

TRANSFERENCE FOR AMENABLE ACTIONS

WALDEMAR HEBISCH AND M. GABRIELLA KUHN

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ABSTRACT. Suppose G acts amenably on a measure space X with quasi-invariant σ -finite measure m . Let σ be an isometric representation of G on $L^p(X, dm)$ and μ a finite Radon measure on G . We show that the operator $\sigma_\mu f(x) = \int_G (\sigma(g)f)(x) d\mu(g)$ has $L^p(X, dm)$ -operator norm not exceeding the $L^p(G)$ -operator norm of the convolution operator defined by μ . We shall also prove an analogous result for the maximal function M associated to a countable family of Radon measures μ_n .

1. INTRODUCTION

Let σ be a uniformly bounded representation of a locally compact group G on some Banach space B and K a convolution operator on $L^p(G)$ given by

$$Kf(x) = \int_G k(y)f(y^{-1}x)dy = k * f(x)$$

with $k \in L^1(G)$. The *transferred operator* K^\sharp is defined by letting

$$K^\sharp(F) = \int_G k(y)(\sigma(y)F)dy .$$

The basic transference theory goes back to the earliest works of R. Coifman and G. Weiss around 1970 and says that if we have an action of an *amenable* group G on some (σ -finite) measure space X and we let $B = L^p(X)$, then

$$(1) \quad \|K^\sharp\|_{L^p(X) \rightarrow L^p(X)} \leq \|K\|_{L^p(G) \rightarrow L^p(G)} .$$

This theory has many different applications (it allows us to convert $L^p(\mathbb{T})$ -Fourier multipliers into $L^p(\mathbb{R})$ -Fourier multipliers, to bound the Riesz transform on $SU(2)$ or, more generally, on compact Lie groups having a “nice” maximal torus) and has been generalized to different contexts: the central role is played by an amenable group.

In 1978 R. Zimmer [8] introduced the notion of *amenable action* for locally compact topological groups G . When X is a homogeneous G space, say $X = G/H$, amenability of the action is equivalent to the amenability of H .

In this paper we shall prove that the above inequality (1) is true when G acts *amenably* on a measure space X with quasi-invariant σ -finite measure. We shall also

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prove an analogous result for the maximal function M_σ with respect to a countable family of Radon measures μ_n on G .

2. NOTATION AND RESULTS

Let G be a second countable locally compact group acting on a Borel space X with quasi-invariant σ -finite measure m . We shall assume that G is acting on the right, that is, $(g, x) \in G \times X \rightarrow g^{-1}x$. Let $r(g, x)$ be the Radon-Nikodym derivative of the G action:

$$(2) \quad r(g, x) = \frac{dm_g(x)}{dm} \quad \text{where } m_g(E) = m(g^{-1}E);$$

equivalently,

$$\int_X |f(g^{-1}x)|r(g, x)dm(x) = \int_X |f(x)|dm(x)$$

for every measurable function f . Recall that r can be chosen to be a strict Borel cocycle, that is, $r(gs, x) = r(g, x)r(s, g^{-1}x)$ for all $g, s \in G$ and $x \in X$.

We shall assume that m is a probability measure, so that

$$\int_X r(g, x)dm(x) = 1 \quad \text{for all } g \in G.$$

This can be done by choosing appropriately a representative in the measure class of m .

Let α be a cocycle, that is, a Borel function from $G \times X \rightarrow \mathbf{C}$ such that:

$$\alpha(gs, x) = \alpha(g, x)\alpha(s, g^{-1}x).$$

In the sequel we will assume that α is unitary, that is, $|\alpha(g, x)| = 1$.

For any given $g \in G$ define a norm preserving map $\sigma(g) : L^p(X) \rightarrow L^p(X)$ by letting

$$\sigma(g)f(x) = f(g^{-1}x)(r(g, x))^{\frac{1}{p}}\alpha(g, x).$$

Denote by $L^p(G \times X)$ the usual L^p space with respect to the product measure $dg \times dm$ where dg is the left invariant Haar measure on G .

