

ON ROURKE'S EXTENSION OF GROUP PRESENTATIONS AND A CYCLIC VERSION OF THE ANDREWS–CURTIS CONJECTURE

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ABSTRACT. In 1979, Rourke proposed to extend the set of cyclically reduced defining words of a group presentation \mathcal{P} by using operations of cyclic permutation, inversion and taking double products. He proved that iterations of these operations yield all cyclically reduced words of the normal closure of defining words of \mathcal{P} if the group, defined by the presentation \mathcal{P} , is trivial. We generalize this result by proving it for every group presentation \mathcal{P} with an obvious exception. We also introduce a new, “cyclic”, version of the Andrews–Curtis conjecture and show that the original Andrews–Curtis conjecture with stabilizations is equivalent to its cyclic version.

1. TWO THEOREMS

Let a group \mathcal{G} be defined by a presentation \mathcal{P} in terms of generators and defining relations

$$(1) \quad \mathcal{P} = \langle a_1, \dots, a_m \mid R = 1, R \in \mathcal{R} \rangle,$$

where \mathcal{R} is a set of words in the alphabet $\mathcal{A}^{\pm 1} = \{a_1^{\pm 1}, \dots, a_m^{\pm 1}\}$, whose elements $R \in \mathcal{R}$ are assumed to be cyclically reduced words and are called *defining words* of \mathcal{G} . Recall that this means that $\mathcal{G} = F(\mathcal{A})/N(\mathcal{R})$ is the quotient group of the free group $F(\mathcal{A})$ in \mathcal{A} by the normal closure $N(\mathcal{R})$ of \mathcal{R} in $F(\mathcal{A})$.

In [7], Rourke introduced the two operations $+, \sim$ over a set $\mathcal{R} \subset F(\mathcal{A})$ of cyclically reduced words so that \mathcal{R}^+ denotes the set of all cyclic permutations of words in \mathcal{R} and \mathcal{R}^{-1} , and \mathcal{R}^\sim stands for the union of \mathcal{R} and the set of all cyclically reduced words obtained from all possible products R_1R_2 , where $R_1, R_2 \in \mathcal{R}$.

Let \mathcal{R}_∞ be the $(+, \sim)$ -closure of \mathcal{R} , that is, \mathcal{R}_∞ is the minimal set such that $\mathcal{R} \subseteq \mathcal{R}_\infty$ and $\mathcal{R}_\infty^+ = \mathcal{R}_\infty$, $\mathcal{R}_\infty^\sim = \mathcal{R}_\infty$. Clearly, \mathcal{R}_∞ can be obtained from \mathcal{R} by iterations of operations $+, \sim$.

The main result of Rourke's article [7] states that if the group \mathcal{G} is trivial, then $\mathcal{A} \subset \mathcal{R}_\infty$, that is, \mathcal{R}_∞ is the set of all cyclically reduced words of $F(\mathcal{A})$. This result was reproved by Scarabotti [8]. It is also observed in [7] that, in general, the set \mathcal{R}_∞ is “distinctly smaller” than the set $\widehat{N}(\mathcal{R})$ of all cyclically reduced words of the normal closure $N(\mathcal{R})$ of \mathcal{R} . For example, if $\mathcal{A} = \{a, b\}$ and $\mathcal{R} = \{a\}$, then

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$\mathcal{R}_\infty = \{a^k \mid k \in \mathbb{Z}\} \neq \widehat{N}(\mathcal{R})$. More generally, if there is a letter $a \in \mathcal{A}$ such that a, a^{-1} do not occur in words of \mathcal{R} , then, obviously, $\mathcal{R}_\infty \neq \widehat{N}(\mathcal{R})$. The first result of this article claims that the existence of such a letter is the only obstruction to the equality $\mathcal{R}_\infty = \widehat{N}(\mathcal{R})$.

Theorem 1. *Let $F(\mathcal{A})$ be the free group in an alphabet \mathcal{A} and let $\mathcal{R} \subset F(\mathcal{A})$ be a set of cyclically reduced words. Then the Rourke closure \mathcal{R}_∞ of \mathcal{R} is the set $\widehat{N}(\mathcal{R})$ of all cyclically reduced words of the normal closure $N(\mathcal{R})$ of \mathcal{R} in $F(\mathcal{A})$ unless there exists a letter $a \in \mathcal{A}$ such that both a, a^{-1} do not occur in words of \mathcal{R} .*

It is rather interesting to apply the Rourke idea of using only cyclic permutations (in place of arbitrary conjugations) in the context of the Andrews–Curtis conjecture; see [1], [2], [3], [5].

