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EQUICOMPACT SETS OF OPERATORS DEFINED ON BANACH SPACES

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ABSTRACT. Let X and Y be Banach spaces. We say that a set $\mathcal{M} \subset \mathcal{K}(X,Y)$ ($\mathcal{K}(X,Y)$ denotes the space of all compact operators from X into Y) is *equicompact* if there exists a null sequence $(x_n^*)_n$ in X^* such that $\|Tx\| \leq \sup_n |x_n^*(x)|$ for all $x \in X$ and all $T \in \mathcal{M}$. It is easy to show that collectively compactness and equicompactness are dual concepts in the following sense: $\mathcal{M}$ is equicompact iff $\mathcal{M}^* = \{T^* : T \in \mathcal{M}\}$ is collectively compact. We study some properties of equicompact sets and, among other results, we prove: 1) a set $\mathcal{M} \subset \mathcal{K}(X,Y)$ is equicompact iff each bounded sequence $(x_n)_n$ in X has a subsequence $(x_{k(n)})_n$ such that $(Tx_{k(n)})_n$ is a converging sequence uniformly for $T \in \mathcal{M}$; 2) if Y does not have finite cotype and $\mathcal{M} \subset \mathcal{K}(X,Y)$ is a maximal equicompact set, then, given $\varepsilon > 0$ and a finite set $\{x_1, \dots, x_n\}$ in X, there is an operator $S \in \mathcal{M}$ such that $\|Tx_i\| \leq (1+\varepsilon)\|Sx_i\|$ for $i = 1, \dots, n$ and all $T \in \mathcal{M}$.

1. INTRODUCTION

Let us consider (real or complex) Banach spaces X and Y. As usual $\mathcal{K}(X,Y)$ will denote the vector space of all compact linear maps endowed with the operator norm. We say that a set $\mathcal{M} \subset \mathcal{K}(X,Y)$ is *equicompact* if there exists a null sequence $(x_n^*)_n$ in X^* so that

$$\|Tx\| \leq \sup_n |x_n^*(x)|$$

for all $x \in X$ and all $T \in \mathcal{M}$. We recall that a set $\mathcal{M} \subset \mathcal{K}(X,Y)$ is called collectively compact iff $\bigcup_{T \in \mathcal{M}} T(B_X)$ has compact closure. A standard proof using separations theorems and the well-known fact that a compact set in a Banach space is contained in the closure of the convex hull of a null sequence of the space allows us to state that $\mathcal{M}$ is equicompact iff $\mathcal{M}^* = \{T^* \in \mathcal{K}(Y^*, X^*) : T \in \mathcal{M}\}$ is collectively compact. We study some properties of equicompact sets. Among other results we have proved the following:

(1) A set $\mathcal{M} \subset \mathcal{K}(X,Y)$ is equicompact iff $\mathcal{M}$ satisfies the next property (invoked as property *(P))*:

(P) "For every bounded sequence $(x_n)_n$ in X there exists a subsequence $(x_{k(n)})_n$ so that $(Tx_{k(n)})_n$ is a converging sequence uniformly for $T \in \mathcal{M}$."

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(2) Given a null sequence $(x_n^*)_n$ in X^*, we will denote by $\mathcal{M}((x_n^*)_n)$ the equicompact set of all compact operators $T \in \mathcal{K}(X,Y)$ satisfying $\|Tx\| \leq \sup_n |x_n^*(x)|$ for all $x \in X$. These sets are absolutely convex and closed in the strong operator topology. If Y does not have finite cotype we prove that, given $\varepsilon > 0$ and a finite set $\{x_1, \ldots, x_n\}$ in X, there exists an operator $Q \in \mathcal{M}((x_n^*)_n)$ so that $\|Tx_i\| \leq (1+\varepsilon)\|Qx_i\|$ for $i = 1, \ldots, n$ and all $T \in \mathcal{M}((x_n^*)_n)$.

We also have obtained a partial converse of (2): if $\mathcal{M} \subset \mathcal{K}(X,Y)$ is countably compact for the strong operator topology (for short, SOT), then $\mathcal{M}$ is equicompact when it verifies the following property (invoked as property (F)):

(F) There exists a positive constant C such that, for every finite set $\{x_1, \ldots, x_n\} \subset X$ there is an operator Q in the closed absolutely convex hull of $\mathcal{M}$ satisfying $\|Tx_i\| \leq C\|Qx_i\|$ for $i = 1, \ldots, n$ and all $T \in \mathcal{M}$.