Denote by λ the left regular representation of G acting on $L^p(G)$. Define a representation $\lambda \otimes \sigma$ on $L^p(G \times X)$ by the rule

$$(\lambda \otimes \sigma)(g)F(h, x) = F(g^{-1}h, g^{-1}x)(r(g, x))^{\frac{1}{p}}\alpha(g, x).$$

Choose any finite Radon measure μ on G . Define operators $(\lambda \otimes \sigma)_\mu$, λ_μ and σ_μ acting respectively on $L^p(G \times X)$, $L^p(G)$ and $L^p(X)$ according to the following rules:

$$(\lambda \otimes \sigma)_\mu F(h, x) = \int_G \lambda \otimes \sigma(g)F(h, x)d\mu(g) \int_G F(g^{-1}h, g^{-1}x)(r(g, x))^{\frac{1}{p}}\alpha(g, x)d\mu(g),$$

$$\lambda_\mu f(h) = \int_G \lambda(g)f(h)d\mu(g) = \int_G f(g^{-1}h)d\mu(g),$$

$$\sigma_\mu f(x) = \int_G \sigma(g)f(x)d\mu(g) = \int_G f(g^{-1}x)(r(g, x))^{\frac{1}{p}}\alpha(g, x)d\mu(g).$$

We shall denote by $\|\cdot\|_p$ the L^p norm in any of the above-defined L^p spaces and by $\|\cdot\|_{p,p}$ the norm of any of the above-defined operators.

When G is *amenable* it is known (see e.g. [2], [3]) that $\|\sigma_\mu\|_{p,p} \leq \|\lambda_\mu\|_{p,p}$. This phenomenon is called transference.

When $p = 2$ and G acts amenably on X the same inequalities are true: see [1], Theorem 3.2.3 (for discrete groups see also [5] for ergodic actions and [6] for general amenable actions including semisimple algebraic groups).

In this paper we shall prove transference for all p ($1 \leq p < \infty$).

We refer to Zimmer’s book [9] for the general definition of amenable action.

For the purposes of this paper we need the following property of amenable G -spaces:

Lemma 2.1 (C. Anantharaman-Delaroche, [1], Theorem 3.1.5). *Assume that G is acting amenably on a standard Borel space with quasi-invariant measure m .*

Then there exists a sequence ϕ_i of nonnegative functions $\phi_i(g, x)$ in $L^1(G \times X)$ such that for all $f \in L^1(X)$ and any compact subset $K \subset G$ we have

- a) $\int_G \phi_i(g, x) dg = 1$ for almost all $x \in X$,
- b) $\lim_i (\sup_{k \in K} \int_G \int_X |\phi_i(k^{-1}g, k^{-1}x) - \phi_i(g, x)| |f(x)| dm(x) dg) = 0$.

Remark 2.2. When G is discrete and ergodic on X , the existence of such a sequence was essentially the proof (given in [5]) that the quasi-regular representation of G on X is weakly contained into the regular representation of G . In [1] it is proved that the above condition is in fact equivalent to the amenability of the G action on X for any locally compact second-countable group.

Fix once and for all a sequence ϕ_i of nonnegative functions satisfying a) and b) of Lemma 2.1.

For every $f \in L^p(X)$ and $i \in \mathbb{N}$ define a function $\tilde{f}_i \in L^p(G \times X)$ according to the following rule:

$$\tilde{f}_i(g, x) = (\phi_i(g, x))^{\frac{1}{p}} f(x)$$

so that

$$\widetilde{\sigma_\mu f}_i(g, x) = (\phi_i(g, x))^{\frac{1}{p}} (\sigma_\mu f)(x) .$$

Lemma 2.3. *Let μ be any finite Radon measure and $f \in L^p(X)$. Let \tilde{f}_i and $\widetilde{\sigma_\mu f}_i$ be as above.*

Then we have

$$\lim_i \|(\lambda \otimes \sigma)_\mu \tilde{f}_i - \widetilde{\sigma_\mu f}_i\|_p = 0.$$

