

LINKING IN HILBERT SPACE

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ABSTRACT. We present the most general definition of the linking of sets in a Hilbert space and, drawing on the theory given in earlier papers by Schechter and Tintarev, give a necessary and sufficient geometric condition for linking when one set is compact.

1. INTRODUCTION

Many nonlinear problems in the physical and social sciences can be reduced to finding critical points (minima, maxima and minimax points) of functionals (real-valued functions on various spaces). The first critical points to be studied were maxima and minima, and much of the activity in the calculus of variations has been devoted to the finding of such points. A more difficult problem is to find critical points that are neither maxima nor minima. In [14] our goal was to prove a theorem of the form:

Theorem 1.1. *Let G be a C^1 -functional on E , and let A, B be subsets of E such that A has a certain relationship to B . Assume that*

$$(1.1) \quad a_0 := \sup_A G < b_0 := \inf_B G.$$

Then there is a sequence $\{u_k\} \subset E$ and an $a \geq b_0$ such that

$$(1.2) \quad G(u_k) \rightarrow a, \quad G'(u_k) \rightarrow 0.$$

The reason for the requirement that A have a relationship to B is that the theorem is obviously false if A and B are two arbitrary sets. The problem facing us was to find a very general relationship that would make the theorem true. We defined a relationship which indeed made the theorem true and was the most general known hypothesis that did so. We termed this relationship “linking.” (Previous authors used this term to describe more restrictive definitions.) We suspect that our definition of linking is the most general possible, and in this paper we obtain a condition which is both necessary and sufficient for Theorem 1.1 to hold under

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mild assumptions on A . Our starting point is to define linking directly as existence of critical levels. We have

Definition 1.2. Let A, B be subsets of a Banach space E . We shall say that A links B if the class of $G \in C_U^1(E; \mathbb{R})$ satisfying

$$(1.3) \quad a_0 := \sup_A G < b_0 := \inf_B G$$

is non-empty, and for every such G there is a sequence $\{u_k\} \subset E$ and a constant c such that

$$(1.4) \quad b_0 \leq c < \infty$$

and

$$(1.5) \quad G(u_k) \rightarrow c, \quad G'(u_k) \rightarrow 0.$$

(In this definition, as well as in the rest of the paper, we consider functionals of the class $C_U^m \subset C^m$, i.e. functionals whose Fréchet derivatives up to the order m are uniformly continuous on bounded sets.) The importance of the concept of linking stems from the fact that in many applications, a sequence satisfying (1.5) (called a Palais-Smale sequence) leads to a critical point of G . In particular, this is true if any sequence satisfying (1.5) has a convergent subsequence. (If this is the case, then G is said to satisfy the Palais-Smale condition.) Thus, a useful method of finding critical points of G is to find two subsets A, B of E such that A links B and (1.5) holds.

There are several sufficient conditions given in the literature which imply that a set A links a set B in this sense (cf., e.g., [2, 3, 6, 7, 8, 9, 13, 14, 15]). The most comprehensive is that of [14]. In this paper we give a characterization of linking sets that is both necessary and sufficient under mild assumptions on A . Thus, under these circumstances, our characterization of linking sets is the most general possible. We use the following maps.

Definition 1.3. We shall say that a map $\varphi : E \rightarrow E$ is of class Φ if it is a homeomorphism and both φ, φ^{-1} are bounded on bounded sets. If, furthermore, $\varphi, \varphi^{-1} \in C_U^1(E; E)$, we shall say that $\varphi \in \Phi_U$. If, in addition, the linear span of $(\varphi - id)(E)$ is finite-dimensional, we shall say that $\varphi \in \Phi_0$. If $\varphi \in \Phi$ (resp. Φ_U, Φ_0) and $\text{supp}(\varphi - id) \subset E \setminus B$, we shall say that $\varphi \in \Phi(B)$ (resp. $\Phi_U(B), \Phi_0(B)$).

We will denote the flow $E \times \mathbb{R} \rightarrow E$ generated by a bounded locally Lipschitz vector field $X : E \rightarrow E$ as $\exp(tX)$.