Recall that if $\mathcal{W} = (W_1, \dots, W_p)$ is a p -tuple of reduced words in $\mathcal{A}^{\pm 1}$, then Nielsen operations over \mathcal{W} of types (T1)–(T2) are defined as follows.

- (T1) For some i , $W_i \rightarrow W_i^{-1}$ and $W_k \rightarrow W_k$ for all k , $k \neq i$.
- (T2) For some i , $W_i \rightarrow W_i W_j$, $j \neq i$, and $W_k \rightarrow W_k$ for all k , $k \neq i$.

We also consider an operation of the third type (T3) so that

- (T3) For some i , $W_i \rightarrow S W_i S^{-1}$, where S is a word in $\mathcal{A}^{\pm 1}$, and $W_k \rightarrow W_k$ for all k , $k \neq i$.

Recall that extended Nielsen operations, termed *EN-operations*, are those of types (T1)–(T3); see [2].

The Andrews–Curtis conjecture, or *AC-conjecture*, states that *if a balanced presentation*

$$(2) \quad \mathcal{P} = \langle a_1, \dots, a_m \mid R_1 = 1, \dots, R_m = 1 \rangle$$

defines the trivial group, then the m -tuple $\mathcal{R} = (R_1, \dots, R_m)$ can be brought to $\mathcal{A} = (a_1, \dots, a_m)$ by a finite sequence of EN-operations.

Consider a new, “cyclic”, version of an EN-operation of type (T3) so that

- (CT3) for some i , $W_i \rightarrow \overline{W}_i$, where \overline{W}_i is a cyclic permutation of W_i , and $W_k \rightarrow W_k$ for all k , $k \neq i$.

Thus, in place of an arbitrary conjugation of W_i , we can now use only a cyclic permutation of W_i . Such redefined operations will be called cyclically extended Nielsen operations, or, briefly, *CEN-operations*, of types (CT1)–(CT3) (operations of types (CT1)–(CT2) are identical to those of types (T1)–(T2)).

Recall that EN-operations apply to tuples of (reduced) words and, for the cyclic version (as in the definition of Rourke’s closure), it is more natural to require that CEN-operations apply to tuples of cyclically reduced words. Hence, when making a CEN-operation of type (CT2), we always cyclically reduce the resulting new word $W_i W_j$.

Now the cyclic version of the Andrews–Curtis conjecture, or *CAC-conjecture*, states that *if the presentation (2) defines the trivial group, then the m -tuple $\mathcal{R} = (R_1, \dots, R_m)$ can be brought to $\mathcal{A} = (a_1, \dots, a_m)$ by a finite sequence of CEN-operations.*

Recall that there is another, more general, version of the AC-conjecture, or *AC-conjecture with stabilizations*, in which a fourth type of operations, called *stabilizations*, is allowed.

- (T4) Add (or remove) a new letter b both to the alphabet \mathcal{A} and to the set \mathcal{R} of defining words (when removing, b and b^{-1} must not occur in all other words of \mathcal{R}).

A stabilization that increases the number of letters in \mathcal{A} is termed *positive*, otherwise, it is *negative*.

Quite analogously, we consider the cyclic version of the AC-conjecture with stabilizations, or *CAC-conjecture with stabilizations*, in which we use only cyclic permutations (versus arbitrary conjugations) and cyclically reduce the resulting words after each CEN-operation. Observe that, in general, the CAC-conjecture (with stabilizations) is stronger than the original AC-conjecture (with stabilizations, respectively) in the sense that if the CAC-conjecture (with stabilizations) holds for a presentation \mathcal{P} , then the AC-conjecture (with stabilizations, respectively) also holds for this \mathcal{P} .

Theorem 2. *The original Andrews–Curtis conjecture with stabilizations is equivalent to its cyclic version with stabilizations. More specifically, if (2) is a balanced presentation of the trivial group and the defining words R_1, \dots, R_m are cyclically reduced, then the m -tuple $\mathcal{R} = (R_1, \dots, R_m)$ can be turned into $\mathcal{A} = (a_1, \dots, a_m)$ by a finite sequence of operations (T1)–(T4) if and only if \mathcal{R} can be turned into \mathcal{A} by a finite sequence of operations (CT1)–(CT3), (T4). In addition, if \mathcal{R} can be transformed into \mathcal{A} by operations (T1)–(T3) and $2s$ stabilizations, then \mathcal{R} can be transformed into \mathcal{A} by operations (CT1)–(CT3) and $2(m + 2s + 1)$ stabilizations.*

2. PROOF OF THEOREM 1

Since one direction in Theorem 1 is obvious, we will be proving that if every letter $a \in \mathcal{A}$ occurs in words of $\mathcal{R}^{\pm 1}$, then $\mathcal{R}_\infty = \widehat{N}(\mathcal{R})$. If \mathcal{A} has a single letter, then Theorem 1 is obvious and we can assume that $|\mathcal{A}| > 1$.