Proof. First, we will assume that μ has compact support F . Choose $\varepsilon > 0$ and fix f . By Lemma 2.1 (with $K = F^{-1}$) we can find N such that for all $i \geq N$,

$$\begin{aligned} \int_G \phi_i(g, x) dg &= 1, \\ \sup_{k \in F} \int_G \int_X |\phi_i(kg, kx) - \phi_i(g, x)| |f(x)|^p dm(x) dg &< \varepsilon. \end{aligned}$$

We need to compute the norm in $L^p(G \times X)$ of the quantity

$$\begin{aligned} &((\lambda \otimes \sigma)_\mu \tilde{f}_i - \widetilde{\sigma_\mu f}_i)(g, x) \\ &= \int_F \{(\phi_i(k^{-1}g, k^{-1}x))^{\frac{1}{p}} - (\phi_i(g, x))^{\frac{1}{p}}\} \{r(k, x)\}^{\frac{1}{p}} \alpha(k, x) f(k^{-1}x) d\mu(k) . \end{aligned}$$

Denote by $|\mu|$ the measure total variation of μ . Using Minkowski's integral inequality we get

$$\begin{aligned}
 & \left\{ \int_{G \times X} |(\lambda \otimes \sigma)_\mu \tilde{f}_i - \widetilde{\sigma_\mu f_i}|^p(g, x) d(g) dm(x) \right\}^{\frac{1}{p}} \\
 & \leq \int_F d|\mu|(k) \left\{ \int_{G \times X} |(\phi_i(k^{-1}g, k^{-1}x))^{\frac{1}{p}} \right. \\
 (3) \quad & \left. - (\phi_i(g, x))^{\frac{1}{p}} r(k, x) |f(k^{-1}x)|^p d(g) dm(x) \right\}^{\frac{1}{p}} \\
 & = \int_F \left\{ \int_{G \times X} |\phi_i(kg, kx)^{\frac{1}{p}} - \phi_i(g, x)^{\frac{1}{p}}|^p |f(x)|^p d(g) dm(x) \right\}^{\frac{1}{p}} d|\mu|(k) \\
 & \leq \int_F \left\{ \int_{G \times X} |\phi_i(kg, kx) - \phi_i(g, x)| |f(x)|^p d(g) dm(x) \right\}^{\frac{1}{p}} d|\mu|(k) \\
 & \leq \varepsilon^{\frac{1}{p}} \|\mu\|
 \end{aligned}$$

where for the last inequality (3) we used $|a^{\frac{1}{p}} - b^{\frac{1}{p}}|^p \leq |a - b|$ (valid for positive a and b) and $\|\mu\|$ stays for $|\mu|(G)$.

So, we showed that for fixed compactly supported μ and given f we can choose N such that for all $i \geq N$,

$$\|(\lambda \otimes \sigma)_\mu \tilde{f}_i - \widetilde{\sigma_\mu f_i}\|_p \leq \varepsilon^{\frac{1}{p}} \|\mu\|,$$

which gives our claim for compactly supported μ .

Now, let K_n be an increasing sequence of compact sets such that $G = \bigcup_n K_n$. Let μ_n denote the restriction of μ to K_n .

$$\|\mu - \mu_n\| = \int_{K_n^c} d|\mu| \rightarrow 0$$

because $|\mu|$ is finite and countably additive. For fixed n ,

$$\begin{aligned}
 & \limsup_i \|(\lambda \otimes \sigma)_\mu \tilde{f}_i - \widetilde{\sigma_\mu f_i}\|_p \\
 & \leq \limsup_i \left(\|(\lambda \otimes \sigma)_{\mu_n} \tilde{f}_i - \widetilde{\sigma_{\mu_n} f_i}\|_p + \|f\|_p \|\mu - \mu_n\| \right) \\
 & = \|f\|_p \|\mu - \mu_n\|.
 \end{aligned}$$

Since $\|\mu - \mu_n\|$ is arbitrarily small when n is large we get the claim. □

We are now able to prove our result:

Theorem 2.4. *Let G be a second-countable locally compact group acting amenably on a standard Borel space with quasi-invariant σ -finite measure m . Fix a finite Radon measure μ on G . Define, as before,*

$$\sigma_\mu f(x) = \int_G f(g^{-1}x) (r(g, x))^{\frac{1}{p}} \alpha(g, x) d\mu(g) .$$

Then the operator norm of σ_μ acting on $L^p(X)$ is bounded by the operator norm of λ_μ acting on $L^p(G)$. In other words,

$$(4) \quad \|\sigma_\mu\|_{p,p} \leq \|\lambda_\mu\|_{p,p} .$$

Proof. Define

$$(UF)(g, x) = F(g, gx) (r(g^{-1}, x))^{\frac{1}{p}} \alpha(g^{-1}, x) = \sigma(g^{-1})F(g, x)$$

for every $F \in L^p(G \times X)$. Observe that U intertwines $\lambda \otimes \sigma$ to $\lambda \otimes i$ where

$$(\lambda \otimes i)(g)F(h, x) = F(g^{-1}h, x).$$

Observe that U^{-1} is also well defined:

$$U^{-1}F(g, x) = \sigma(g)F(g, x)$$

and $\|UF\|_p = \|F\|_p = \|U^{-1}F\|_p$, so that $\|(\lambda \otimes \sigma)_\mu\|_{p,p} = \|(\lambda \otimes i)_\mu\|_{p,p}$. Since obviously $\|(\lambda \otimes i)_\mu\|_{p,p} \leq \|\lambda_\mu\|_{p,p}$, it is enough to compare the norm of σ_μ with the norm of $(\lambda \otimes \sigma)_\mu$.

Fix $f \in L^p(X)$. Let \tilde{f}_i and $\widetilde{\sigma_\mu f_i}$ be as in Lemma 2.3. Observe that

$$\|f\|_p = \|\tilde{f}_i\|_p \quad \text{and} \quad \|\sigma_\mu f\|_p = \|\widetilde{\sigma_\mu f_i}\|_p.$$

Since

$$\begin{aligned} \|\widetilde{\sigma_\mu f_i}\|_p &\leq \|(\lambda \otimes \sigma)_\mu \tilde{f}_i\|_p + \|(\lambda \otimes \sigma)_\mu \tilde{f}_i - \widetilde{\sigma_\mu f_i}\|_p \\ &\leq \|(\lambda \otimes \sigma)_\mu\|_{p,p} \|\tilde{f}_i\|_p + \|(\lambda \otimes \sigma)_\mu \tilde{f}_i - \widetilde{\sigma_\mu f_i}\|_p \\ &= \|(\lambda \otimes \sigma)_\mu\|_{p,p} \|f\|_p + \|(\lambda \otimes \sigma)_\mu \tilde{f}_i - \widetilde{\sigma_\mu f_i}\|_p, \end{aligned}$$

we also have

$$\|\sigma_\mu f\|_p \leq \|(\lambda \otimes \sigma)_\mu\|_{p,p} \|f\|_p + \|(\lambda \otimes \sigma)_\mu \tilde{f}_i - \widetilde{\sigma_\mu f_i}\|_p.$$

Taking the limit as $i \rightarrow \infty$ we get

$$\|\sigma_\mu f\|_p \leq \|(\lambda \otimes \sigma)_\mu\|_{p,p} \|f\|_p,$$

which gives

$$\|\sigma_\mu f\|_p \leq \|(\lambda \otimes \sigma)_\mu\|_{p,p} \|f\|_p = \|(\lambda \otimes i)_\mu\|_{p,p} \|f\|_p \leq \|\lambda_\mu\|_{p,p} \|f\|_p. \quad \square$$

