To prove that $H \in T_{\mathfrak{L}(G)}(p)$, consider a Sylow $p$-subgroup $S$ of $G$ containing $H$. If $S$ is not cyclic, then we may consider a smallest non-cyclic subgroup $P$ of $S$ containing $H$. Every maximal subgroup of $P$ containing $H$ is cyclic and hence $[P/\phi(P)] \simeq M_{p+1}$; thus $H \in R_{\mathfrak{L}(G)}(p)$. So suppose that $S$ is cyclic. Since $G$ is not $p$-nilpotent, we have $S \leq C_G(S) < N_G(S)$ (see [2], 10.1.8), and for some prime $q \neq p$ there exists an element $g \in N_G(S)$ with order $q^n$ inducing an automorphism of order $q$ in $S$. Then $\phi(\langle g \rangle) = \langle g^q \rangle$ centralizes $S$, and in particular the subgroup $\langle H, \phi(\langle g \rangle)\rangle$ is cyclic; furthermore, $\langle H, g\rangle/\phi(\langle g \rangle)$ is non-abelian of order $pq$ and hence $[\langle H, g\rangle/\phi(\langle g \rangle)] \simeq M_{p+1}$. So if $\phi(\langle g \rangle) \neq \{1\}$, then $H \in S_{\mathfrak{L}(G)}(p)$; and if $\phi(\langle g \rangle) = \{1\}$, then $\langle H, g\rangle \in P(\mathfrak{L}(G))$ and $H \in R_{\mathfrak{L}(G)}(p)$. In all cases, $H \in T_{\mathfrak{L}(G)}(p)$ as we wanted to show. □

We can now prove the following result, which provides a purely lattice-theoretic description of the order of a finite group having no normal Sylow complement, in particular of any finite perfect group.

**Theorem 3.3.** *Let $G$ be a finite group having no normal Sylow complement. Then* $|G| = \prod_{p\in\mathbb{P}} p^{\ell_p(\mathfrak{L}(G))}$.

*Proof.* It follows from Lemma 3.2 that for each prime number $p$, the $p$-elements of the lattice $\mathfrak{L}(G)$ are precisely the $p$-subgroups of $G$. In particular, if $P$ is any Sylow $p$-subgroup of $G$, we have that $|P| = p^{\ell_p(\mathfrak{L}(G))}$. The theorem follows. □

For an arbitrary finite group $G$, the order of $G$ cannot be recognized in $\mathfrak{L}(G)$. But we can describe the number of prime factors of $|G|$ in $\mathfrak{L}(G)$.

**Theorem 3.4.** *Let $G$ be a finite group. Then the weight $||\mathfrak{L}(G)||$ of the subgroup lattice of $G$ is the number of prime factors of the order of $G$ (with multiplicity).*

*Proof.* Let $R$ be the soluble residual of $G$. Then $R = r(\mathfrak{L}(G))$ and Theorem 3.3 yields that

$$\sum_{p\in\mathbb{P}} \ell_p([r(\mathfrak{L}(G))/0]) = \sum_{p\in\mathbb{P}} \ell_p(\mathfrak{L}(R))$$

is the number of prime factors of $|R|$. Since $G/R$ is soluble, the number of prime factors of $|G/R|$ is just the length of the lattice $\mathfrak{L}(G/R)$. The number of prime factors of $|G|$ is the sum of these two numbers, and hence it is $||\mathfrak{L}(G))||$. □

The above theorem has the following obvious consequence.

**Corollary 3.5.** *Let $H$ be a subgroup of the finite group $G$. Then the number of prime factors of the index $|G : H|$ (with multiplicity) is $||\mathfrak{L}(G)|| - ||\mathfrak{L}(H)||$.*

## 4. Subgroups of finite index

It is well known that if $a$ and $b$ are modular elements of a lattice $\mathfrak{L}$, then also $a \vee b$ is a modular element of $\mathfrak{L}$; in the case of the lattice of all subgroups of a group $G$, it has been proved that the join of any collection of modular subgroups of $G$ is likewise a modular subgroup (see [1], Proposizione 1.2). As G. Zacher pointed out to one of the authors, this property also holds for arbitrary algebraic lattices (recall that a complete lattice $\mathfrak{L}$ is called *algebraic* if each element of $\mathfrak{L}$ is a join of compact elements).

