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MORITA EQUIVALENCES BETWEEN SOME BLOCKS FOR p-SOLVABLE GROUPS

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Abstract. We prove that any Morita equivalence between some blocks with Abelian defect groups and cyclic inertia quotients for p-solvable groups is basic.

1. Let $\mathcal{O}$ be a complete discrete valuation ring of characteristic zero with an algebraically closed residue field k of characteristic p; further we assume that the quotient $\mathcal{K}$ of $\mathcal{O}$ is a splitting field for all finite groups below. Throughout the paper, all $\mathcal{O}$-modules are finitely generated and $\mathcal{O}$-algebras are $\mathcal{O}$-free; an $\mathcal{O}$-subalgebra of any $\mathcal{O}$-algebra A does not necessarily contain the identity element 1_A of A and any homomorphism between two $\mathcal{O}$-algebras does not necessarily preserve their identity elements. For an $\mathcal{O}$-algebra A, we denote by $J(A)$ the Jacobson radical of A and by A^* the multiplicative group of all invertible elements of A.

For a ring A, we denote by A° its opposite ring; if A is an $\mathcal{O}$-algebra, so is A°. Let G be a finite group. Obviously the map $G \longrightarrow G$, $x \mapsto x^{-1}$ induces an $\mathcal{O}$-algebra isomorphism $^\circ$ between $\mathcal{O}G$ and $(\mathcal{O}G)^\circ$; in particular, $(\mathcal{O}G)^\circ$ becomes an $\mathcal{O}G$-interior algebra (see 3) through $^\circ$. Let b be a block idempotent of the group algebra $\mathcal{O}G$ and let b° be the image of b in $(\mathcal{O}G)^\circ$ through $^\circ$. Then the algebra isomorphism $^\circ$ between $\mathcal{O}G$ and $(\mathcal{O}G)^\circ$ induces an $\mathcal{O}G$-interior algebra isomorphism (see 3) between $\mathcal{O}Gb^\circ$ and $(\mathcal{O}Gb)^\circ$, still denoted by $^\circ$ for convenience. Let G' be another group and let b' be a block idempotent of the group algebra $\mathcal{O}G'$. The $\mathcal{O}$-algebra isomorphism $^\circ$ above between block algebras applied to G' and b' always allows us to identify an $(\mathcal{O}Gb, \mathcal{O}G'b')$-bimodule M with an $\mathcal{O}(G\times G')$-module associated to the block $b \otimes b'^\circ$ ($\mathcal{O}(G \times G') \cong \mathcal{O}G \otimes_{\mathcal{O}} \mathcal{O}G'$). From now on, we always use the identification without further notice.

An indecomposable $\mathcal{O}(G\times G')$-module M associated with $b\otimes b'^\circ$ induces a Morita equivalence between $\mathcal{O}Gb$ and $\mathcal{O}G'b'$ if

$$M \otimes_{\mathcal{O}G'b'} M^* \cong \mathcal{O}Gb$$

as $\mathcal{O}(G \times G)$-modules and

$$M^* \otimes_{\mathcal{O}Gb} M \cong \mathcal{O}G'b'$$

as $\mathcal{O}(G' \times G')$-modules, where M^* is the dual of M. Let P'' be a vertex of the $\mathcal{O}(G \times G')$-module M and let the $\mathcal{O}P''$-module S'' be a source module of M. If S''

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is an *endo-permutation* $\mathcal{O}P''$-module (see [12]), then we say that M induces a *basic Morita equivalence* (see [8]) between $\mathcal{O}Gb$ and $\mathcal{O}G'b'$.

It is well known that all defect pointed groups of the block b of G over $\mathcal{O}$ exactly form a conjugacy class under the G-conjugation action (see [6]). For a defect pointed group P_γ on $\mathcal{O}Gb$, we denote by $N_G(P_\gamma)$ the stabilizer of P_γ under the G-conjugation action, by $E_G(P_\gamma)$ the usual inertia quotient $N_G(P_\gamma)/PC_G(P)$ of the block b, and by $(\mathcal{O}G)_\gamma$ a source algebra of $\mathcal{O}Gb$ corresponding to P_γ, where $C_G(P)$ is the centralizer of P in G. Here we would like to remind readers to see Puig's work [6] for more on defect pointed groups and source algebras, etc.

Theorem 2. *Let G and G' be p-solvable groups and let b and b' be blocks of G and G', respectively, with Abelian defect groups and cyclic inertia quotients. Then any Morita equivalence between $\mathcal{O}Gb$ and $\mathcal{O}G'b'$ is basic.*

3. Now we are ready to introduce some concepts for a remark on Theorem 2. Let G be a finite group and let $\mathcal{O}G$ be the group algebra of G over $\mathcal{O}$. An *$\mathcal{O}G$-interior* algebra A is an $\mathcal{O}$-algebra A endowed with a unitary $\mathcal{O}$-algebra homomorphism $\rho : \mathcal{O}G \to A$. For any $g \in G$ and $a \in A$, we define the left multiplication of g on A by $ga = \rho(g)a$ and the right multiplication of g on A by $ag = a\rho(g)$. Let A' be another $\mathcal{O}G$-interior algebra. An $\mathcal{O}$-algebra homomorphism $f : A \to A'$ is said to be *$\mathcal{O}G$-interior* if for any $g \in G$ and $a \in A$,

$$f(ga) = gf(a) \quad \text{and} \quad f(ag) = f(a)g \quad .$$

Furthermore f is an *embedding* of $\mathcal{O}G$-interior algebras A and A' if f is injective and $\mathrm{Im}(f) = f(1_A)A'f(1_A)$. We can also endow the tensor product $A \otimes_{\mathcal{O}} A'$ with an $\mathcal{O}G$-interior algebra structure through the homomorphism $\mathcal{O}G \longrightarrow A \otimes_{\mathcal{O}} A'$, $g \in G \longrightarrow g1_A \otimes g1_{A'}$.