Remark 2.5. As an application take $G = SL(2, R)$ and $X = P^1$, the projective line. Since $X \simeq SL(2, R)/P$ where $P = \begin{pmatrix} a & 0 \\ b & a^{-1} \end{pmatrix}$, X is an amenable G space. The G action is given by fractional linear transformations

$$g^{-1}x = \frac{ax + b}{cx + d} \quad \text{for} \quad g^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

and the Radon–Nikodým derivative $r(g, x)$ is given by $r(g, x) = \frac{1}{(cx+d)^2}$. For every $f \in L^p(X)$ define

$$R_t f(x) = \frac{f(g^{-1}x)}{(cx + d)^{\frac{2}{p} + it}}$$

or, more generally,

$$R_\mu f(x) = \int_G \frac{f(g^{-1}x)}{(cx + d)^{\frac{2}{p} + it}} d\mu(g).$$

Then R_t is an isometry for every real t and $1 \leq p < \infty$, while $\|R_\mu\| \leq \|\lambda_\mu\|$.

More generally, we can apply our result to boundaries of symmetric spaces: $X = G/MAN$ (so we can transfer to the corresponding principal series representations).

We can also handle symmetric spaces: $X = G/K$ (however, since K is compact, a more direct argument works better).

Remark 2.6. Assume that μ (not necessarily given by a Radon measure) is a bounded convolver of $L^p(G)$ for some p ($1 \leq p < \infty$). Suppose also that G is amenable: another phenomenon related to transference says that such a μ is also a bounded convolver of $L^2(G)$ (see [4]). We want to remark that such a phenomenon *does not* hold in the case of amenable actions: take the free group F_r on r generators ($r \geq 2$) acting on itself; this action is amenable since the subgroup $\{e\}$ is compact and $X = F_r/e$. It is well known that there exists a bounded convolver of $\ell^p(F_r)$ that is not a convolver of $\ell^q(F_r)$ for any $q \neq p$; see [7]. Moreover, in the same paper, it is shown that for every positive M there exists a function f with finite support such that $\|\lambda_f\|_{2,2} > M\|\lambda_f\|_{p,p}$ (λ denotes, as usual, the left regular representation of F_r).

Theorem 2.7. *Let G and X be as in Theorem 2.4. Fix any sequence $\{\mu_n\}$ of finite Radon measures on G . Define*

$$M_\sigma f(x) = \sup_n |\sigma_{\mu_n} f|(x) = \sup_n \left| \int_G f(g^{-1}x)(r(g,x))^{\frac{1}{p}} \alpha(g,x) d\mu_n(g) \right|.$$

Analogously define

$$M_\lambda f(h) = \sup_n |\lambda_{\mu_n} f|(h) = \sup_n \left| \int_G f(g^{-1}h) d\mu_n(g) \right|.$$

Then the operator norm of M_σ acting on $L^p(X)$ is bounded by the operator norm of the corresponding M_λ acting on $L^p(G)$.

Sketch of the proof. Notation as in Theorem 2.4. It is enough to show that, for every positive integer L ,

$$\|M_{\{\sigma,L\}} f\|_p \leq \|M_{\{\lambda,L\}}\|_{p,p} \|f\|_p$$

where

$$M_{\{\sigma,L\}} f(x) = \sup_{1 \leq n \leq L} |\sigma_{\mu_n} f|(x)$$

and $M_{\{\lambda,L\}}$ is defined analogously. Let

$$\begin{aligned} \widetilde{M_{\{\sigma,L\}} f_i}(g,x) &= (\phi_i(g,x))^{\frac{1}{p}} M_{\{\sigma,L\}} f(x) \\ &= (\phi_i(g,x))^{\frac{1}{p}} \sup_{1 \leq n \leq L} \left| \int_G f(g^{-1}x)(r(g,x))^{\frac{1}{p}} \alpha(g,x) d\mu_n(g) \right| \\ &= \sup_{1 \leq n \leq L} \left| (\phi_i(g,x))^{\frac{1}{p}} \int_G f(g^{-1}x)(r(g,x))^{\frac{1}{p}} \alpha(g,x) d\mu_n(g) \right| \\ &= \sup_{1 \leq n \leq L} |\widetilde{\sigma_{\mu_n} f_i}|(x). \end{aligned}$$