**Lemma 4.1.** *Let $\mathfrak{L}$ be an algebraic lattice, and let $X$ be a non-empty set of modular elements of $\mathfrak{L}$. Then also $\sup X$ is a modular element of $\mathfrak{L}$.*

*Proof.* Put $a = \sup X$, and let $b$ be any element of $\mathfrak{L}$. Consider an element $y$ of the interval $[a \vee b/a]$, and let $(y_i)_{i \in I}$ be a collection of compact elements of $\mathfrak{L}$ such that $y = \sup\limits_{i \in I} y_i$. For each $i \in I$ there exists a finite subset $X_i$ of $X$ such that $y_i \leq x_i \vee b$, where $x_i = \sup X_i$; clearly, $x_i$ is a modular element of $\mathfrak{L}$, and hence

$$y_i \leq y \wedge (x_i \vee b) = x_i \vee (b \wedge y) \leq a \vee (b \wedge y).$$

Thus $y \leq a \vee (b \wedge y)$, and so $a \vee (b \wedge y) = y$.

Suppose now that $z$ is an element of the interval $[b/a \wedge b]$, and put $c = (a \vee z) \wedge b$. Let $(c_j)_{j \in J}$ be a collection of compact elements of $\mathfrak{L}$ for which $c = \sup\limits_{j \in J} c_j$, and for each $j \in J$ let $X'_j$ be a finite subset of $X$ such that $c_j \leq x'_j \vee z$, where $x'_j = \sup X'_j$. Since $x'_j$ is a modular element of $\mathfrak{L}$, we have

$$c_j \leq (x'_j \vee z) \wedge b = z \vee (x'_j \wedge b) = z,$$

so that $c \leq z$ and hence $z = c = (a \vee z) \wedge b$. It follows that $a$ is a modular element of $\mathfrak{L}$ (see [4], Theorem 2.1.5). □

Let $\mathfrak{L}$ be an algebraic lattice, and let $a$ be any element of $\mathfrak{L}$. The largest modular element $m$ of $\mathfrak{L}$ such that $m \leq a$ is called the *modular core* of $a$ in $\mathfrak{L}$, and is denoted by $core_{\mathfrak{L}} a$. Clearly, the element $a$ is modular if and only if $a = core_{\mathfrak{L}} a$; note also that if $core_{\mathfrak{L}} a < a$, then $a$ cannot be modular in the lattice $[I/core_{\mathfrak{L}} a]$. If $a$ and $b$ are elements of $\mathfrak{L}$ such that $a < b$, the modular core of $a$ in $[b/0]$ will also be denoted by $core_b a$.

Let $\mathfrak{L}$ be an infinite algebraic lattice. A maximal element $a$ of $\mathfrak{L}$ is called *$f$-maximal* if the interval $[a/0]$ is infinite and $a$ satisfies one of the following conditions:

(1) $a$ is not modular in $\mathfrak{L}$ and $[I/core_{\mathfrak{L}} a]$ is a finite lattice;
(2) there exists an automorphism $\varphi$ of $\mathfrak{L}$ such that $a \wedge a^{\varphi}$ is a modular element of $\mathfrak{L}$ and $[I/a \wedge a^{\varphi}]$ is a finite lattice with length 2 and at least 3 atoms;
(3) for each automorphism $\varphi$ of $\mathfrak{L}$, the element $a \wedge a^{\varphi}$ is modular in $\mathfrak{L}$ and $[I/a \wedge a^{\varphi}] = \{a \wedge a^{\varphi}, a, a^{\varphi}, I\}$.

It follows from the definition that if $a$ is any $f$-maximal element of $\mathfrak{L}$, the lattice $[I/core_{\mathfrak{L}} a]$ is finite; note also that both conditions (2) and (3) above force the element $a$ to be modular in $\mathfrak{L}$.

