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POROSITY AND HYPERCYCLIC OPERATORS

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Abstract. We study if the set of hypercyclic vectors of a hypercyclic operator is the complement of a σ-porous set. This leads to interesting results for both points of view: a limitation of the size of hypercyclic vectors, and new examples of first category sets which are not σ-porous.

1. Introduction

Let X be a separable F-space (namely a topological vector space whose topology is induced by a complete invariant metric ρ). An operator T on X is said to be hypercyclic provided there exists a vector x in X such that the orbit $Orb(x) = \{T^n x;\ n \geq 0\}$ of x under the action of T is dense in X. Such a vector is called a *hypercyclic* vector for T, and we denote by $HC(T)$ the set of its hypercyclic vectors.

Many operators have been proved to be hypercyclic. The first example goes back to Birkhoff [Bi]. Let $\mathcal{H}(\mathbb{C})$ be the space of holomorphic functions on $\mathbb{C}$, endowed with the topology of uniform convergence on compact sets. The usual metric on $\mathcal{H}(\mathbb{C})$ is given by

$$\rho(f,g) = \sum_{n\geq 1} \frac{1}{2^n}\left(\frac{\sup_{|z|\leq n}|f(z)-g(z)|}{1+\sup_{|z|\leq n}|f(z)-g(z)|}\right)$$

for $f, g \in \mathcal{H}(\mathbb{C})$. Let T be the operator of translation by one on $\mathcal{H}(\mathbb{C})$, defined by $Tf(z) = f(z+1)$. Then T is hypercyclic.

The first hypercyclic operator in the context of Banach spaces was exhibited by Rolewicz [Rol]. Let B be the backward shift on ℓ^p $(1 \leq p < +\infty)$ or c_0:

$$B(x_1, x_2, \dots) = (x_2, x_3, \dots).$$

B itself is not hypercyclic since it is a contraction, but any scalar multiple λB, for $\lambda > 1$, is hypercyclic on ℓ^p or c_0. More generally, Salas has characterized in [Sal] the hypercyclic weighted shifts. An excellent account on the subject of hypercyclicity up to 1999 can be found in [Gre].

A strange phenomenon appears during the study of $HC(T)$ for a given hypercyclic operator T: as soon as T is hypercyclic, the set of its hypercyclic vectors is very large. Indeed, $HC(T)$ is topologically generic (it contains a dense G_δ set) and algebraically generic (it contains a dense subspace, except 0, see [Bou]). A natural

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question arises: can we replace the statement that $[HC(T)]^c$ is of first category by a more restrictive one, using by instance the geometric notion of σ-porous sets?

Definition 1. Let (E,d) be a metric space. A subset $A \subset E$ is said to be *porous* at x provided there exists a positive number λ such that, for any $\delta > 0$, there exists $y \in E$ satisfying

$$0 < d(y,x) < \delta \text{ and } B(y, \lambda d(y,x)) \cap A = \varnothing.$$

A is *porous* if it is porous at all points of A; it is λ-porous if it is porous at all points with the same constant λ. Porosity is a more restricted notion than nowhere dense; it is something like nowhere dense with estimates.

A set A is said to be σ-porous if it is a countable union of porous sets. Our reference on σ-porous sets is [Za1]. σ-porous sets are very thin. On $\mathbb{R}^n$, a σ-porous set is of first category and has measure 0. However, the question above is meaningful because L. Zajicek proved that in any complete metric space without any isolated points (in particular, in any F-space), the notion of first category and σ-porous sets are distinct [Za2].

Let us return to the case of hypercyclic operators. We call a σ-porous hypercyclic operator an operator whose set of non-hypercyclic vectors is σ-porous. In Section 2, we prove that in two important classes of hypercyclic operators (weighted shifts, composition operators), we can find natural examples of non-σ-porous hypercyclic operators. On the contrary, there exist σ-porous hypercyclic operators, and in Section 3 we exhibit two examples of such operators. The first one is in the context of Banach spaces, but the σ-porosity is trivial. The second one is the translation operator of Birkhoff.