Definition 1.4. We shall say that a map $\psi : E \times [0, 1] \rightarrow E$ is a flow concatenation, $\psi \in \Psi$ (resp. Ψ_U, Ψ_0), if there exist a finite collection of bounded locally Lipschitz vector fields $X_i : E \rightarrow E$ (resp. $X_i \in C_U^1(E; E); \dim X_i(E) < \infty$), such that $\psi(\cdot, 0) = id, 0 = t_0 < t_1 < \dots < t_n = 1$ and either

$$(1.6) \quad \psi(\cdot, t) = \psi(\cdot, t_k) \circ \exp((t - t_k)X_{k+1}), \quad t \in [t_k, t_{k+1}]$$

or

$$(1.7) \quad \psi(\cdot, t) = \exp((t - t_k)X_{k+1}) \circ \psi(\cdot, t_k), \quad t \in [t_k, t_{k+1}].$$

Definition 1.5. We shall say that a set A is **chained to a set** B if

$$(1.8) \quad \inf_{x \in B} \|\varphi(x)\| \leq \sup_{x \in A} \|\varphi(x)\|, \quad \varphi \in \Phi_0(B).$$

Definition 1.6. We shall say that a set A is **free from a set B** if it is not chained to it, i.e., if there is a $\varphi \in \Phi_0(B)$ such that

$$(1.9) \quad \sup_{x \in A} \|\varphi(x)\| < \inf_{x \in B} \|\varphi(x)\|$$

holds.

Our main theorem is

Theorem 1.7. *Let E be a Hilbert space, let $A, B \subset E$ and assume that A is compact and $d(A, B) > 0$. Then A links B if and only if it is chained to B . The necessity remains true also if A is not required to be compact.*

Definition 1.8.

$$(1.10) \quad \Psi_0(A, B) := \{\psi \in \Psi_0 : \psi(A, 1) \text{ is free from } B\}.$$

Remark 1.1. If A, B are bounded, then it is easy to see that $\Psi_0(A, B)$ is not empty. In particular, it contains a map $\exp(tX)$, where $X(u) = y$, $y \in E \setminus \{0\}$ and $\|y\|$ is sufficiently large. It is also easy to see that the class $\Psi_0(A, B)$ is not empty when the set A is compact. Fix a point $y \in A$ and let P_n be a monotone sequence of n -dimensional orthogonal projectors on E , strongly convergent to I . By the compactness of A , $\text{diam}(I - P_n)A \rightarrow 0$ as $n \rightarrow \infty$. Moreover, for any $\epsilon > 0$ there are an n and an $M > 0$ such that $y + (I - P_n)A + e^{-M}(P_n A - y) \subset V_\epsilon(y)$. (An open ball of radius R centered at $y \in E$ will be denoted by $V_R(y)$.) Fix $\epsilon > 0$ (and thus n), so that $V_{2\epsilon}(y) \cap B = \emptyset$. Then $\psi(x, t) = (I - P_n)x + y + e^{-tM}(P_n x - y)$ is in $\Psi_0(A, B)$.

We also have

Theorem 1.9. *Under the hypotheses of Theorem 1.7, A links B if and only if $\psi(A, (0, 1)) \cap B \neq \emptyset$ for every $\psi \in \Psi_0(A, B)$.*

The necessity in Theorem 1.7 follows from

Lemma 1.10. *If A links B , then it is chained to B .*

Proof. Assume that $\varphi \in \Phi_0(B)$ satisfies (1.9), and let $G(u) = \|\varphi(u)\|^2$. Then by the definition of the class $\Phi_0(B)$, $G \in C^1_U(E, \mathbb{R})$, $\sup G(A) < \inf G(B)$ and G has no critical level $c \geq \inf_{x \in B} G(x) > 0$. To show the latter, assume that there is a sequence u_k satisfying (1.5). Then we have, for any bounded sequence $v_k \in E$,

$$(1.11) \quad (\varphi'(u_k)v_k, \varphi(u_k)) \rightarrow 0.$$

Let $v_k := (\varphi'(u_k))^{-1}(\varphi(u_k))$. Then $(\varphi'(u_k)v_k, \varphi(u_k)) = G(u_k) \rightarrow c$. However, the sequence v_k is bounded: $G(u_k) \rightarrow c$ implies that $\varphi(u_k)$ is bounded, which implies that u_k , and consequently $(\varphi'(u_k))^{-1}$ and v_k are also bounded. Hence, $G'(u_k) \rightarrow 0$ implies $G(u_k) \rightarrow 0$, showing that $c = 0$. Thus, A does not link B . \square

The sufficiency part of Theorem 1.7 will be proved in the next section. Theorem 1.9 will be proved there as well. We note that if the values of A and B are separated by $G \in C^1(E, \mathbb{R})$ (i.e., (1.3) holds), then $d(A, B) > 0$.