Remark 4. Keep the same assumption as in Theorem 2. Let M be an $\mathcal{O}(G \times G')$-module inducing a Morita equivalence between $\mathcal{O}Gb$ and $\mathcal{O}G'b'$ in Theorem 2, let P'' be a vertex of it and let the $\mathcal{O}P''$-module S'' be a source of it. By Theorem 2, the $\mathcal{O}P''$-module S'' is an endo-permutation module. Let σ and σ' be the projections of P'' into G and G', respectively, and set $P = \sigma(P'')$ and $P' = \sigma'(P'')$. Then by [8, Cor. 7.4 and Th. 6.9], the group homomorphisms $\sigma : P'' \longrightarrow P$ and $\sigma' : P'' \longrightarrow P'$ are isomorphisms, P and P' are defect groups of b and b', respectively, and there exist defect pointed groups P_γ of b and $P'_{\gamma'}$ of b' such that we have the following $\mathcal{O}P$-interior algebra embedding:

$$(\mathcal{O}G)_\gamma \longrightarrow \mathrm{Res}_{\sigma^{-1}}(\mathrm{End}_{\mathcal{O}}(S'')) \otimes_{\mathcal{O}} \mathrm{Res}_{\sigma' \circ \sigma^{-1}}((\mathcal{O}G')_{\gamma'}) \quad .$$

5. Before beginning the proof of Theorem 2, we first start with the structure of source algebras of block algebras for p-solvable groups. It is well known that blocks of finite p-solvable groups were first systematically investigated by P. Fong [1], who determined their character theory; a complete characterization of their source algebras has been given in [8]; similar references also include [3] and [5].

Let G be a finite p-solvable group and let b be a block of the group algebra $\mathcal{O}G$ with a defect pointed group P_γ; suppose that P is Abelian. Then from [5, Prop. 3.1 and Prop. 3.5], one can easily conclude that there exist an indecomposable endo-permutation $\mathcal{O}P$-module N, a p'-subgroup L of $\mathrm{Aut}(P)$ and an $\mathcal{O}^*$-group $\hat{L}$ with $\mathcal{O}^*$-quotient L such that we have the following embedding of $\mathcal{O}P$-interior algebras:

$$(\mathcal{O}G)_\gamma \longrightarrow \mathrm{End}_{\mathcal{O}}(N) \otimes_{\mathcal{O}} \mathcal{O}_*(P \rtimes \hat{L}), \tag{5.1}$$

where $\hat{L}$ acts on P through the composition of the natural surjection $\hat{L} \longrightarrow L$ with the inclusion $L \subset \mathrm{Aut}(P)$. The embedding (5.1) is also Puig's description [7] of source algebras of block algebras for p-solvable groups in the abelian defect group case. Here a so-called $\mathcal{O}^*$-group $\hat{L}$ with $\mathcal{O}^*$-quotient L is just a central extension of L by $\mathcal{O}^*$ (see [7]); if $\mathcal{O} = k$, then we call $\hat{L}$ a k^*-group with k^*-quotient L (see [9]).

Remark 6. We can choose the $\mathcal{O}^*$-group $\hat{L}$ in (5.1) to be a k^*-group $\hat{L}^k$ with k^*-quotient L. Indeed, by [11, Chap. III, Prop. 8], there exists a canonical decomposition

$$\mathcal{O}^* \cong (1 + J(\mathcal{O})) \times k^* \quad ;$$

so we can identify k^* as a subgroup of $\mathcal{O}^*$ and then the canonical surjective homomorphism $\hat{L}/k^* \longrightarrow \hat{L}/\mathcal{O}^*$induces the following short exact sequence:

$$1 \longrightarrow \mathcal{O}^*/k^* \longrightarrow \hat{L}/k^* \longrightarrow L \longrightarrow 1.$$

Since L is a p'-group and $\mathcal{O}^*/k^* \cong 1 + J(\mathcal{O})$, by [9, Prop. 4.6], the sequence splits and there exists a subgroup $\hat{L}^k$ of $\hat{L}$ which is a k^*-group with k^*-quotient L.

Remark 7. Keep the same notation as in 5. Choose $i \in \gamma$ and set $(\mathcal{O}G)_\gamma = i\mathcal{O}Gi$. Denote by $(\mathcal{O}G)_\gamma^P$ the centralizer of Pi in $(\mathcal{O}G)_\gamma$ and by $N_{(\mathcal{O}G)_\gamma^*}(Pi)$ the normalizer of the group Pi in $(\mathcal{O}G)_\gamma^*$. On the one hand, by [9, 2.15.7, 2.15.9 and 2.16.3], there exists a canonical isomorphism between $E_G(P_\gamma)$ and $N_{(\mathcal{O}G)_\gamma^*}(Pi)/P((\mathcal{O}G)_\gamma^P)^*$. On the other hand, by [11, Chap. III, Prop. 8], there exists a canonical decomposition

$$((\mathcal{O}G)_\gamma^P)^* \cong (i + J((\mathcal{O}G)_\gamma^P)) \times k^*$$

extending the unique decomposition $\mathcal{O}^* \cong (1 + J(\mathcal{O})) \times k^*$. In this case, k^* can be identified with a subgroup of $((\mathcal{O}G)_\gamma^P)^*$ and then the group

$$N_{(\mathcal{O}G)_\gamma^*}(Pi)/P(i + J((\mathcal{O}G)_\gamma^P))$$

is a central extension of $E_G(P_\gamma)$ by k^*, denoted by $\hat{E}_G(P_\gamma)^\circ$.

In the embedding (5.1), if we choose $\hat{L}$ to be a k^*-group, then by [10, Prop. 5.11] and [4, 2.12.4], $\hat{E}_G(P_\gamma)^\circ$ is isomorphic to $\hat{L}$ as k^*-groups.