Let $X = Y$ denote the equality of elements of the free group $F(\mathcal{A})$ in \mathcal{A} (also called \mathcal{A} -words or just words) and let $X \equiv Y$ stand for the graphical (= letter-by-letter) equality of words.

Lemma. *Let B be a reduced \mathcal{A} -word. Then there exists a word $R \in \mathcal{R}_\infty$ such that $R \equiv BC$ with some C .*

Proof. Pick a letter $a \in \mathcal{A}^{\pm 1}$ and let R be a word in \mathcal{R}_∞ so that $R \equiv aD$ (such a word R can already be found in \mathcal{R}^+). Note that if E is a reduced word such that $E = D^3$, then $a^3E \in \mathcal{R}_\infty$, in particular, a^3E is cyclically reduced.

Now, by induction on the length $|B|$ of a word B , we will prove the following modified claim that obviously implies the Lemma: Suppose B is a reduced \mathcal{A} -word and $B \equiv b_1B_1 \equiv B_2b_2$, where $b_1, b_2 \in \mathcal{A}^{\pm 1}$. Then there exists a word $R \in \mathcal{R}_\infty$ such that $R \equiv b_1Bb_2C$. Note that this claim is actually proved above when $|B| = 1$. To make the induction step, we assume $|B| > 1$ and let $B \equiv b_1B_1 \equiv B_2b_2 \equiv B_3b_3b_2$, where $b_1, b_2, b_3 \in \mathcal{A}^{\pm 1}$. By the induction assumption, there are words R and S in \mathcal{R}_∞ for B_2 and b_2 , respectively, such that $R \equiv b_1B_2b_3C$ and $S \equiv b_2^3D$. Multiplying cyclic permutations $\bar{R} \equiv b_3Cb_1B_2$, $\bar{S} \equiv b_2^2Db_2$ of R , S , we get the word

$$\bar{R}\bar{S} \equiv b_3Cb_1B_2b_2^2Db_2 \equiv b_3Cb_1Bb_2Db_2$$

which is cyclically reduced and hence is in \mathcal{R}_∞ . Its cyclic permutation $b_1Bb_2Db_2b_3C$ is a word of the form required for B and the Lemma is proved. \square

Recall that a *van Kampen* (or *disk*) *diagram* Δ over presentation (1) is a planar, connected and simply connected 2-complex that is equipped with a labelling function φ from the set of oriented edges of Δ to $\mathcal{A}^{\pm 1}$ and has the following two properties (for more details see [4, 6]):

- (L1) If e is an oriented edge of Δ , then $\varphi(e^{-1}) = \varphi(e)^{-1}$.
- (L2) If Π is a face of Δ , then $\varphi(\partial\Pi)$ is a cyclic permutation of a word in $\mathcal{R} \cup \mathcal{R}^{-1}$.

Now suppose that W is a cyclically reduced nonempty word and $W = 1$ in the group \mathcal{G} defined by presentation (1), that is, $W \in \hat{N}(\mathcal{R})$. Then there is a van Kampen diagram $\Delta = \Delta(W)$ (see [4, 6]) over (1) such that $\varphi(\partial\Delta) \equiv W$, where the boundary $\partial\Delta$ of Δ is negatively (clockwise) oriented. We will prove that $W \in \mathcal{R}_\infty$ by induction on the number $|\Delta(2)|$ of faces in Δ .

As in [4, 6], assume that the boundary $\partial\Pi$ of a face Π of Δ is positively (counterclockwise) oriented. Pick a face Π in Δ so that there is an edge $e \in \partial\Pi$ with $e^{-1} \in \partial\Delta$. Denote $\partial\Pi = eu$, $\partial\Delta = e^{-1}d$; see Figure 1.

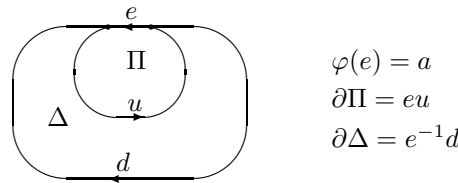


FIGURE 1

Taking e and Π out of Δ , we will get a diagram Δ_1 such that $\partial\Delta_1 = du$ and $|\Delta_1(2)| = |\Delta(2)| - 1$.