Our notation is standard. If X is a Banach space, B_X will denote its closed unit ball. If I is an arbitrary index set, we will write $\ell_a^1(I,X)$ (respectively, $\ell^\infty(I,X)$) for the Banach space of all absolutely summable (respectively, bounded) X-valued functions defined on I, endowed with the norm $\|\xi\| = \sum_{i\in I} \|\xi(i)\|$ (respectively, $\|\xi\| = \sup\{\|\xi(i)\| \colon i \in I\}$) for each $\xi \in \ell_a^1(I,X)$ (respectively, $\xi \in \ell^\infty(I,X)$).

2. Equicompact sets

If $\mathcal{M} \subset \mathcal{K}(X,Y)$ is bounded, we can consider the continuous linear map $U\colon \ell_a^1(\mathcal{M},X) \longrightarrow Y$ defined by $U(\xi) = \sum_{T\in\mathcal{M}} T(\xi(T))$ for all $\xi \in \ell_a^1(\mathcal{M},X)$.

Proposition 2.1. *The following statements are equivalent for a set $\mathcal{M} \subset \mathcal{K}(X,Y)$:*

a) $\mathcal{M}$ is collectively compact.
b) The operator U is compact.

Proof. Given $S \in \mathcal{M}$ and $x \in B_X$, denote by $\xi_{S,x}$ the element of $\ell_a^1(\mathcal{M},X)$ defined by

$$\xi_{S,x}(T) = \begin{cases} 0 & \text{if } T \neq S, \\ x & \text{if } T = S. \end{cases}$$

It is clear that $U(\xi_{S,x}) = Sx$; then $H = U\{\xi_{S,x} \colon S \in \mathcal{M}, x \in B_X\} = \bigcup_{T\in\mathcal{M}} T(B_X)$. On the other hand, for every $\xi \in B_{\ell_a^1(\mathcal{M},X)}$ we have:

$$U(\xi) = \sum_{T\in\mathcal{M}} T(\xi(T)) = \sum_{\substack{T\in\mathcal{M} \\ \xi(T)\neq 0}} \|\xi(T)\| T\left(\frac{\xi(T)}{\|\xi(T)\|}\right).$$

Therefore, $U(B_{\ell_a^1(\mathcal{M},X)}) \subset \overline{\text{co}}\,(H)$. So we have obtained $H \subset U(B_{\ell_a^1(\mathcal{M},X)}) \subset \overline{\text{co}}\,(H)$ and this concludes the proof. □

Now we consider the operator $V\colon X \longrightarrow \ell^\infty(\mathcal{M},Y)$ defined by $(Vx)(T) = Tx$ for all $T \in \mathcal{M}$ and $x \in X$. For the proof of the following proposition, notice that the equivalence $a) \Leftrightarrow b)$ is obvious and $b) \Leftrightarrow c)$ is straightforward.

Proposition 2.2. *The following statements are equivalent for a set $\mathcal{M} \subset \mathcal{K}(X,Y)$:*

a) $\mathcal{M}$ is equicompact.
b) The operator V is compact.
c) $\mathcal{M}$ has property (P).

Now we are ready to face our main result:

Theorem 2.3. *If $\mathcal{M} \subset \mathcal{K}(X,Y)$ is bounded, the following statements are equivalent:*

a) $\mathcal{M}$ is collectively compact.
b) $\mathcal{M}^*$ has property (P).

Proof. The adjoint map of $U\colon \ell^1_a(\mathcal{M},X) \longrightarrow Y$ is the operator $U^*\colon Y^* \longrightarrow \ell^\infty(\mathcal{M},X^*)$ defined by $(U^*y^*)(T) = T^*y^*$, for all $T \in \mathcal{M}$ and $y^* \in Y^*$. In fact, we have:

$$\langle \xi, U^*y^* \rangle = \langle U\xi, y^* \rangle = \sum_{T\in\mathcal{M}} \langle T(\xi(T)), y^* \rangle = \sum_{T\in\mathcal{M}} \langle \xi(T), T^*y^* \rangle$$

for all $\xi \in \ell^1_a(\mathcal{M},X)$ and $y^* \in Y^*$. A call to Propositions 2.1 and 2.2 concludes the proof. □

The next lemma easily yields the dual equivalence

$$\mathcal{M} \text{ has property } (P) \qquad \Longleftrightarrow \qquad \mathcal{M}^* \text{ is collectively compact.}$$

Lemma 2.4. *If $(x^*_n)_n$ is a null sequence in X^* and $\mathcal{M}$ is a subset of $\mathcal{K}(X,Y)$ such that*

$$\|Tx\| \le \sup_n |x^*_n(x)|$$

for all $x \in X$ and all $T \in \mathcal{M}$, then

$$\|T^{**}x^{**}\| \le \sup_n |x^{**}(x^*_n)|$$

for all $x^{**} \in X^{**}$ and $T \in \mathcal{M}$.