2. PROOF OF SUFFICIENCY

In this section we prove sufficiency in the statement of Theorem 1.7, namely

Theorem 2.1. *Let E be a Hilbert space, let $A, B \subset E$, let $G \in C_V^1(H)$ such that $\sup G(A) < \inf G(B)$ and assume that A is compact and is chained to B . If*

$$(2.1) \quad c := \inf_{\psi \in \Psi_0(A, B)} \sup_{x \in A, t \in [0, 1]} G(\psi(x, t)),$$

then $\inf G(B) \leq c < \infty$ and (1.5) holds. In particular, A links B .

The proof of Theorem 2.1 will be based on several lemmas. First, we have

Lemma 2.2. *If the set A is chained to B , then for every map $\psi \in \Psi_0$ such that $\psi(A, 1)$ is free from B , we have $\psi(A, (0, 1)) \cap B \neq \emptyset$.*

Proof. Assume that there is a $\psi \in \Psi_0$ such that $\psi(A, 1)$ is free from B and $\psi(A, (0, 1)) \cap B = \emptyset$. Let X_i be the bounded locally Lipschitz vector fields that define ψ according to a particular sequence of left and right concatenations (1.6), (1.7). Let $\chi : E \rightarrow [0, 1]$ be a locally Lipschitz function supported in the complement of B that equals 1 on $\psi(A, [0, 1])$. Let now ψ_1 be the function (1.6), (1.7) with every X_i replaced by χX_i . Then $\psi_1|_{A \times [0, 1]} = \psi|_{A \times [0, 1]}$ and $\psi(\cdot, 1) \in \Phi_0(B)$. Thus $\psi(\cdot, 1)$ frees A from B . \square

Next, we let

$$(2.2) \quad G_s := G^{-1}([c - s, c + s]), s > 0.$$

We have

Lemma 2.3. *Let $G \in C_V^1(E)$. Assume that there exists an $\epsilon > 0$ such that*

$$(2.3) \quad u \in G_{2\epsilon} \Rightarrow \|G'(u)\| \geq 2\epsilon.$$

Assume that $A \subset E$ is a compact set such that

$$(2.4) \quad \max G(A) \leq c + \epsilon/2,$$

and let $R > 0$ be such that $A \subset V_R(0)$. Then there exists a map $X \in C_{0U}^\infty(E; E)$ with finite-dimensional range and a $\rho > 0$, depending on G , ϵ and R , but independent of A and subject to the conditions above, such that

$$(2.5) \quad \text{supp } X \subset G_{2\epsilon},$$

$$(2.6) \quad \exp(-tX)A \subset V_{R+\rho}(0), \quad t \in [0, \rho],$$

and

$$(2.7) \quad \max G(\exp(-\rho X)A) \leq \max\{c - \epsilon/2, \max G(A) - \rho\epsilon\}.$$

In proving Lemma 2.3, we shall use

Lemma 2.4. *Let $x_i \in E$, $i = 1, \dots, N$, let $\rho > 0$ and let*

$$(2.8) \quad C_r := \bigcup_{i=1}^N \overline{V_r(x_i)}, \quad r > 0.$$

Then there exists a $\chi \in C_{0U}^\infty(E; [0, 1])$ such that

$$(2.9) \quad x \in C_{2\rho} \Rightarrow \chi(x) = 1$$

and

$$(2.10) \quad x \notin C_{3\rho} \Rightarrow \chi(x) = 0.$$

Proof. Let $Y \subset E$ be the span of the x_i , $i = 1, \dots, N$, and let P be the orthogonal projector onto Y . Let $\eta \in (0, \rho/6)$, and let, using the sets (2.8), relative to the Euclidean space $Y \times \mathbb{R}$ with the norm $\|(y, t)\|^2 = \|y\|^2 + t^2$,

$$(2.11) \quad \psi(x) := \frac{d(x, (Y \times \mathbb{R}) \setminus C_{3\rho-\eta})}{d(x, (Y \times \mathbb{R}) \setminus C_{3\rho-\eta}) + d(x, C_{2\rho+\eta})}, \quad x \in Y \times \mathbb{R}.$$

Let now χ_N be a convolution of ψ with a standard radially symmetric mollifier on $Y \times \mathbb{R}$ supported in a ball of radius η , and let

$$(2.12) \quad \chi(x) = \chi_N(Px, \|(I - P)x\|).$$

It is easy to see that χ has the required properties. \square

We now give the proof of Lemma 2.3.