If $\varphi(\partial\Delta_1) \equiv \varphi(d)\varphi(u) = 1$ in $F(\mathcal{A})$, then $\varphi(\partial\Delta) \equiv \varphi(\partial\Pi)^{-1} \equiv W$. Hence, $W \in \mathcal{R}_\infty$, as required.

Now assume that $\varphi(\partial\Delta_1) \neq 1$ in $F(\mathcal{A})$. Note that the words $\varphi(d)$, $\varphi(u)$ are subwords of cyclically reduced words W , $\varphi(\partial\Pi)$, respectively, and hence $\varphi(d)$, $\varphi(u)$ are reduced. Since the product $\varphi(d)\varphi(u)$ need not be (cyclically) reduced, it follows that the words $D \equiv \varphi(d)^{-1}$ and $U \equiv \varphi(u)$ can be represented in one of the following three forms (F1)–(F3).

- (F1) $D \equiv BD_1E$ and $U \equiv BU_1E$, where $D_1U_1^{-1}$ is a nonempty cyclically reduced word (some of B, D_1, U_1, E may be empty).
- (F2) $D \equiv BE$ and $U \equiv BSU_0S^{-1}E$, where U_0 is a nonempty cyclically reduced word (some of B, S, E may be empty).
- (F3) $D \equiv BSD_0S^{-1}E$ and $U \equiv BE$, where D_0 is a nonempty cyclically reduced word (some of B, S, E may be empty).

Observe that there is a diagram Δ_0 such that $|\Delta_0(2)| \leq |\Delta(2)| - 1$ and $\varphi(\partial\Delta_0) \equiv D_1U_1^{-1}$ in Case (F1) or $\varphi(\partial\Delta_0) \equiv U_0$ in Case (F2) or $\varphi(\partial\Delta_0) \equiv D_0$ in Case (F3).

Since $\varphi(\partial\Delta_0)$ is cyclically reduced, it follows from the induction assumption that $\varphi(\partial\Delta_0) \in \mathcal{R}_\infty$. Denote $\varphi(e) = a$, where $a \in \mathcal{A}^{\pm 1}$. In Cases (F1)–(F2), we can see that W is a cyclic permutation of the product of some cyclic permutations of $\varphi(\partial\Delta_0)$ and $\varphi(\partial\Pi)^{-1}$; see Figure 2(a) in Case (F1) and Figure 2(b) in Case (F2).

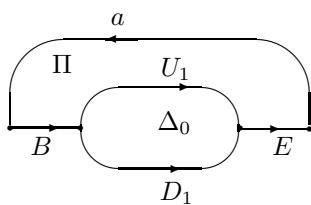


FIGURE 2(a)

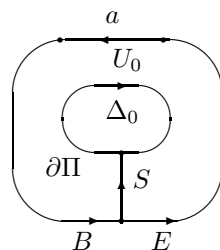


FIGURE 2(b)

In Case (F3), W is a cyclic permutation of the cyclically reduced product of words $\varphi(\partial\Pi)^{-1} \equiv B^{-1}a^{-1}E^{-1}$ and $SD_0^{-1}S^{-1}$; see Figure 3(a).

It follows from the Lemma, applied to a word of the form xSy , where $x, y \in \mathcal{A}^{\pm 1}$ are letters such that xE, D_0y are reduced (recall $|\mathcal{A}| > 1$), that there is a word $R \in \mathcal{R}_\infty$ such that $R \equiv ST$ for some T of the form $T \equiv yT_0x$ and the word $EaBSD_0T$ is cyclically reduced. Since the word $EaBSD_0T$ is the product of cyclic permutations of $\varphi(\partial\Pi)$ and $TS\varphi(\partial\Delta_0)^{-1}$, and $TS\varphi(\partial\Delta_0)^{-1}$ in its turn is the product of cyclic permutations of $R \equiv ST$ and $D_0 \equiv \varphi(\partial\Delta_0)^{-1}$ (see Figure 3(b)), it follows that $EaBSD_0T \in \mathcal{R}_\infty$. Now we observe that W^{-1} is a cyclic permutation of the product of some cyclic permutations of $EaBSD_0T$ and $R^{-1} \equiv T^{-1}S^{-1}$. Thus, $W \in \mathcal{R}_\infty$ and Theorem 1 is proved. \square