Proof. Given $T \in \mathcal{M}$, $\varepsilon > 0$ and $x^{**} \in B_{X^{**}}$, choose $y^* \in B_{Y^*}$ so that $\|T^{**}x^{**}\| = |y^*(T^{**}x^{**})|$. By hypothesis, there exists $n_0 \in \mathbb{N}$ such that $\|x^*_n\| < \varepsilon/4$ for all $n \ge n_0$. Now we consider the weak* neighborhood $W = W(x^*_1, \ldots, x^*_{n_0}, T^*y^*; \varepsilon/2)$ of 0; there exists $x \in B_X$ satisfying $x \in x^{**} + W$. Then we have:

$$\begin{aligned}
\|T^{**}x^{**}\| &= |\langle T^*y^*, x^{**}\rangle| \\
&\le |\langle T^*y^*, x^{**}\rangle - \langle T^*y^*, x\rangle| + |\langle T^*y^*, x\rangle| \\
&< \frac{\varepsilon}{2} + \sup_n |\langle x^*_n, x\rangle| \\
&\le \frac{\varepsilon}{2} + \sup_n |\langle x^*_n, x\rangle - \langle x^*_n, x^{**}\rangle| + \sup_n |\langle x^*_n, x^{**}\rangle| \\
&< \varepsilon + \sup_n |\langle x^*_n, x^{**}\rangle|
\end{aligned}$$

for all $\varepsilon > 0$. Letting $\varepsilon \to 0$, we obtain $\|T^{**}x^{**}\| \le \sup_n |x^{**}(x^*_n)|$ for all $x^{**} \in X^{**}$ and all $T \in \mathcal{M}$. □

Remark 2.5. A set $\mathcal{M} \subset \mathcal{L}(X,Y)$ is *sequentially weak–norm equicontinuous* (or *uniformly completely continuous*) if, for every weakly null sequence $(x_n)_n$ in X, $\lim_n \|Tx_n\| = 0$ uniformly for $T \in \mathcal{M}$. It is obvious that every equicompact set is uniformly completely continuous (for short, u.c.c.). If X does not contain a copy of ℓ^1, then Rosenthal's theorem about ℓ^1 tells us that each bounded sequence in X has a weakly Cauchy subsequence. So, in this case, every u.c.c. set has property (P) and, therefore, it is equicompact. That is to say, if $X \not\supseteq \ell^1$, then the following statements are equivalent for a bounded set $\mathcal{M} \subset \mathcal{K}(X,Y)$:

a) $\mathcal{M}$ is equicompact.
b) $\mathcal{M}$ is u.c.c.
c) $\mathcal{M}^*$ is collectively compact.

The equivalence stated in Remark 2.5 and [4, theorem 2.2] yields directly the recent Mayoral's theorem [3]:

Theorem (F. Mayoral, 2001)**.** *If X does not contain a copy of ℓ^1, a set $\mathcal{M} \subset \mathcal{K}(X,Y)$ is relatively compact iff $\mathcal{M}$ is u.c.c. and, for every $x \in X$, the set $\mathcal{M}(x) = \{Tx \colon T \in \mathcal{M}\}$ is relatively compact in Y.*

Nevertheless, for an arbitrary Banach space X, a u.c.c. set $\mathcal{M} \subset \mathcal{K}(X,Y)$ is equicompact if, in addition, every ℓ^1-sequence $(x_n)_n$ in X has a subsequence $(x_{k(n)})_n$ such that $(Tx_{k(n)})_n$ is uniformly convergent for $T \in \mathcal{M}$.