Proof. Let us first define ρ . Let ω_R be the sum of respective moduli of continuity for G' and $G'/\|G'\|$ on $\overline{V_{R+1}(0)} \cap G_{2\epsilon}$. Let $\rho \in [0, 1]$ satisfy

$$(2.13) \quad \omega_{R+1}(\rho) < 1/4.$$

It is easy to see that there exists an open cover $\{\mathcal{O}_\alpha\}$ of $\overline{V_{R+1}(0)} \cap G_{2\epsilon}$ such that for every α ,

$$(2.14) \quad \sup_{u \in \mathcal{O}_\alpha, v \in \overline{V_{R+1}(0)} \cap G_{2\epsilon}} \left\| \frac{G'(u)}{\|G'(u)\|} - \frac{G'(v)}{\|G'(v)\|} \right\| \leq 1/4$$

and

$$(2.15) \quad \inf_{u \in \mathcal{O}_\alpha, v \in \overline{V_{R+1}(0)} \cap G_{2\epsilon}} \left(G'(u), \frac{G'(v)}{\|G'(v)\|} \right) \geq \epsilon.$$

Let us now consider a renamed locally finite refinement of $\{\mathcal{O}_\alpha\}$, which obviously inherits (2.14), (2.15) and an associated locally Lipschitz partition of unity $\{\chi_\alpha\}$. Select an arbitrary $v_\alpha \in \mathcal{O}_\alpha \cap \overline{V_{R+1}(0)} \cap G_{2\epsilon}$ and set

$$(2.16) \quad \tilde{X}(u) := \sum_{\alpha} \chi_\alpha \frac{G'(v_\alpha)}{\|G'(v_\alpha)\|}, \quad u \in \overline{V_{R+1}(0)} \cap G_{2\epsilon}.$$

Then $\|\tilde{X}\| \leq 1$, and from (2.14), (2.15) it follows that whenever $u \in \overline{V_{R+1}(0)} \cap G_{2\epsilon}$, we have

$$(2.17) \quad (\tilde{X}(u), G'(u)) \geq \epsilon$$

and

$$(2.18) \quad \left\| \tilde{X}(u) - \frac{G'(u)}{\|G'(u)\|} \right\| \leq 1/4.$$

We can now define

$$(2.19) \quad X(u) := \chi_0((G(u) - c)/\epsilon)\tilde{X}(u),$$

where $\chi_0 \in C_0^\infty(\mathbb{R}; [0, 1])$, $\chi_0 = 1$ on $[-1, 1]$ and $\text{supp } \chi_0 \subset [-2, 2]$. Let

$$(2.20) \quad A_\rho := \left(\bigcup_{t \in [0, \rho]} \exp(-tX)A \right) \cap G_{\epsilon/2}.$$

Since $\|X\| \leq 1$, $A_\rho \subset V_{R+\rho}(0)$. Compactness of A_ρ is immediate. Let $\{V_\rho(u_i)\}$, $u_i \in A_\rho$, be a finite cover of A_ρ . Let χ_ρ be the cut-off function relative to the collection $\{B_\rho(u_i)\}$, as provided by Lemma 2.4. Let

$$(2.21) \quad X_\rho(u) := \chi_\rho(u) \frac{\sum_i \chi_0\left(\frac{\|u-u_i\|}{3\rho}\right) \frac{G'(u_i)}{\|G'(u_i)\|}}{\sum_i \chi_0\left(\frac{\|u-u_i\|}{3\rho}\right)}, \quad u \in C_{5\rho}.$$

Note that $\text{supp } X_\rho \subset C_{3\rho}$, so that X_ρ admits a C^∞ -extension by zero to the whole of E . Note also that whenever $d(u, A_\rho) \leq \rho$, there is an i such that $u \in \overline{V_{2\rho}(u_i)}$, so that $u \in C_{2\rho}$ and $\chi_\rho(u) = 1$. In other words,

$$(2.22) \quad \chi_\rho(u) = 1 \text{ whenever } d(u, A_\rho) \leq \rho.$$

Then, if $d(u, A_\rho) \leq \rho$ and $u \in G_\epsilon$, by (2.18), (2.21),

$$(2.23) \quad \|X_\rho(u) - X(u)\| \leq \frac{1}{4} + \frac{1}{4} = \frac{1}{2}.$$

