ERRATA TO “HECKE ALGEBRAS OF SEMIDIRECT PRODUCTS”

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There is a mistake in Lemma 1.3 of our paper [3] that invalidates the proofs of [3, Lemma 1.10] and [3, Theorem 1.9]. We are grateful to Iain Raeburn, Jacqui Ramagge and Udo Baumgartner for communicating this to us and for suggesting that Lemma [11] below should be used in place of [3, Lemma 1.3] to fix the results. It is also necessary to normalize the products mentioned in part (i) of [3, Theorem 1.9]. Indeed, the correct linear basis for the Hecke algebra there is the set

\[ \left\{ \frac{1}{R(n)} \mu_n^* [n] \mu_s : s, t \in \Sigma \text{ and } n \in N \right\}. \]

**Lemma 1** (cf. [2, Corollary I. 4.5]). If \( \Gamma_0 x \Gamma_0 y \Gamma_0 = \Gamma_0 xy \Gamma_0 \), then \([x][y] = [xy]\). The following lemma should replace [3, Lemma 1.10].

**Lemma 2.** For \( s, t, \tau, \sigma \in \Sigma \) and \( n, m \in N \),

\[ [t^{-1} n s] = [t^{-1} m \sigma] \iff \mu^*_t \frac{[n]}{R(n)} \mu_s = \mu^*_s \frac{[m]}{R(m)} \mu_\tau, \text{ in } H(\Gamma, \Gamma_0). \]

**Proof.** First notice that the partial products in \( \mu^*_t \frac{[n]}{R(n)} \mu_s \) are supported on single double cosets. To compute them we use Lemma [11] to get

\[ [t^{-1}][m] = \frac{R(\tau^{-1}) R(m)}{R(\tau^{-1} m)} [t^{-1} m] \quad \text{and} \quad [t^{-1} m][\sigma] = \frac{R(\tau^{-1} m) R(\sigma)}{R(\tau^{-1} m \sigma)} [t^{-1} m \sigma], \]

and then combine the results to obtain the triple product

\[ \mu^*_t \frac{[m]}{R(m)} \mu_\sigma = \frac{[t^{-1}][m][\sigma]}{[\tau^{-1}][m][\sigma]} = \frac{1}{R(\tau)^{1/2} R(m) R(\sigma)^{1/2}} \frac{R(\tau^{-1}) R(m)}{R(\tau^{-1} m)} \frac{R(\tau^{-1} m) R(\sigma)}{R(\tau^{-1} m \sigma)} [t^{-1} m \sigma]. \]

Since the triple product is supported on a single double coset, the implication \((\Leftarrow)\) follows. Next we simplify the coefficient, using \( R(\tau^{-1}) = L(\tau) = 1 \), to obtain

\[ \mu^*_t \frac{[m]}{R(m)} \mu_\sigma = \left( \frac{R(\sigma)}{R(\tau)} \right)^{1/2} \frac{[t^{-1} m \sigma]}{R(\tau^{-1} m \sigma)}. \]

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Suppose now that $[\tau^{-1}m\sigma] = [t^{-1}ns]$. Taking quotients modulo $N$ we see that $\tau^{-1} = t^{-1}s$ in $G = \Gamma/N$ so there exist elements $\gamma$ and $r$ in $S = \Sigma/N$ such that $\gamma\tau = rt$ and $\gamma\sigma = rs$ because $S$ is directed. Since $R$ is a homomorphism on $\Sigma$, 
\[
\frac{R(\sigma)}{R(\tau)} = \frac{R(\gamma\sigma)}{R(\gamma\tau)} = \frac{R(rs)}{R(rt)} = \frac{R(s)}{R(t)},
\]
and the implication ($\Rightarrow$) now follows from (7). \qed

To correct the proof of [3, Theorem 1.9] we need a direct proof of the (rescaled) key identity
\[
(\star) \quad \mu_x\mu_x^m\mu_y^n = \mu_x\mu_x^m\mu_y^n. \tag{\star}
\]
Suppose $\mu_x^m = \mu_x^m\mu_y^n$ and multiplying on the left by $\mu_x$ and on the right by $\mu_y$ shows that this holds in $H(\Gamma, \Gamma_0)$ because of the relation (\star). Thus (\star) also holds for the universal (tilded) generators. Multiplying this now on the right by $\tilde{\mu}_y$ and simplifying, yields $\tilde{\mu}_x \tilde{\mu}_y = \tilde{\mu}_x \tilde{\mu}_y$, as desired. It follows that the canonical homomorphism $H(\Gamma, \Gamma_0) \to H(\Gamma, \Gamma_0)$ maps the spanning set $\{\frac{1}{\Gamma(\Gamma_0)}\tilde{\mu}_x \tilde{\mu}_y\}$ of the universal algebra of the relations one-to-one and onto the linear basis $\{\frac{1}{\Gamma(\Gamma_0)}\mu_x \mu_y\}$ of the Hecke algebra, hence is an isomorphism.

Note: A very interesting generalization of the results of [3] to group extensions has been obtained by Baumgartner et al. in [1] which implicitly provides, in the particular case of split extensions, a correction to the error they found in [3].

References

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