Lemma 8. *Let P be a common p-subgroup of finite groups G and G' and let b (resp. b') be a block of G (resp. G') and let P_γ (resp. $P_{\gamma'}$) be a defect pointed group of b (resp. b'). If there exist an indecomposable $\mathcal{O}P$-module S and an embedding of $\mathcal{O}P$-interior algebras $(\mathcal{O}G)_\gamma \longrightarrow \mathrm{End}_{\mathcal{O}}(S) \otimes_{\mathcal{O}} (\mathcal{O}G')_{\gamma'}$, then there exists an $\mathcal{O}(G \times G')$-bimodule M inducing a basic Morita equivalence between $\mathcal{O}Gb$ and $\mathcal{O}G'b'$.*

Proof. Let $\Delta(P)$ be the diagonal subgroup of $P \times P$ and let σ and σ' be the projections of $\Delta(P)$ into the first and second factor; obviously σ and σ' are group isomorphisms. Considering the $\mathcal{O}P$-module S as an $\mathcal{O}\Delta(P)$-module through the isomorphism σ, we can restate the $\mathcal{O}P$-interior algebra embedding as the following:

$$(\mathcal{O}G)_\gamma \longrightarrow \mathrm{Res}_{\sigma^{-1}}(\mathrm{End}_{\mathcal{O}}(S)) \otimes_{\mathcal{O}} \mathrm{Res}_{\sigma' \circ \sigma^{-1}}((\mathcal{O}G')_{\gamma'}) \quad .$$

Then [8, 6.12.2 and Theorem 6.9 and Theorem 7.2] imply that there exists an indecomposable $\mathcal{O}(G \times G')$-bimodule M inducing a basic Morita equivalence between $\mathcal{O}Gb$ and $\mathcal{O}G'b'$.

Proposition 9. *If the $\mathcal{O}(G \times G')$-module M (resp. $\mathcal{O}(G' \times G'')$-module M') induces a basic Morita equivalence between block algebras $\mathcal{O}Gb$ and $\mathcal{O}G'b'$ (resp. between* *block algebras $\mathcal{O}Gb'$ and $\mathcal{O}G'b''$), then $M\otimes_{\mathcal{O}G'}M'$ induces a basic Morita equivalence between $\mathcal{O}Gb$ and $\mathcal{O}G''b''$.*

Proof. Let $\mathcal{O}Q''$-module S'' and $\mathcal{O}Q'''$-module S''' be sources of the $\mathcal{O}(G\times G')$-module M and of the $\mathcal{O}(G'\times G'')$-module M', respectively. Let σ and σ' be the projections of Q'' into G and G' and let ρ and ρ' be the projections of Q''' into G' and G''; by [8, Cor. 7.4], all these projections are group isomorphisms. Since the images $\sigma'(Q'')$ and $\rho(Q''')$ are defect groups of b' by [8, Theorem 6.9], with a suitable choice of Q'' and Q''', we can assume that the images $\mathrm{Im}(\sigma')$ and $\mathrm{Im}(\rho)$ coincide. Set $P=\mathrm{Im}(\sigma)$, $P'=\mathrm{Im}(\rho)$ and $P''=\mathrm{Im}(\rho')$. Let π and π' be the projections of $P\times P''$ into P and P''. It is easy to check that we have the $\mathcal{O}(G\times G'')$-module isomorphism

$$\begin{aligned}&\mathrm{Ind}_{P\times P''}^{G\times G''}(\mathcal{O}G'\otimes_{\mathcal{O}}\mathrm{Res}_{\sigma^{-1}\circ\pi}(S'')\otimes\mathrm{Res}_{\rho'^{-1}\circ\pi'}(S'''))\\&\qquad\qquad\cong\mathrm{Ind}_{Q''}^{G\times G'}(S'')\otimes_{\mathcal{O}G'}\mathrm{Ind}_{Q'''}^{G'\times G''}(S''')\quad,\end{aligned}$$

mapping $(a\otimes b)\otimes(c\otimes s''\otimes s''')$ onto $((a\otimes c)\otimes s'')\otimes((1\otimes b)\otimes s''')$, where $a\in\mathcal{O}G$, $b\in\mathcal{O}G'', c\in\mathcal{O}G', s''\in S''$, $s'''\in S'''$, and $\mathcal{O}G'$ as an $\mathcal{O}(P\times P'')$-module is defined by

$$(x,y)d=((\rho\circ\rho'^{-1}\circ\pi')(x,y))d((\sigma'\circ\sigma^{-1}\circ\pi)(x^{-1},y^{-1}))$$

for $x\in P, y\in P''$ and $d\in\mathcal{O}G'$. Moreover as an $\mathcal{O}(P\times P'')$-module,

$$\mathcal{O}G'\cong\bigoplus_{z\in P'\backslash G'/P'}\mathrm{Ind}_{N_z}^{P\times P''}(\mathcal{O})\quad,$$

where $P'\setminus G'/P'$ denotes a set of all representatives of the double cosets of P' in G' and

$$N_z=\{((\sigma'\circ\sigma^{-1})(x),z((\sigma'\circ\sigma^{-1})(x))z^{-1})|x\in(\sigma\circ\sigma'^{-1})(z^{-1}P'z\cap P')\}$$

for any $z\in P'\setminus G'/P'$. So now we can conclude that the order of a vertex of any indecomposable direct summand of $\mathrm{Ind}_{Q''}^{G\times G'}(S'')\otimes_{\mathcal{O}G'}\mathrm{Ind}_{Q'''}^{G'\times G''}(S''')$ is less than or equal to $|P|=|P'|=|P''|$. Obviously $M\otimes_{\mathcal{O}G'}M'$ is an indecomposable direct summand of $\mathrm{Ind}_{Q''}^{G\times G'}(S'')\otimes_{\mathcal{O}G'}\mathrm{Ind}_{Q'''}^{G'\times G''}(S''')$ as $\mathcal{O}(G\times G'')$-modules, thus the order of its vertex is less than or equal to $|P|=|P'|=|P''|$. Then by [8, Theorem 6.9], the order of any vertex of $M\otimes_{\mathcal{O}G'}M'$ is forced to be equal to $|P|=|P''|$. Finally [8, Cor. 7.4] implies that any source module of $M\otimes_{\mathcal{O}G'}M'$ as $\mathcal{O}(G\times G'')$ is an endo-permutation module.