These results seem interesting, because this is the first limitation on the size of hypercyclic vectors. Observe also that it is not so easy to exhibit a first category set which is not σ-porous. So Section 2 supplies new examples of sets of this type.

2. EXAMPLES OF NON-σ-POROUS HYPERCYCLIC OPERATORS

We begin by several lemmas. The first one is a very useful technical tool in the theory of σ-porous sets. A proof can be found in [MMPZ].

Lemma 1. *Let $0 < \lambda < 1$. Then any σ-porous set A can be expressed as the union of a sequence of sets (A_n) such that A_n is porous at any point with constant greater than λ.*

We shall say that A is σ-λ-porous to express that $A = \bigcup_{n\geq 1} A_n$, where each A_n is λ-porous. Lemma 1 says that each σ-porous set is in fact σ-λ-porous, for any $\lambda \in]0,1[$. Our second lemma is a way to prove that a family of closed sets contains no σ-1/2-porous sets. It is known in the literature as Foran's lemma (see [Za1]), and we formulate it for the special case of σ-1/2-porosity :

Lemma 2. *Suppose that $\mathcal{F}$ is a nonempty family of nonempty closed sets such that for each $F \in \mathcal{F}$, and each open ball $B(x,r)$ with $F \cap B(x,r) \neq \varnothing$, there exists $G \in \mathcal{F}$ such that $G \cap B(x,r) \neq \varnothing$, $G \cap B(x,r) \subset F \cap B(x,r)$ and $F \cap B(x,r)$ is 1/2-porous at no point of $G \cap B(x,r)$. Then no set from $\mathcal{F}$ is σ-1/2-porous.*

Combining Lemma 1 and Lemma 2, no set of such a family is in fact σ-porous. The last lemma is a very easy result on complex numbers.

Lemma 3. *Let u, v be complex numbers, and $0 < \lambda < 1$. Then there exists $w \in \mathbb{C}$ such that*

$$|v - w| = \lambda |v - u| \text{ and } |w| \geq \lambda |u|.$$

Proof. We may write $v = |v| e^{i\theta}$. We set $w = (|v| + \lambda |u - v|) e^{i\theta}$. We then have

$$|w - v| = \lambda |u - v|.$$

On the other hand, by using the reverse triangle inequality, one obtains

$$|w| \geq |v| + \lambda |u| - \lambda |v| \geq \lambda |u|.$$

□

The rest of this section will be devoted to the proof of our main results. Let T be a hypercyclic bilateral bounded weighted shift acting on $\ell^2(\mathbb{Z})$ and $(w_n)_{n \in \mathbb{Z}}$ its weight sequence. Namely, $Te_n = w_n e_{n-1}$ where, as usual, $(e_n)_{n \in \mathbb{Z}}$ is the canonical basis of $\ell^2(\mathbb{Z})$.

Theorem 1. *If the series $\sum_{n \geq 1} \frac{1}{(w_1 \ldots w_n)^2}$ converges, then the set of non-hypercyclic vectors for T is not σ-porous.*

For instance, if $w_n = 2$ for any $n \geq 1$ and $w_n = \frac{1}{2}$ for any $n \leq 0$, T is a non-σ-porous hypercyclic operator. As will be clear after the proof, this Theorem remains true for a unilateral backward shift acting on $\ell^2(\mathbb{N})$ satisfying the same condition. Hence, $2B$ is a non-σ-porous hypercyclic operator.