Let $\mathfrak{L}$ be an infinite algebraic lattice, and let $a$ and $b$ be elements of $\mathfrak{L}$ such that $a < b$ and $a$ is $f$-maximal in $[b/0]$; since the lattices $[a/core_b a]$ and $[b/core_b a]$ are finite, we can define the *lattice index* $||b : a||$ of $a$ in $b$ by the position

$$||b : a|| = ||[b/core_b a]|| - ||[a/core_b a]||.$$

In particular, if $a$ is an $f$-maximal and modular element of $[b/0]$, we have $||b:a|| = |||[b/a]||| = 1$.

Let $G$ be an infinite group; a subgroup $M$ of $G$ is called *$f$-maximal* if $M$ is an $f$-maximal element of the lattice $\mathfrak{L}(G)$. Actually, the $f$-maximal subgroups of $G$ are precisely the maximal subgroups of finite index; in fact, the following lattice characterization of the finiteness of the index of a subgroup holds.

**Lemma 4.2.** *Let $G$ be an infinite group, and let $H$ be a proper subgroup of $G$. Then $H$ has finite index in $G$ if and only if there exists a finite chain $H = H_0 < H_1 < \ldots < H_t = G$ such that $H_i$ is an $f$-maximal subgroup of $H_{i+1}$ for each $i = 0, 1, \ldots, t-1$.*

*Proof.* Suppose first that the index $|G : H|$ is finite, and let

$$H = H_0 < H_1 < \ldots < H_t = G$$

be a maximal chain of subgroups between $H$ and $G$. Then the subgroup $H_i$ is infinite and maximal in $H_{i+1}$ for each $i = 0, 1, \ldots, t-1$; moreover, since $|H_{i+1} : H_i|$ is finite, we have that $H_i$ is an $f$-maximal subgroup of $H_{i+1}$ (see [3], Satz 3). The converse statement follows from the same result. □

We also need the following known result; it shows that if $M$ is an $f$-maximal subgroup of an infinite group $G$, then either $core_{\mathfrak{L}(G)}M = M$ or $core_{\mathfrak{L}(G)}M = core_G M$ (the usual core of $M$ in $G$ in the group-theoretical sense).

**Lemma 4.3** (see [3], Lemma 3)**.** *Let $G$ be a group, and let $M$ be a maximal subgroup of finite index of $G$. If $M$ is not modular in $G$, then the largest modular subgroup of $G$ contained in $M$ is normal in $G$.*

**Theorem 4.4.** *Let $G$ be an infinite group, and let $H$ be a proper subgroup of finite index of $G$. Then the number of prime factors of $|G : H|$ (with multiplicity) is the sum $\sum_{i=0}^{t-1} ||H_{i+1} : H_i||$, where*

$$H = H_0 < H_1 < \ldots < H_t = G$$

*is a finite chain of subgroups such that $H_i$ is an $f$-maximal subgroup of $H_{i+1}$ for each $i = 0, 1, \ldots, t-1$.*

*Proof.* Assume first that $H_i$ is a modular subgroup of $H_{i+1}$ for some non-negative integer $i < t$, so that $||H_{i+1} : H_i|| = 1$ as we already observed; on the other hand, it is well known that in this case the index $|H_{i+1} : H_i|$ is a prime number (see [4], Lemma 5.1.2). Suppose now that $H_i$ is not modular in $H_{i+1}$, and let $K_i$ be the normal core of $H_i$ in $H_{i+1}$. By Lemma 4.3, $K_i$ is the largest modular subgroup of $H_{i+1}$ contained in $H_i$, and hence we have

$$\begin{aligned} ||H_{i+1} : H_i|| &= |||[H_{i+1}/K_i]||| - |||[H_i/K_i]||| \\ &= ||\mathfrak{L}(H_{i+1}/K_i)|| - ||\mathfrak{L}(H_i/K_i)||. \end{aligned}$$

Since $H_{i+1}/K_i$ is a finite group, it follows from Corollary 3.5 that the lattice index $||H_{i+1} : H_i||$ is the number of prime factors of $|H_{i+1}/K_i : H_i/K_i| = |H_{i+1} : H_i|$. The theorem is proved. □

**Corollary 4.5.** *Let $\varphi$ be a projectivity between the groups $G$ and $\bar{G}$, and let $H$ be a subgroup of finite index of $G$. Then the indices $|G : H|$ and $|\bar{G} : H^{\varphi}|$ have the same number of prime factors.*

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