10. Let G and G' be finite p-solvable groups and let b and b' be blocks of the group algebras $\mathcal{O}G$ and $\mathcal{O}G'$ with defect pointed groups P_γ and $P'_{\gamma'}$, respectively. We also assume that P and P' are Abelian and that $E_G(P_\gamma)$ and $E_{G'}(P'_{\gamma'})$ are cyclic. Since G is p-solvable, by (5.1) and Remarks 6 and 7, there exists an indecomposable endo-permutation $\mathcal{O}P$-module S, a p'-subgroup L of $\mathrm{Aut}(P)$ and a k^*-group $\hat{L}$ with k^*-quotient L such that there exists the following embedding of $\mathcal{O}P$-interior algebras:

$$(\mathcal{O}G)_\gamma\longrightarrow\mathrm{End}_{\mathcal{O}}(S)\otimes_{\mathcal{O}}\mathcal{O}_*(P\rtimes\hat{L})\quad,\tag{10.1}$$

where the action of $\hat{L}$ on P lifts that of L on P and $\hat{L}$ is isomorphic to $\hat{E}_G(P_\gamma)^\circ$ as k^*-groups. Since $E_G(P_\gamma)$ is cyclic, $\hat{E}_G(P_\gamma)^\circ$ is isomorphic to $k^*\times L$. Therefore the embedding (10.1) can be restated as

$$(\mathcal{O}G)_\gamma \longrightarrow \mathrm{End}_{\mathcal{O}}(S) \otimes_{\mathcal{O}} \mathcal{O}(P \rtimes L) \quad .$$

Set $H = P \rtimes L$; by Lemma 8, there exists an $\mathcal{O}(G \times H)$-module N inducing a basic Morita equivalence between $\mathcal{O}Gb$ and $\mathcal{O}H$.

Similarly for G' and its block b', there also exists an $\mathcal{O}(G' \times H')$-module N' inducing a basic Morita equivalence between $\mathcal{O}G'b'$ and $\mathcal{O}H'$, where H' is the semi-direct product $P' \rtimes L'$ and L' is a p'-subgroup of $\mathrm{Aut}(P')$. Suppose that the $\mathcal{O}(G \times G')$-module M induces a Morita equivalence between $\mathcal{O}Gb$ and $\mathcal{O}G'b'$. Then the $\mathcal{O}(H \times H')$-module $N^* \otimes_{\mathcal{O}G} M \otimes_{\mathcal{O}G'} N'$ induces a Morita equivalence between $\mathcal{O}H$ and $\mathcal{O}H'$. Moreover by Proposition 9, the Morita equivalence induced by M between $\mathcal{O}Gb$ and $\mathcal{O}G'b'$ being basic is equivalent to the Morita equivalence induced by $N^* \otimes_{\mathcal{O}G} M \otimes_{\mathcal{O}G'} N'$ between $\mathcal{O}H'$ and $\mathcal{O}H$ being basic. Therefore Theorem 2 is reduced to prove that any Morita equivalence between $\mathcal{O}(P \rtimes L)$ and $\mathcal{O}(P' \rtimes L')$ is basic.

Note that if P is an Abelian group and E is an Abelian p'-subgroup of $\mathrm{Aut}(P)$, then $P \rtimes E$ is a p-constrained group, and the group algebra $\mathcal{O}(P \rtimes E)$ itself is a block algebra with the unique defect pointed group $P_{\{1\}}$. In greater generality, we will prove in Theorem 14 below that any Morita equivalence between such group algebras $\mathcal{O}(P \rtimes E)$ is basic, thus Theorem 2 is proved. In order to do so, we also need [8, Theorem B] to hold over $\mathcal{O}$ for the group algebra $\mathcal{O}(P \rtimes E)$.

Lemma 11. *Assume that $\mathcal{O}$ contains a primitive p-th unity root ζ and A is an $\mathcal{O}$-algebra. Then the group $1 + (\zeta - 1)^2 A$ contains no nontrivial torsion p-element.*

Proof. Assume that $1 + a \in 1 + (\zeta - 1)^2 A$ is an element of order p. Then we have that $1 = (1 + a)^p = 1 + pa + pa^2 b + a^p$ for some $b \in A$. Since $p\mathcal{O} = (\zeta - 1)^{p-1}\mathcal{O}$, denoting by h the highest number such that $a \in (\zeta - 1)^h A$, then we have that pa does not belong to $(\zeta - 1)^{p+h} A$. But since $h \geq 2$, $pa^2 b + a^p$ belongs to $(\zeta - 1)^{p+h} A$. So it is a contradiction.

For a finite group G, by $I(\mathcal{O}G)$ we always denote the augmentation ideal of the group algebra $\mathcal{O}G$.

Lemma 12. *Assume that P is a p-group and E is a p'-subgroup of* $\mathrm{Aut}(P)$. *Then the image of any p-subgroup Q of $1 + I(\mathcal{O}(P \rtimes E))$ centralizing P in*

$$\mathcal{O}(P \rtimes E) \Big/ I(\mathcal{O}P)\mathcal{O}(P \rtimes E) \cong \mathcal{O}E$$

is trivial.

Proof. Assume that $\mathcal{O}$ contains a primitive p-th unity root ζ; then $p\mathcal{O} = (\zeta - 1)^{p-1}\mathcal{O}$.

Obviously P acts on the group $P \rtimes E$ by conjugation and $P \rtimes E$ is divided into a disjoint of P-orbits on $P \rtimes E$. Let I be a set of representatives of all these P-orbits and let J_g be a set of representatives of all left cosets of $C_P(g)$ in P for any $g \in I$; then any element $a \in C_{\mathcal{O}(P \rtimes E)}(P)$ can be written in the sum

$$\sum_{g \in I} a_g \sum_{n \in J_g} ngn^{-1}, \quad \text{where } a_g \in \mathcal{O} \quad .$$

Assume that p is odd and consider the sum $\sum_{n\in P/C_P(g)} ngn^{-1}$. If $P = C_P(g)$, then by our assumption, $g \in Z(P)$, and if $|P : C_P(g)| = p^\alpha > 1$, then

$$\sum_{n\in J_g} ngn^{-1} \in I(\mathcal{O}P)\mathcal{O}(P \rtimes E) + p^\alpha\mathcal{O}(P \rtimes E) \quad .$$

Since Q is contained in $1 + I(\mathcal{O}(P \rtimes E))$, the image of Q in

$$\mathcal{O}(P \rtimes E)\Big/ I(\mathcal{O}P)\mathcal{O}(P \rtimes E) \cong \mathcal{O}E$$

is contained in

$$1 + p\mathcal{O}E \subset 1 + (\zeta - 1)^2\mathcal{O}E \quad .$$

Thus by Lemma 11, the image of Q in $\mathcal{O}E$ is trivial.