Proof. Fix (x^j) a dense sequence in ℓ^2, with $x^1 = 0$. The set of non-hypercyclic vectors for T is

$$\begin{aligned} [HC(T)]^c &= \bigcup_{j,k \geq 1} \left\{ x \in \ell^2;\ \forall n \geq 1,\ \left\| T^n x - x^j \right\| \geq \frac{1}{k} \right\} \\ &:= \bigcup_{j,k \geq 1} F_{j,k}. \end{aligned}$$

In particular, one has the following inclusion:

$$G = \left\{ x = (x_n) \in \ell^2;\ \forall n \geq 1,\ |w_1 \ldots w_n x_n| \geq 1 \right\} \subset F_{1,1} \subset [HC(T)]^c.$$

It is enough to prove that G is not a σ-porous set, or by Lemma 1, a σ-1/2-porous set. For $\varepsilon = (\varepsilon_n)_{n \geq 1}$ a sequence of positive numbers in $\ell^2(\mathbb{N})$, denote by F_ε the nonempty closed set

$$F_\varepsilon = \left\{ x = (x_n) \in \ell^2;\ \forall n \geq 1,\ |x_n| \geq \varepsilon_n \right\},$$

and by $\mathcal{F}$ the set of all F_ε. Observe that $G = F_\varepsilon$, with $\varepsilon_n = \frac{1}{w_1 \ldots w_n}$. Therefore, it is enough to prove that $\mathcal{F}$ satisfies the assumptions of Lemma 2. Now, take ε to be such a sequence, $x \in \ell^2$ and $r > 0$ such that $B(x, r) \cap F_\varepsilon \neq \varnothing$. We should first build another sequence ε'. Let y belong to $B(x, r) \cap F_\varepsilon$, and let $0 < \mu < 1$ such that $\|x - y\| < \mu r$. One may fix $N \geq 1$ such that one has

$$\sum_{n \geq N} \left(|x_n| + 3\varepsilon_n \right)^2 < (1 - \mu) r^2.$$

On the other hand, we consider $\delta > 0$ sufficiently small so that the following inequality holds:

$$\sum_{1 \leq n < N} \left(|x_n - y_n| + \delta \right)^2 < \mu r^2.$$

We define $\varepsilon' = (\varepsilon'_n)$ by setting

$$\begin{cases} \varepsilon'_n &= \varepsilon_n + \delta \quad \text{if } 1 \leq n < N, \\ \varepsilon'_n &= 3\varepsilon_n \quad \text{otherwise.} \end{cases}$$

We finally define $z = (z_n)$ by setting

$$\begin{cases} z_n &= x_n \quad \text{if } n \leq 0, \\ z_n &= (|y_n| + \delta)\, e^{i\theta_n} \quad \text{if } 1 \leq n < N, \\ z_n &= \varepsilon'_n \quad \text{otherwise,} \end{cases}$$

where θ_n is defined by $y_n = |y_n| e^{i\theta_n}$. It is straightforward to check that z belongs to $F_{\varepsilon'}$ and that $F_{\varepsilon'}$ is a subset of F_ε. Moreover,

$$\|z - x\|^2 \leq \sum_{1 \leq n < N} (|y_n - x_n| + \delta)^2 + \sum_{n \geq N} (|x_n| + 3\varepsilon_n)^2 < r^2,$$

and so $F_{\varepsilon'} \cap B(x, r)$ is nonempty. It remains to prove that $F_\varepsilon \cap B(x, r)$ is 1/2-porous at no point of $F_{\varepsilon'} \cap B(x, r)$. To this end, take u in $F_{\varepsilon'} \cap B(x, r)$ and take any v in $B\left(u, \min\left(\delta, \frac{r - \|x-u\|}{2}\right)\right)$. We claim that

$$B\left(v, \frac{\|u - v\|}{2}\right) \cap F_\varepsilon \cap B(x, r) \neq \varnothing.$$