3. Dominated sets of operators

The simplest examples of equicompact sets are the sets dominated by a compact operator, that is to say, the sets for which there exists an operator $S \in \mathcal{K}(X,Y)$ such that $\|Tx\| \leq \|Sx\|$ for all $x \in X$ and all $T \in \mathcal{M}$. We are going to prove that a maximal equicompact set $\mathcal{M}$ is dominated by an operator $Q_H \in \mathcal{M}$ on every finite set $H = \{x_1, \dots, x_n\} \subset X$. But a more general class of sets $\mathcal{M} \subset \mathcal{L}(X,Y)$ enjoys this property. That is why we now consider the class of (Z,S)-dominated sets.

Given a Banach space Z and a linear map $S \colon X \longrightarrow Z$, we say that a set $\mathcal{M} \subset \mathcal{L}(X,Y)$ is (Z,S)-*dominated* if $\|Tx\| \leq \|Sx\|$ for all $x \in X$ and all $T \in \mathcal{M}$.

Examples.

(1) If $\mathcal{M} \subset \mathcal{K}(X,Y)$ is an equicompact set satisfying $\|Tx\| \leq \sup_n |x_n^*(x)|$ for all $x \in X$ and all $T \in \mathcal{M}$, with $(x_n^*)_n$ a null sequence in X^*, consider the map $S \colon X \longrightarrow c_0$ defined by $Sx = (x_n^*(x))_n$. Then, $\mathcal{M}$ is (c_0, S)-dominated.

(2) Let $\Pi_p(X,Y)$ be the space of p-summing operators from X into Y endowed with the norm $\pi_p(T) = \sup\{(\sum_n \|Tx_n\|^p)^{1/p} : (x_n)_n \in B_{\ell_w^p(X)}\}$, where $\ell_w^p(X)$ is the Banach space of the weakly p-summable sequences in X. A set $\mathcal{M} \subset \Pi_p(X,Y)$ is said to be uniformly p-dominated if there exists a positive Radon measure μ on B_{X^*} such that

$$\|Tx\|^p \leq \int_{B_{X^*}} |x^*(x)|^p \, d\mu(x^*)$$

for all $x \in X$ and all $T \in \mathcal{M}$. Put $Z = L^p(\mu, B_{X^*})$ and define $S \colon X \longrightarrow Z$ by $(Sx)(x^*) = x^*(x)$ for all $x^* \in B_{X^*}$ and all $x \in X$. Then every uniformly p-dominated set is (Z,S)-dominated.

If X and Y are Banach spaces, we will denote by $\mathcal{M}(Z,S)$ the set of all operators $T \in \mathcal{L}(X,Y)$ satisfying $\|Tx\| \leq \|Sx\|$ for all $x \in X$. Note that $\mathcal{M}(Z,S)$ is absolutely convex and closed in $\mathcal{K}(X,Y)$ for the SOT topology. The next theorem shows that $\mathcal{M} = \mathcal{M}(Z,S)$ is dominated by an operator $Q \in \mathcal{M}$ on every finite set $\{x_1, \dots, x_n\} \subset X$ when Y does not have finite cotype.

Theorem 3.1. *Let Y be a Banach space that does not have finite cotype. Given $\varepsilon > 0$, for every finite set $\{x_1, \ldots, x_n\} \subset X$ there exists $Q \in \mathcal{M}(Z, S)$ such that*

$$\|Tx_i\| \leq (1+\varepsilon)\|Qx_i\|$$

for $i = 1, \ldots, n$ and all $T \in \mathcal{M}(Z, S)$.

Proof. Since Y does not have finite cotype, Y contains ℓ_n^∞ uniformly ($\ell_n^\infty = (\mathbb{R}^n, \|.\|_\infty)$). By [2, theorem 14.1], for every $\varepsilon > 0$ and $n \in \mathbb{N}$, there is an isomorphism J_n from ℓ_n^∞ onto a subspace of Y satisfying $\|J_n^{-1}\| = 1$ and $\|J_n\| \leq 1+\varepsilon$ for all $n \in \mathbb{N}$.

Given $\{x_1, \ldots, x_n\} \subset X$, choose $z_i^* \in B_{Z^*}$ so that $|z_i^*(Sx_i)| = \|Sx_i\|$ for $i = 1, \ldots, n$. Put $y_i = J_n e_i$, $(e_i)_{i=1}^n$ being the unit basis of ℓ_n^∞. We define an operator $Q\colon X \longrightarrow Y$ by

$$Qx = \frac{1}{1+\varepsilon} J_n((\langle Sx, z_i^*\rangle)_1^n).$$

Then we have:

$$\|Qx\| \leq (1+\varepsilon)^{-1}\|J_n\|\|(\langle Sx, z_i^*\rangle)_1^n\|_\infty \leq \|Sx\| \sup_i \|z_i^*\| \leq \|Sx\|$$

and this proves that $Q \in \mathcal{M}(Z, S)$.