Assume that p is equal to 2. If $|P : C_P(g)| = 2^\alpha > 1$, where $\alpha \neq 1$, then as in the case of p being odd, it is easy to check that the class sum $\sum_{n\in J_g} ngn^{-1} \in I(\mathcal{O}P)\mathcal{O}(P \rtimes E) + 4\mathcal{O}(P \rtimes E)$. Now suppose that $|P : C_P(g)| = 2$. Consider the quotient group $P/\Phi(P)$ of P by the Frattini subgroup and let $g_{p'}$ be the p'-part of g. Suppose $C_P(g_{p'}) = C_P(g)$. In that case, $g_{p'}$ induces a nontrivial p'-automorphism of $P/\Phi(P)$ and we have a $g_{p'}$-stable short exact sequence as the following:

$$1 \longrightarrow C_P(g)/\Phi(P) \longrightarrow P/\Phi(P) \longrightarrow P/C_P(g) \longrightarrow 1 \quad .$$

Then by Maschke's Theorem,

$$P/\Phi(P) \cong C_P(g)/\Phi(P) \bigoplus P/C_P(g)$$

as a $\langle g_{p'}\rangle$-module. Since $g_{p'}$ centralizes $C_P(g)/\Phi(P)$ and $P/C_P(g)$, so does $g_{p'}$ on $P/\Phi(P)$ and further on P. By the assumption on $P \rtimes E$, we have that $g_{p'} = 1$. This contradicts the equality $C_P(g_{p'}) = C_P(g)$. Thus $C_P(g_{p'}) = P$ since $|P : C_P(g)| = 2$. By the assumption on $P \rtimes E$ again, $g_{p'} \in P$, $g_{p'} = 1$ and $g \in P$. Therefore the class sum $\sum_{n\in J_g} ngn^{-1} \in \mathcal{O} + I(\mathcal{O}P)\mathcal{O}(P \rtimes E)$. Now it is clear that the image of Q in $\mathcal{O}(P \rtimes E)\Big/ I(\mathcal{O}P)\mathcal{O}(P \rtimes E) \cong \mathcal{O}E$ is contained in

$$1 + 4\mathcal{O}E = 1 + (-1 - 1)^2\mathcal{O}E \quad .$$

Then by Lemma 11 again, the image of Q in $\mathcal{O}E$ is trivial.

Finally suppose that $\mathcal{O}$ does not contain a p-th primitive unity root and let $\hat{\mathcal{O}} = \mathcal{O}[\zeta]$, where ζ is a p-th primitive unity root. By the hypothesis, Q is contained in $1 + I(\hat{\mathcal{O}}(P \rtimes E))$ and the image of Q in

$$\hat{\mathcal{O}}(P \rtimes E)\Big/ I(\hat{\mathcal{O}}P)\hat{\mathcal{O}}(P \rtimes E) \cong \hat{\mathcal{O}} \otimes_{\mathcal{O}} \mathcal{O}(P \rtimes E)\Big/ I(\mathcal{O}P)\mathcal{O}(P \rtimes E)$$

is trivial; thus the image of Q in $\mathcal{O}(P \rtimes E)\Big/ I(\mathcal{O}P)\mathcal{O}(P \rtimes E)$ is trivial.

Proposition 13. *Assume that P is a p-group and E is a p'-subgroup of* Aut(P). *Then any p-subgroup Q of $1 + I(\mathcal{O}(P \rtimes E))$ centralizing P is a subgroup of P.*

Proof. Without loss of generality, we assume that $Q = \langle c\rangle$.

Consider $\mathcal{O}(P \rtimes E)$ as an $\mathcal{O}(P \rtimes E \times P \times Q)$-module defined by $(x, y, z)a = xay^{-1}z^{-1}$ for $x \in P \rtimes E$, $y \in P$, $z \in Q$ and $a \in \mathcal{O}(P \rtimes E)$. By Lemma 12, the image of Q in $\mathcal{O}(P \rtimes E)/I(\mathcal{O}P)\mathcal{O}(P \rtimes E) \cong \mathcal{O}E$ is trivial and $P \times P \times Q/P \times 1 \times 1$

acts trivially on $\mathcal{O}E$. By [8, Theorem A1.2], $\mathrm{Res}_{P\times P\times Q}(\mathcal{O}(P\rtimes E))$ is a permutation module and thus

$$\mathrm{Res}_{P\times P\times Q}(\mathcal{O}(P\rtimes E))\cong\bigoplus_{j=1}^{j=|E|}\mathrm{Ind}_{U_j}^{P\times P\times Q}\mathcal{O}_{U_j}\quad ,$$

where U_j is a subgroup of $P\times P\times Q$ and $\mathcal{O}_{U_j}$ is the trivial $\mathcal{O}U_j$-module. Since $\mathcal{O}(P\rtimes E)$ is projective as left and right $\mathcal{O}P$-modules, it is easily concluded that

$$U_j\cap P\times 1\times 1=1\quad\text{ and }\quad U_j\cap 1\times P\times 1=1$$

for any j and that

$$|U_j|=|P||Q|$$

for any j.

We claim that for any j, there exists some $z_j\in P$ and some $\theta_j\in\mathrm{Aut}(P)$ such that

$$U_j=\{(x,\theta_j(x)z_j^{-i},c^i)|x\in P,i\in\mathbb{Z}\}\quad .$$

Let $\{a_l\}_{1\le l\le|P|}$ be a $P\times P\times Q$-stable $\mathcal{O}$-basis of $\mathrm{Ind}_{U_j}^{P\times P\times Q}\mathcal{O}_{U_j}$ and assume that the stabilizer of a_1 is U_j. On the one hand, since $U_j\cap 1\times P\times 1=1$, for any $x\in P$, there exists a unique $x'\in P$ such that $xa_1=a_1x'$; so we can define a map

$$\theta:P\longrightarrow P,\quad x\in P\mapsto x'.$$

Since $U_j\cap P\times 1\times 1=1$, it is easily checked that the map θ is a group isomorphism and that $\{(x,\theta(x),1)|x\in P\}$ is a subgroup of U_j. On the other hand, for any $i\in\mathbb{Z}$, there exists a unique $z_j\in P$ such that we have that $a_1z_j^i=a_1c^i$; so $\{(1,z_j^{-i},c^i)|i\in\mathbb{Z}\}$ is a subgroup of U_j. Therefore