First, observe that by construction $B\left(v, \frac{\|u-v\|}{2}\right) \subset B(x, r)$. Second, by Lemma 3, for any $n \geq N$, one can find w_n such that $|w_n - v_n| \leq \dfrac{|u_n - v_n|}{3}$ and $|w_n| \geq \dfrac{|u_n|}{3}$. For $n < N$, we set $w_n = v_n$. Clearly, $w = (w_n)$ belongs to $B\left(v, \frac{\|u-v\|}{2}\right)$. Now, if $1 \leq n < N$, one has

$$|w_n| \geq |v_n| \geq |u_n| - \delta \geq \varepsilon_n.$$

On the contrary, if $n \geq N$, one has

$$|w_n| \geq \frac{|u_n|}{3} \geq \varepsilon_n.$$

Thus, w belongs to $B\left(v, \frac{\|u-v\|}{2}\right) \cap F_\varepsilon$, and this achieves the proof. $\square$

The techniques used in Theorem 1 may also be applied to prove that some classical hypercyclic composition operators are non-σ-porous hypercyclic operators. Let ϕ be a hyperbolic or a parabolic automorphism of the unit disk $\mathbb{D}$ (for background materials on this, we refer to [Sh]). It is well known that ϕ induces a bounded composition operator $C_\phi(f) = f \circ \phi$ on the classical Hardy space $H^2(\mathbb{D})$ and that C_ϕ is hypercyclic.

Theorem 2. *C_ϕ is a non-σ-porous hypercyclic operator.*

Proof. We restrict ourselves to the case where ϕ has fixed points in $\{-1, 1\}$, with Denjoy-Wolff point 1. Hence, one may write $\phi(z) = (z + r)/(1 + rz)$, where $r = (a-1)/(a+1)$ and $a > 1$ if ϕ is hyperbolic, or $\phi(z) = [(s - 2i)z - s]/(sz - s - 2i)$ where s is a nonzero real number if ϕ is parabolic. For $n \in \mathbb{Z}$, denote by $z_n = \phi_n(0)$, where $\phi_n = \phi \circ \cdots \circ \phi$ is the n-th iterate of ϕ if $n \geq 0$, $\phi_n = \phi^{-1} \circ \cdots \circ \phi^{-1}$ if $n < 0$. One has $z_n = (a^n - 1)/(a^n + 1)$ in the hyperbolic case, $z_n = ns/(ns + 2i)$

in the parabolic one. By a result of Hurst [Hu] (see also [NRW]), C_ϕ is similar to an operator of the form

$$T = \begin{pmatrix} W & 0 \\ A & B \end{pmatrix}$$

where W is the weighted bilateral shift on $\ell^2(\mathbb{Z})$ with weight sequence $w_n = \dfrac{\sqrt{1-|z_n|^2}}{\sqrt{1-|z_{n+1}|^2}}$. In the hyperbolic case, the sequence (w_n) has limit $\sqrt{a}$ at $+\infty$, and therefore $\sum_{n\geq 1} \dfrac{1}{(w_1\dots w_n)^2} < +\infty$. In the parabolic case, one has $w_n = \dfrac{\sqrt{(n+1)^2s^2+4}}{\sqrt{n^2s^2+4}}$, so that

$$(w_1\dots w_n)^2 = \frac{(n+1)^2s^2+4}{s^2+4},$$

and the same is true. Now,

$$[HC(T)]^c \supset \left\{\big((x_n),(y_n)\big) \in \ell^2\times l^2;\ \forall n\geq 1,\ |w_1\dots w_n x_n| \geq 1\right\}.$$

As in Theorem 1, one proves that this set is not σ-porous, and this gives the result. □

3. Examples of σ-porous operators

In view of the results of the previous section, one may expect that the complement of the set of hypercyclic vectors is never a σ-porous set. This is far from being the case. Indeed, C. Read has exhibited in [Re] an operator on ℓ^1 such that all vectors, except zero, are hypercyclic, and clearly $\{0\}$ is σ-porous!

This example is in some sense trivial. We give next a concrete example of a hypercyclic operator, whose set of non-hypercyclic vectors is a σ-porous set which is not reduced to $\{0\}$.