Finally, we need to prove that $\|Tx_i\| \leq (1+\varepsilon)\|Qx_i\|$ for $i = 1, \ldots, n$ and all $T \in \mathcal{M}(Z, S)$. Put $y_i^* = e_i^* \circ J_n^{-1}$, $(e_i^*)_{i=1}^n$ being the unit basis of $(\ell_n^\infty)^* \simeq \ell_n^1$. Note that $\|y_i^*\| \leq 1$ for $i = 1, \ldots, n$. We also denote by y_i^* a Hahn-Banach extension of $e_i^* \circ J_n^{-1}$ to Y. It is easy to show that $\langle y_i, y_j^*\rangle = \delta_{ij}$. We have:

$$\begin{aligned} \|Qx_j\| &\geq |\langle Qx_j, y_j^*\rangle| \\ &= (1+\varepsilon)^{-1} \left| \sum_{i=1}^n \langle Sx_j, z_i^*\rangle\langle y_i, y_j^*\rangle \right| \\ &= (1+\varepsilon)^{-1} \left|\langle Sx_j, z_j^*\rangle\right| \\ &= (1+\varepsilon)^{-1}\|Sx_j\| \\ &\geq (1+\varepsilon)^{-1}\|Tx_j\| \end{aligned}$$

for all $T \in \mathcal{M}(Z, S)$ and $j = 1, \ldots, n$. □

Corollary 3.2. *If the Banach space Y does not have finite cotype, every equicompact subset of $\mathcal{K}(X, Y)$ may be uniformly dominated, on each finite subset of X, by a compact operator.*

Now we give a partial converse of the last theorem in case $\mathcal{M} \subset \mathcal{K}(X, Y)$ is a SOT-countably compact set. Note that the absolutely convex hull of an equicompact set is equicompact, too. So, without loss of generality, we can suppose from now on that $\mathcal{M}$ is absolutely convex.

Theorem 3.3. *Let $\mathcal{M}$ be an absolutely convex subset of $\mathcal{K}(X, Y)$ enjoying property (F). If $\mathcal{M}$ is countably compact for the strong operator topology, then $\mathcal{M}$ is equicompact.*

Proof. (a) First, we prove the theorem in case X is a separable Banach space. Let $(x_n)_n$ be a dense sequence in X. By hypothesis, for every $n \in \mathbb{N}$, there exists $Q_n \in \mathcal{M}$ so that

$$\|Tx_i\| \leq C\|Q_n x_i\|$$

for $i = 1, \dots, n$ and all $T \in \mathcal{M}$. The sequence $(Q_n)_n$ has a cluster point $Q \in \mathcal{M}$ for the SOT. Proceeding by contradiction, it is easy to show that

$$\|Tx_i\| \leq C\|Qx_i\| \tag{1}$$

for $i = 1, \dots, n$ and all $T \in \mathcal{M}$. Now, given $x \in X$ and $\varepsilon > 0$, choose x_i such that $\|x - x_i\| < \varepsilon/K$, where $K = \sup_{T\in\mathcal{M}} \|T\|$. Using (1), we have

$$\begin{aligned} \|Tx\| &\leq \|Tx - Tx_i\| + \|Tx_i\| \\ &< \varepsilon + C\|Qx_i\| \\ &\leq \varepsilon + C(\|Qx_i - Qx\| + \|Qx\|) \\ &\leq \varepsilon + C\varepsilon + C\|Qx\| \end{aligned}$$

for all $T \in \mathcal{M}$. Letting $\varepsilon \to 0$ we conclude $\|Tx\| \leq C\|Qx\|$ for all $T \in \mathcal{M}$.