$$U_j\ge\{(x,\theta_j(x)z_j^{-i},c^i)|x\in P,i\in\mathbb{Z}\}\quad .$$

Obviously the cardinality of the latter set is equal to $|P||Q|$. So

$$U_j=\{(x,\theta_j(x)z_j^{-i},c^i)|x\in P,i\in\mathbb{Z}\}.$$

Obviously $\mathcal{O}(P\rtimes E)$ as an $\mathcal{O}(P\rtimes E\times P\times Q)$-module is indecomposable and relatively projective to $P\times P\times Q$. Then by Higman's Criterion on relatively projective modules, $\mathcal{O}(P\rtimes E)$ is a direct summand of

$$\mathrm{Ind}_{P\times P\times Q}^{P\rtimes E\times P\times Q}\Big(\mathrm{Res}_{P\times P\times Q}(\mathcal{O}(P\rtimes E))\Big)\cong\mathrm{Ind}_{P\times P\times Q}^{P\rtimes E\times P\times Q}\Big(\bigoplus_{j=1}^{j=|E|}\mathrm{Ind}_{U_j}^{P\times P\times Q}\mathcal{O}_{U_j}\Big)$$

as an $\mathcal{O}(P\rtimes E\times P\times Q)$-module. Therefore there exists some j such that

$$\mathcal{O}(P\rtimes E)\cong\mathrm{Ind}_{U_j}^{P\rtimes E\times P\times Q}\mathcal{O}_{U_j}\quad .$$

This means that there exists $u\in\mathcal{O}(P\rtimes E)$ such that $\mathcal{O}(P\rtimes E)u=\mathcal{O}(P\rtimes E)$ and u is stable under the action of U_j; that is, u is a unit of $\mathcal{O}(P\rtimes E)$ and $u=uz_j^{-1}c$; thus $c\in P$.

Theorem 14. *Let P (resp. P') be an Abelian p-group and E (resp. E') be an Abelian p'-subgroup of* $\mathrm{Aut}(P)$ *(resp.* $\mathrm{Aut}(P')$*). Set $G=P\rtimes E$ and $G'=P'\rtimes E'$. If the $\mathcal{O}(G\times G')$-module M induces a Morita equivalence between group algebras $\mathcal{O}G$ and $\mathcal{O}G'$, then the following hold:*

(14.1) $\mathrm{Dim}_{\mathcal{K}}(V')=\mathrm{Dim}_{\mathcal{K}}((\mathcal{K}\otimes_{\mathcal{O}}M)\otimes_{\mathcal{K}G'}V')$ *for any simple $\mathcal{K}G'$-module V'; in particular, there exists an $\mathcal{O}$-algebra isomorphism $\rho:\mathcal{O}G'\longrightarrow\mathcal{O}G$ such that M*

as an $\mathcal{O}(G\times G')$*-module is isomorphic to* $\mathcal{O}G$*, where* $\mathcal{O}G$ *as an* $\mathcal{O}(G\times G')$*-module is defined by* $(x,y)a = xa\rho(y^{-1})$ *for* $x\in G$*,* $y\in G'$ *and* $a\in \mathcal{O}G$*.*

(14.2) *Let* P'' *be a vertex of the* $\mathcal{O}(G\times G')$*-module* M *and let the* $\mathcal{O}P''$*-module* S *be a source module of* M*. Then* $\mathrm{Rank}_{\mathcal{O}}(S)=1$*. That is to say, the Morita equivalence induced by* M *between* $\mathcal{O}G'$ *and* $\mathcal{O}G$ *is basic. In particular,* G' *is isomorphic to* G*.*

Proof. Let V' be a $\mathcal{K}G'$-module. Then it is well known that there is a full $\mathcal{O}G'$-lattice W' in V'. Since all simple modules of kG' have dimension 1,

$$\mathrm{Dim}_{\mathcal{K}}(V') = \mathrm{Rank}_{\mathcal{O}}(W') = \mathrm{Dim}_k(k\otimes_{\mathcal{O}} W')$$

is equal to the number of the composition factors (counting multiplicities) in a composition chain of $k\otimes_{\mathcal{O}} W'$. Since the $\mathcal{O}(G\times G')$-module M induces a Morita equivalence between $\mathcal{O}G$ and $\mathcal{O}G'$, so does $\mathcal{K}\otimes_{\mathcal{O}} M$ between $\mathcal{K}G'$ and $\mathcal{K}G$; thus $V=(\mathcal{K}\otimes_{\mathcal{O}}M)\otimes_{\mathcal{K}G'}V'$ is a simple module of $\mathcal{K}G$ and $M\otimes_{\mathcal{O}G'}W'$ is a full $\mathcal{O}G$-lattice in V. Moreover we have

$$\begin{aligned}\mathrm{Dim}_{\mathcal{K}}((\mathcal{K}\otimes_{\mathcal{O}} M)\otimes_{\mathcal{K}G'} V') &= \mathrm{Rank}_{\mathcal{O}}(M\otimes_{\mathcal{O}G'} W')\\ &= \mathrm{Dim}_k((k\otimes_{\mathcal{O}} M)\otimes_{kG'} (k\otimes_{\mathcal{O}} W'))\\ &= \mathrm{Dim}_k(k\otimes_{\mathcal{O}} W') = \mathrm{Dim}_{\mathcal{K}}(V')\end{aligned}$$

since $k\otimes_{\mathcal{O}} M$ induces a Morita equivalence between kG and kG', which preserves composition chains of kG'-modules. It is more or less known that

$$\mathcal{K}\otimes_{\mathcal{O}} M \cong \bigoplus_{V'\in\mathrm{Irr}(G')} ((\mathcal{K}\otimes_{\mathcal{O}} M)\otimes_{\mathcal{K}G'} V')\otimes_{\mathcal{K}} V'^{*}$$

as a $\mathcal{K}(G\times G')$-module, where V'^{*} denotes the dual of V' as a $\mathcal{K}G'$-module and $\mathrm{Irr}(G')$ denotes the set of all simple $\mathcal{K}G'$-modules. Since

$$\mathrm{Dim}_{\mathcal{K}}(V') = \mathrm{Dim}_{\mathcal{K}}((\mathcal{K}\otimes_{\mathcal{O}} M)\otimes_{\mathcal{K}G'} V'),$$

identifying $G\times 1$ with G,

$$\mathrm{Res}_G(\mathcal{K}\otimes_{\mathcal{O}} M) \cong \bigoplus_{V'\in\mathrm{Irr}(G')} \mathrm{Dim}_{\mathcal{K}}(V')((\mathcal{K}\otimes_{\mathcal{O}} M)\otimes_{\mathcal{K}G'} V') \cong \mathcal{K}G.$$