Theorem 3. *Let T be the operator of translation by one on $\mathcal{H}(\mathbb{C})$, equipped with the usual metric. T is a σ-porous hypercyclic operator.*

In one sense, one can say that the operator of translation is more hypercyclic than twice the backward shift: it has more hypercyclic vectors. Before proceeding with the proof of Theorem 3, one should introduce some notation. If K is a compact subset of $\mathbb{C}$, and if f belongs to $\mathcal{H}(\mathbb{C})$, we denote by $\|f\|_K$ the supremum of $|f(z)|$ on K. Moreover, K_n means the closed ball of center 0 and of radius n in $\mathbb{C}$. The following elementary lemma will be useful (recall that the distance ρ has been defined in the introduction).

Lemma 4. *Let f, g be in $\mathcal{H}(\mathbb{C})$ and r be a positive integer. The following inequalities hold.*

(1) $\rho(f,g) \leq \|f-g\|_{K_r} + \dfrac{1}{2^r}$.

(2) *Suppose that $2^r\rho(f,g) < \alpha < 1$. Then one has*

$$\|f-g\|_{K_r} \leq \frac{\alpha}{1-\alpha}.$$

Proof. (1) We have

$$\begin{aligned}\rho(f,g) &\leq \sum_{n=1}^{r}\frac{1}{2^n}\times\frac{\|f-g\|_{K_r}}{1+\|f-g\|_{K_r}}+\sum_{n=r+1}^{+\infty}\frac{1}{2^n}\\ &\leq \|f-g\|_{K_r}+\frac{1}{2^r}.\end{aligned}$$

(2) From the inequality

$$\rho(f,g)\geq\frac{1}{2^r}\times\frac{\|f-g\|_{K_r}}{1+\|f-g\|_{K_r}},$$

we deduce

$$\|f-g\|_{K_r}\leq\frac{2^r\rho(f,g)}{1-2^r\rho(f,g)}.$$

The monotonicity of the function $x\mapsto x/(1-x)$ on $]0,1[$ gives the conclusion. □

Let us now proceed with the proof of Theorem 3. We denote by (P_j) an exhaustive sequence of all polynomials with coefficients in $\mathbb{Q}+i\mathbb{Q}$. It is plain that

$$[HC(T)]^c=\bigcup_{j\geq 1}\bigcup_{k\geq 2}F(j,k),$$

where $F(j,k)$ is the closed set defined by

$$F(j,k)=\left\{f\in\mathcal{H}(\mathbb{C});\forall n\geq 1,\rho(T^nf,P_j)\geq\frac{1}{k}\right\}.$$

We shall prove that each $F(j,k)$ is porous. Given a couple (j,k), we set $p=\left[\frac{\ln 2k}{\ln 2}\right]+1$, where $[x]$ means the integer part of x. We claim that $F(j,k)$ is λ-porous, with λ any positive number satisfying the following inequality:

$$\frac{2^{3p+2}\lambda}{1-2^{3p+2}\lambda}<\frac{1}{4k}.$$

Observe that if λ is close enough to 0, the previous condition is fulfilled, and that it implies $2^{3p+2}\lambda<1$. Now fix $f\in F(j,k)$ and $\delta>0$. Let N be an integer such that

$$\frac{1}{2^N}\leq\frac{\delta}{k}<\frac{1}{2^{N-1}}.$$

By Mergelyan's Theorem, there exists a polynomial g_0 such that

$$\left\{\begin{aligned}\|f-g_0\|_{K_N} &< \frac{\delta}{k},\\ \|T^{-N-2p}P_j-g_0\|_{N+2p+K_p} &< \frac{1}{4k}.\end{aligned}\right.$$