Again we have

$$\|M_{\{\sigma,L\}} f\|_p = \|\widetilde{M_{\{\sigma,L\}} f_i}\|_p.$$

Remember the definition of $U^{-1} : L^p(G \times X) \rightarrow L^p(G \times X)$:

$$(U^{-1})F(g,x) = F(g,g^{-1}x)(r(g,x))^{\frac{1}{p}} \alpha(g,x) = \sigma(g)F(g,x).$$

Put

$$VF(g,x) = F(g,gx)(r(g^{-1},x))^{\frac{1}{p}}$$

so that

$$(V^{-1})F(g, x) = F(g, g^{-1}x)(r(g, x))^{\frac{1}{p}}$$

and V is an isometry.

Observe that

$$V^{-1}(\sup_n |F_n|) = \sup_n (|U^{-1}F_n|).$$

For every $f \in L^p(G \times X)$ write $f = U^{-1}F$ and evaluate

$$\begin{aligned} M_{\{\lambda \otimes \sigma, L\}} f(g, x) &= \sup_{1 \leq n \leq L} |(\lambda \otimes \sigma)_{\mu_n} f|(g, x) = \sup_{1 \leq n \leq L} |(\lambda \otimes \sigma)_{\mu_n} U^{-1}F|(g, x) \\ &= \sup_{1 \leq n \leq L} |U^{-1}(\lambda \otimes i)_{\mu_n} F|(g, x) = V^{-1} \sup_{1 \leq n \leq L} |(\lambda \otimes i)_{\mu_n} F|(g, x) \\ &= V^{-1}(M_{\{\lambda \otimes i, L\}})F(g, x). \end{aligned}$$

Hence

$$(5) \quad \|M_{\{\lambda \otimes \sigma, L\}}\|_{p,p} = \|M_{\{\lambda \otimes i, L\}}\|_{p,p} \leq \|M_{\{\lambda, L\}}\|_{p,p}.$$

Compute

$$(6) \quad \|M_{\{\sigma, L\}} f\|_p = \|\widetilde{M_{\{\sigma, L\}} f_i}\|_p \leq \|\widetilde{M_{\{\sigma, L\}} f_i} - M_{\{\lambda \otimes \sigma, L\}} \tilde{f}_i\|_p + \|M_{\{\lambda \otimes \sigma, L\}}\|_{p,p} \|f\|_p.$$

Since

$$(7) \quad \left| \sup_{1 \leq n \leq L} |\widetilde{\sigma_{\mu_n} f_i}|(g, x) - \sup_{1 \leq n \leq L} |(\lambda \otimes \sigma)_{\mu_n} \tilde{f}_i|(g, x) \right| \leq \sum_{n=1}^L |\widetilde{\sigma_{\mu_n} f_i} - (\lambda \otimes \sigma)_{\mu_n} \tilde{f}_i|(g, x),$$

one has

$$\|\widetilde{M_{\{\sigma, L\}} f_i} - M_{\{\lambda \otimes \sigma, L\}} \tilde{f}_i\|_p \leq \sum_{n=1}^L \|\widetilde{\sigma_{\mu_n} f_i} - (\lambda \otimes \sigma)_{\mu_n} \tilde{f}_i\|_p \rightarrow 0.$$

Conclude the proof as in Theorem 2.4. □

ADDED AFTER PROOF

After this paper was accepted the authors were informed by A. Nevo that there was an old preprint of S. Durand and R. Feres which, using different language, presents very similar ideas.

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MATHEMATICAL INSTITUTE, WROCLAW UNIVERSITY, PL. GRUNWALDZKI 2/4, 50-384 WROCLAW,
POLAND

E-mail address: `hebisch@math.uni.wroc.pl`

DIPARTIMENTO DI MATEMATICA, UNIVERSITÀ DI MILANO “BICOCCA”, VIA R. COZZI 53, EDIFICIO U5, 20125 MILANO, ITALIA

E-mail address: `kuhn@matapp.unimib.it`