(b) Consider now an arbitrary Banach space X. In order to show that $\mathcal{M}$ is equicompact, we will prove that $\mathcal{M}$ has property (P). Let $(x_n)_n$ be a bounded sequence in X and put $H = \overline{\text{span}}\,\{x_n : n \in \mathbb{N}\}$. If i_H denotes the inclusion map from H into X, then part (a) applied to the set $\mathcal{N} = \{T \circ i_H : T \in \mathcal{M}\} \subset \mathcal{K}(H, Y)$ yields the equicompactness of $\mathcal{N}$ and, therefore, also of $\mathcal{M}$. □

Examples. 1. *A maximal equicompact set of operators valued in ℓ_2 that fails property (F).* Let $\mathcal{M}$ be the set of all operators T from c_0 into ℓ_2 satisfying

$$\|T\alpha\|_2 \leq \sup_n \left| \frac{1}{\sqrt{n}} e_n^*(\alpha) \right|$$

for all $\alpha \in c_0$ ($(e_n^*)_n$ denotes the unit basis of ℓ^1). By contradiction, suppose there exists a positive constant C such that, for every finite set $\{\alpha_1, \dots, \alpha_n\} \subset c_0$, there is an operator $Q \in \mathcal{M}$ such that $\|T\alpha_k\|_2 \leq C\|Q\alpha_k\|_2$ for $k = 1, \dots, n$ and all $T \in \mathcal{M}$. In particular, for every $n \in \mathbb{N}$, there exists $Q_n \in \mathcal{M}$ so that

$$\|Te_k\|_2 \leq C\|Q_n e_k\|_2$$

for $k = 1, \dots, n$ and all $T \in \mathcal{M}$ (here $(e_n)_n$ denotes the unit basis of c_0). Then we have

$$\sum_{k=1}^n \|T_k e_k\|_2^2 \leq C^2 \sum_{k=1}^n \|Q_n e_k\|_2^2 \tag{2}$$

for all $n \in \mathbb{N}$ and all $\{T_1, \dots, T_n\} \subset \mathcal{M}$. Now we consider the operators $T_k \in \mathcal{M}$ defined by $T_k\alpha = \frac{1}{\sqrt{k}}\, e_k^*(\alpha)\, u_k$ for all $\alpha \in c_0$, where $(u_n)_n$ is the unit basis of ℓ_2. Notice that $\|T_k e_k\|_2 = \frac{1}{\sqrt{k}}$; then (2) yields

$$C^{-2} \sum_{k=1}^n \frac{1}{k} \leq \sum_{k=1}^n \|Q_n e_k\|_2^2$$

for all $n \in \mathbb{N}$. Finally, recall that every bounded operator T from c_0 into ℓ_2 is 2-summing and $\pi_2(T) \leq \lambda\|T\|$, for some constant $\lambda > 0$ (see [2, theorem 3.5]). This fact allows us to obtain

$$\pi_2(Q_n)^2 \geq \sum_{k=1}^n \|Q_n e_k\|_2^2 \geq C^{-2} \sum_{k=1}^n \frac{1}{k}$$

for all $n \in \mathbb{N}$. This is a contradiction because $\mathcal{M}$ is bounded for the π_2-norm.

2. It is interesting to show that the countable compactness of $\mathcal{M}$ cannot be omitted in Theorem 3.3. For example, consider the closed unit ball of $\mathcal{L}(c_0, c_0)$.

This ball is the same that $\mathcal{M}(c_0, I)$, I being the identity map on c_0. The proof of Theorem 3.1 shows that $\mathcal{M}(c_0, I)$ has the following property:

> "There exists a positive constant C such that, for every finite set $\{\alpha_1, \ldots, \alpha_n\} \subset c_0$, there is an operator of finite rank $Q \in \mathcal{M}(c_0, I)$ satisfying $\|T\alpha_k\| \leq C\|Q\alpha_k\|$ for $k = 1, \ldots, n$ and all $T \in \mathcal{M}(c_0, I)$."

This implies that the closed unit ball of $\mathcal{K}(c_0, c_0)$, $\mathcal{M}$, has property (F). Nevertheless, we are going to show that $\mathcal{M}$ is not equicompact. For each $\delta = (\delta_n)_n$ belonging to the unit ball of c_0, we denote by T_δ the operator defined by $T_\delta(\alpha_n)_n = (\alpha_n \cdot \delta_n)_n$ for all $(\alpha_n)_n \in c_0$. It is obvious that $T_\delta \in \mathcal{M}$ for all $\delta \in B_{c_0}$. For every $n \in \mathbb{N}$, we have $\|T_{e_n}\| = \|e_n\| = 1$. Since $(e_n)_n$ is weakly null in c_0, this shows that $\mathcal{M}$ is not u.c.c.

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