($N+2p+K_p$ is the closed ball of center $N+2p$ and of radius p.) By Lemma 4, item (1), one has

$$\rho(f,g_0)<\frac{\delta}{k}+\frac{1}{2^N}\leq\frac{2\delta}{k}\leq\delta.$$

We now pick any $g\in\mathcal{H}(\mathbb{C})$ such that $\rho(g,g_0)\leq\lambda\rho(f,g_0)\leq\frac{2\lambda\delta}{k}$ and prove that g does not belong to $F(j,k)$. Indeed, we deduce from

$$2^{N+3p}\rho(g,g_0)\leq 2^{3p+2}\lambda<1$$

and from Lemma 4 item (2) the following inequality:

$$\|g-g_0\|_{K_{N+3p}} \leq \frac{2^{3p+2}\lambda}{1-2^{3p+2}\lambda}.$$

This in turn implies

$$\begin{aligned}\left\|T^{N+2p}g-P_j\right\|_{K_p} &\leq \left\|T^{N+2p}g-T^{N+2p}g_0\right\|_{K_p}+\left\|T^{N+2p}g_0-P_j\right\|_{K_p}\\ &\leq \|g-g_0\|_{K_{N+3p}}+\left\|g_0-T^{-N-2p}P_j\right\|_{N+2p+K_p}\\ &\leq \frac{2^{3p+2}\lambda}{1-2^{3p+2}\lambda}+\frac{1}{4k}\\ &\leq \frac{1}{2k}.\end{aligned}$$

By using Lemma 4, item (1), one last time, one finally gets

$$\rho(T^{N+2p}g,P_j)\leq\frac{1}{2k}+\frac{1}{2^p}<\frac{1}{k}.$$

g does not belong to $F(j,k)$. □

Remarks.

(1) The previous proof actually yields a more precise statement: $[HC(T)]^c$ is a union of closed σ-porous sets.

(2) A key argument in the previous proof is the fact that the metric ρ is non-homogeneous. In particular, with the notation of the proof, there exists a *single* $n\in\mathbb{N}$ such that, for any $g\in\mathcal{H}(\mathbb{C})$ with $\rho(g,g_0)\leq\lambda\rho(f,g_0)$, one has $\rho(T^ng,P_j)<1/k$. In the context of Banach spaces, such an argument seems difficult to use. Indeed, let X be a separable Banach space, T a bounded operator on X, and (x^j) a dense sequence in X with $x^1=0$. Suppose that there exists $u\in X$ such that $\|T^nu\|\to+\infty$ (this is true for a large class of hypercyclic operators). Keeping the notation of the proof of Theorem 1, namely

$$F(j,k)=\left\{x\in X;\ \forall n\geq1,\ \|T^nx-x^j\|\geq1/k\right\},$$

u belongs to $F(1,k)$ provided k is large enough. Assume that $F(1,2k)$ is λ-porous at u. We fix N such that $\|T^nu\|\geq\frac{4}{\lambda k}$ for $n\geq N$. Let $\delta>0$ and $y\in B(u,\delta)$ such that $B(y,\lambda\|y-u\|)\cap F(1,2k)=\varnothing$. Provided δ is sufficiently small, we may assume that $\|T^ny\|\geq\frac{1}{2k}$ for $n\leq N$. Since y does not belong to $F(1,2k)$, there exists $n>N$ such that $\|T^ny\|<\frac{1}{2k}$. Now, set $z=y+\frac{\lambda}{2}(y-u)$. One has

$$\begin{aligned}\|T^nz\| &\geq \frac{\lambda}{2}\|T^nu\|-\left(1+\frac{\lambda}{2}\right)\|T^ny\|\\ &\geq \frac{2}{k}-\frac{3}{4k}\geq\frac{1}{2k}.\end{aligned}$$

Hence, we cannot choose the same n for each z in $B(y,\lambda\|y-u\|)$. For this reason, it seems to be more complicated to build a σ-porous hypercyclic operator on a Banach space, and we end this paper by asking the following question.

Question 1. Does there exist a σ-porous hypercyclic operator on a Hilbert space?

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