Since M is projective as a left $\mathcal{O}G$-module, M is isomorphic to $\mathcal{O}G$ as left $\mathcal{O}G$-modules. Similarly, M is isomorphic to $\mathcal{O}G'$ as right $\mathcal{O}G'$-modules. Therefore there exists $m\in M$ such that $M=\mathcal{O}Gm$ and for any $x'\in\mathcal{O}G'$, there exists $x\in\mathcal{O}G$ such that $xm=mx'$. Moreover if another $y\in\mathcal{O}G$ also fulfils $ym=mx'$, then $(x-y)m=0$ and thus $x=y$. Now we can define an $\mathcal{O}$-algebra homomorphism

$$\rho:\mathcal{O}G'\longrightarrow\mathcal{O}G,\quad x'\mapsto x\quad \text{such that } xm=mx' \quad .$$

If $\rho(x')=0$, then $mx'=0$ and thus $Mx'=0$. Since M is isomorphic to $\mathcal{O}G'$ as right $\mathcal{O}G'$-modules, $x'=0$ and so ρ is injective; in particular, $1\otimes\rho: k\otimes_{\mathcal{O}}\mathcal{O}G'\longrightarrow k\otimes_{\mathcal{O}}\mathcal{O}G$ is injective. Since $\mathrm{Dim}_{\mathcal{K}}((K\otimes_{\mathcal{O}} M)\otimes_{\mathcal{K}G'} V')=\mathrm{Dim}_{\mathcal{K}}(V')$ for any simple $\mathcal{K}G'$-module V', we have $|G|=|G'|$. Therefore $1\otimes\rho$ is an isomorphism and further so is ρ. Consider the $\mathcal{O}(G\times G')$-module $(\mathcal{O}G)_\rho=\mathcal{O}G$ defined by $(x,y)a=xa\rho(y^{-1})$ for $x\in G, a\in\mathcal{O}G$ and $y\in G'$. Then it is easy to check that

$$(\mathcal{O}G)_\rho\cong M \quad \text{as } \mathcal{O}(G\times G')\text{-modules} \quad .$$

Up to now, the proof of (14.1) is done.

Set $Z = C_P(E)$ and $Z' = C_{P'}(E')$; obviously Z and Z' are maximal central p-subgroups of G and G', respectively. Set $\bar{G} = G/Z$ and $\bar{G}' = G'/Z'$. For $x \in G$ and $H \leq G$, denote by $\bar{x}$ and $\bar{H}$ the images of x and H in $\bar{G}$, respectively. Then $\bar{E}$ is isomorphic to a p'-subgroup of $\mathrm{Aut}(\bar{P})$, $C_{\bar{P}}(\bar{E})$ is trivial and $\bar{G}$ is equal to $\bar{P} \rtimes \bar{E}$; similar results also hold for $\bar{G}'$, $\bar{P}'$ and $\bar{E}'$.

Suppose that Z' is nontrivial. Let φ be the character of G' determined by the pullback of the trivial $\mathcal{O}G$-module through ρ and define an $\mathcal{O}$-algebra homomorphism

$$\rho' : \mathcal{O}G' \longrightarrow \mathcal{O}G, \;\; x \in G' \longmapsto \varphi(x^{-1})\rho(x) \quad .$$

Then it is easily checked that ρ' is an isomorphism, that ρ' maps $I(\mathcal{O}G')$ onto $I(\mathcal{O}G)$, and that $\rho'(Z')$ is a p-subgroup of $1 + I(\mathcal{O}G)$ centralized by G. By Proposition 13, $\rho'(Z') \subset Z$. Similarly, $\rho'^{-1}(Z) \subset Z'$ and thus $\rho'(Z') = Z$. Now it is clear that ρ' induces an $\mathcal{O}$-algebra isomorphism

$$\bar{\rho}' : \mathcal{O}\bar{G}' \cong \mathcal{O}G'/I(\mathcal{O}Z')\mathcal{O}G' \longrightarrow \mathcal{O}G/I(\mathcal{O}Z)\mathcal{O}G \cong \mathcal{O}\bar{G} \quad .$$

Consider an $\mathcal{O}(\bar{G} \times \bar{G}')$-module $(\mathcal{O}\bar{G})_{\bar{\rho}'} = \mathcal{O}\bar{G}$ defined by $(\bar{x}, \bar{y})a = \bar{x}\bar{a}\bar{\rho}'(\bar{y}^{-1})$ for $\bar{x} \in \bar{G}, \bar{a} \in \mathcal{O}\bar{G}$ and $\bar{y} \in \bar{G}'$. We claim that $\mathrm{Res}_{\bar{P}\times\bar{P}'}((\mathcal{O}\bar{G})_{\bar{\rho}'})$ is a permutation module. It is clear that $I(\mathcal{O}\bar{P})(\mathcal{O}\bar{G})_{\bar{\rho}'}$ is a submodule of the $\mathcal{O}(\bar{G} \times \bar{G}')$-module $(\mathcal{O}\bar{G})_{\bar{\rho}'}$ and that the quotient module $(\mathcal{O}\bar{G})_{\bar{\rho}'}/(I(\mathcal{O}\bar{P})(\mathcal{O}\bar{G})_{\bar{\rho}'})$ is $\mathcal{O}$-free. Let the $\mathcal{K}\bar{G}'$-module $\bar{V}'$ be an irreducible direct summand of

$$\mathcal{K} \otimes_{\mathcal{O}} \mathrm{Res}_{1\times\bar{G}'}((\mathcal{O}\bar{G})_{\bar{\rho}'}/(I(\mathcal{O}\bar{P})(\mathcal{O}\bar{G})_{\bar{\rho}'})) \quad .$$