Let

$$(2.24) \quad t_1 := \sup\{\theta \leq \rho : m(\theta) := \max_{u \in A, t \in [0, \theta]} \|\exp(-tX_\rho)(u) - \exp(-tX)(u)\| \leq \rho/2\}.$$

Note that the function $m(\theta)$ defined above is a continuous function of $\theta \geq 0$ and $m(0) = 0$, so that $t_1 > 0$ and the value t_1 is attained. If we assume that $t_1 < \rho$, we will easily arrive at a contradiction: if $m(\theta) \leq \rho$ for $\theta \in [0, t_2]$, $t_2 \in (t_1, \theta]$, then by (2.23),

$$(2.25) \quad \begin{aligned} & \|\exp(-t_2X_\rho)(u) - \exp(-t_2X)(u)\| \\ & \leq \int_0^{t_2} \|X_\rho(\exp(-tX_\rho)(u)) - X(\exp(-tX)(u))\| dt \leq \frac{1}{2}t_2 \leq \frac{1}{2}\rho. \end{aligned}$$

We conclude therefore that for all $u \in A$, $t \in [0, \rho]$, $d(\exp(-tX_\rho)(u), A_\rho) \leq \rho$, and therefore, by (2.21),

$$(2.26) \quad \sup G(\exp(-\rho X_\rho)A) \leq \sup G(A) - \rho\epsilon.$$

This completes the proof. □

Corollary 2.5. *There exists a map $\psi \in \Psi_0$ such that*

$$(2.27) \quad \max G \circ \psi(A, 1) \leq c - \epsilon/2.$$

Proof. Let $A_1 = \exp(-\rho X_\rho)A$ and rename X_ρ as X_1 . Let $k \in \mathbb{N}$, $k \in [\frac{1}{2\rho}, \frac{1}{2\rho} + 1]$, and define X_{j+1} as the map X provided by Lemma 2.3 for $A = A_j$, while setting $A_{j+1} := \exp(-\rho X_{j+1})$, $j = 1, \dots, k-1$. Note that $A_j \subset V_{R+j\rho}(0)$, so that $\bigcup_{j=1}^k A_j \subset V_{R+1}(0)$. Thus, the selection of ρ in Lemma 2.3 is independent of j (see (2.13)). Then we can set $\psi(\cdot, 0) = id$ and

$$(2.28) \quad \psi(\cdot, t) := \exp(-tX_{j+1}) \circ \psi(\cdot, j\rho), t \in [j, j+1].$$

The proof is complete. □

We can now give the proof of Theorem 2.1.

Proof. For every $\psi \in \Psi_0(A, B)$ we have $\psi(A, [0, 1]) \cap B \neq 0$ and so $c \geq \inf G(B)$ (Lemma 2.2). If we assume that c is not a critical level, we adopt the deformation ψ from Corollary 2.5 and, with its argument range rescaled to $[0, 1]$, we will rename it ψ_1 . By the corollary, if $\psi \in \Psi_0(A, B)$ satisfies $\sup G(\psi(A \times [0, 1])) \leq c + \epsilon/4$, then $\sup G(\psi_1(\cdot, 1) \circ \psi(\cdot, [0, 1](A))) \leq c - \epsilon/4$. Consider now a map $\tilde{\psi}(\cdot, t) = \psi_1(\cdot, t)$ for $t \in [0, 1]$ and $\tilde{\psi}(\cdot, t) = \psi_1(\cdot, 1)\psi(\cdot, t - 1)$ for $t \in [1, 2]$. Then the function $\tilde{\psi}(\cdot, 2t)$ remains in the class $\Psi_0(A, B)$, but $\sup G(\tilde{\psi}(A \times [0, 1])) \leq c - \epsilon/4$, a contradiction. This completes the proof. \square

We can now give the proof of Theorem 1.7.

Proof. Apply Theorem 2.1 and Lemma 1.10. \square

We can also give the proof of Theorem 1.9.

Proof. The sufficiency in Theorem 1.9 follows from Lemmas 1.10 and 2.2. The necessity follows from the proof of Theorem 2.1. \square

We also have

Theorem 2.6. *If A links B , then*

$$(2.29) \quad \inf_{x \in B} \|\varphi(x)\| \leq \sup_{x \in A} \|\varphi(x)\|, \quad \varphi \in \Phi_U(B),$$

even if A is not compact.

Proof. Lemma 1.10. \square

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