Since $\bar{G}/\bar{P}$ is Abelian, $\mathcal{K} \otimes_{\mathcal{O}} \mathrm{Res}_{1\times\bar{G}'}((\mathcal{O}\bar{G})_{\bar{\rho}'}/(I(\mathcal{O}\bar{P})(\mathcal{O}\bar{G})_{\bar{\rho}'}))$ is a direct sum of irreducible $\mathcal{K}\bar{G}'$-modules with dimension 1. Therefore $\mathrm{Dim}_{\mathcal{K}}(\bar{V}') = 1$. Then the restriction of $\bar{V}'$ to $\bar{P}'$ determines an $\bar{E}'$-stable linear character χ' of $\bar{P}'$. Since $C_{\bar{P}'}(\bar{E}')$ is trivial, by the well-known Glauberman theorem on characters, χ' has to be trivial. Therefore, $\bar{P}'$ acts trivially on $\bar{V}'$. Since $\bar{V}'$ is any direct summand of

$$\mathcal{K} \otimes_{\mathcal{O}} \mathrm{Res}_{1\times\bar{G}'}((\mathcal{O}\bar{G})_{\bar{\rho}'}/(I(\mathcal{O}\bar{P})(\mathcal{O}\bar{G})_{\bar{\rho}'}))$$

as $\mathcal{K}\bar{G}'$-modules, $\bar{P}'$ acts trivially on $\mathcal{K} \otimes_{\mathcal{O}} \mathrm{Res}_{1\times\bar{G}'}((\mathcal{O}\bar{G})_{\bar{\rho}'}/(I(\mathcal{O}\bar{P})(\mathcal{O}\bar{G})_{\bar{\rho}'}))$ and further trivially on $(\mathcal{O}\bar{G})_{\bar{\rho}'}/I(\mathcal{O}\bar{P})(\mathcal{O}\bar{G})_{\bar{\rho}'}$. Equivalently, $\bar{\rho}'(\bar{P}') \subset 1 + I(\mathcal{O}\bar{P})\mathcal{O}\bar{G}$. Since $\mathrm{Res}_{\bar{P}\times 1}((\mathcal{O}\bar{G})_{\bar{\rho}'})$ is free and $\mathrm{Res}_{\bar{P}\times\bar{P}'}((\mathcal{O}\bar{G})_{\bar{\rho}'}/I(\mathcal{O}\bar{P})(\mathcal{O}\bar{G})_{\bar{\rho}'})$ is the direct sum of trivial modules, by [8, Theorem A1.2], $\mathrm{Res}_{\bar{P}\times\bar{P}'}((\mathcal{O}\bar{G})_{\bar{\rho}'})$ is a permutation module.

Consider the $\mathcal{O}(G \times G')$-module $(\mathcal{O}G)_{\rho'} = \mathcal{O}G$ defined by $(x, y)a = xa\rho'(y^{-1})$ for any $x \in G, a \in \mathcal{O}G$ and $y \in G'$. Obviously $I(\mathcal{O}Z)(\mathcal{O}G)_{\rho'}$ is a submodule of $(\mathcal{O}G)_{\rho'}$. Since $\rho'(Z') = Z$, the quotient module $(\mathcal{O}G)_{\rho'}/I(\mathcal{O}Z)(\mathcal{O}G)_{\rho'}$ becomes an $\mathcal{O}(\bar{G}\times\bar{G}')$-module and is isomorphic to $(\mathcal{O}\bar{G})_{\bar{\rho}'}$. Since $\mathcal{O}G$ is free as left $\mathcal{O}Z$-module, by [8, Theorem A1.2] again, we have that $\mathrm{Res}_{P\times P'}((\mathcal{O}G)_{\rho'})$ is a permutation module. Denote by $\mathcal{O}_\varphi$ the $\mathcal{O}G'$-module $\mathcal{O}$ obtained through the homomorphism φ and define $(\mathcal{O}G)_{\rho'} \otimes_{\mathcal{O}} \mathcal{O}_\varphi$ as an $\mathcal{O}(G\times G')$-module by G acting on the left of $(\mathcal{O}G)_{\rho'}$ and trivially on the left of $\mathcal{O}_\varphi$ and G' acting diagonally on the right of $(\mathcal{O}G)_{\rho'} \otimes_{\mathcal{O}} \mathcal{O}_\varphi$. Then it is easy to check that $(\mathcal{O}G)_{\rho'} \otimes_{\mathcal{O}} \mathcal{O}_\varphi \cong (\mathcal{O}G)_\rho$ as $\mathcal{O}(G \times G')$-modules and therefore that any source module of $M = (\mathcal{O}G)_\rho$ as an $\mathcal{O}(G \times G')$-module has $\mathcal{O}$-rank 1.

Suppose that Z' is trivial. Then it follows by a proof similar to the nontrivial case of Z' that any source module of M is of $\mathcal{O}$-rank 1. Now by [8, Cor. 7.4], we reach the conclusion that the Morita equivalence between $\mathcal{O}G$ and $\mathcal{O}G'$ induced by M is basic.

Finally we prove that G is isomorphic to G'. Let σ and σ' be projections of P'' into G and G', respectively. Then by [8, Cor. 7.4 and Th. 6.9], $P = \sigma(P'')$, $P' = \sigma'(P'')$, the group homomorphisms $\sigma : P'' \longrightarrow P$ and $\sigma' : P'' \longrightarrow P'$ are isomorphisms; moreover there exists the $\mathcal{O}P$-interior algebra embedding

$$\mathcal{O}G \longrightarrow \mathrm{Res}_{\sigma^{-1}}(\mathrm{End}_{\mathcal{O}}(S)) \otimes_{\mathcal{O}} \mathrm{Res}_{\sigma' \circ \sigma^{-1}}(\mathcal{O}G') \quad ,$$

which actually is an isomorphism. Without loss of the generality, we can assume that S is the trivial $\mathcal{O}P''$-module. Then we obtain an isomorphism between $\mathcal{O}G$ and $\mathcal{O}G'$, mapping P onto P'. Now G being isomorphic to G' follows from [9, 2.15.7, 2.16.1 and 2.16.3].

15. Finally Theorem 2 follows from (14.2) and the second paragraph in 10.

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