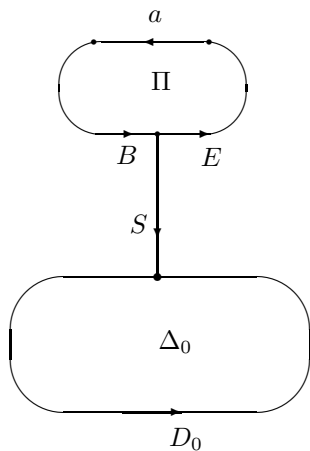


FIGURE 3(a)

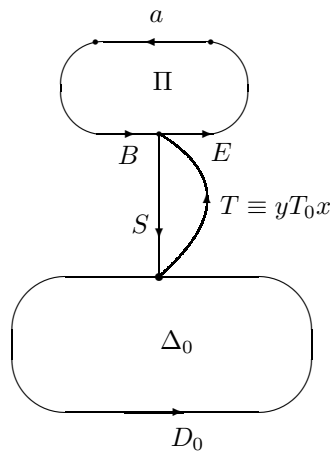


FIGURE 3(b)

3. PROOF OF THEOREM 2

Recall that the input and output of an EN-operation are tuples of (reduced) words while those of a CEN-operation are tuples of cyclically reduced words. Since CEN-operations are more restrictive and a cyclic permutation of a word can be realized by an EN-operation of type (T3), it follows that we only need to show that if $\mathcal{R} = (R_1, \dots, R_m)$ can be turned into an $\mathcal{A} = (a_1, \dots, a_m)$ by a sequence of EN-operations and $2s$ stabilizations, then \mathcal{R} can also be turned into \mathcal{A} by a sequence of CEN-operations and at most $2(m + 2s + 1)$ stabilizations.

Furthermore, it is clear that, in the process of getting \mathcal{A} from \mathcal{R} by EN-operations and $2s$ stabilizations, we can do all s positive stabilizations in the very beginning and all s negative stabilizations in the very end. Therefore, we can avoid stabilizations altogether and assume that the $(m + s)$ -tuple $(\mathcal{R}, \mathcal{B}) = (R_1, \dots, R_m, b_1, \dots, b_s)$, where b_1, \dots, b_s are all new letters that were introduced by s positive stabilizations, can be turned into the $(m + s)$ -tuple $(\mathcal{A}, \mathcal{B}) = (a_1, \dots, a_m, b_1, \dots, b_s)$ by a sequence of EN-operations $\sigma_1, \sigma_2, \dots, \sigma_\ell$ of types (T1)–(T3).

Now, with the original $(m + s)$ -tuple $\mathcal{W}(0) = (\mathcal{R}, \mathcal{B})$, we associate a $(2m + 2s + 1)$ -tuple

$$(3) \quad \mathcal{U}(0) = (R_1, \dots, R_m, b_1, \dots, b_s, x_1R_1, \dots, x_mR_m, x_{m+1}b_1, \dots, x_{m+s}b_s, y).$$

Clearly, $\mathcal{U}(0)$ can be obtained from \mathcal{R} by $m + 2s + 1$ positive stabilizations and by CEN-operations (CT1)–(CT2).

Let

$$\mathcal{W}(k) = (W_1, \dots, W_{m+s})$$

be the $(m + s)$ -tuple obtained from $\mathcal{W}(0)$ by the first $k \geq 0$ EN-operations $\sigma_1, \dots, \sigma_k$. Now, by induction on $k \geq 0$ (the base $k = 0$ is trivial), assume that a $(2m + 2s + 1)$ -tuple $\mathcal{U}(k) = (U_1, \dots, U_{2m+2s+1})$ of cyclically reduced words is already constructed so that U_i is freely conjugate (that is, conjugate in a free group) to W_i for $1 \leq i \leq m + s$ and, for $i > m + s$, we have

$$(4) \quad U_{m+s+1} \equiv x_1R_1, \dots, U_{2m+s} \equiv x_mR_m, U_{2m+s+1} \equiv x_{m+1}b_1, \dots, U_{2m+2s} \equiv x_{m+s}b_s, U_{2m+2s+1} \equiv y.$$

Let $\mathcal{W}(k + 1)$ be obtained from $\mathcal{W}(k)$ by the next EN-operation σ_{k+1} . Our goal is to find a sequence of CEN-operations that would turn $\mathcal{U}(k)$ into $\mathcal{U}(k + 1)$, where $\mathcal{U}(k + 1)$ corresponds to $\mathcal{W}(k + 1)$ in the same fashion as $\mathcal{U}(k)$ corresponds to $\mathcal{W}(k)$.

If σ_{k+1} has type (T1), that is, $W_i \rightarrow W_i^{-1}$, then we make a similar CEN-operation over $\mathcal{U}(k)$ which is $U_i \rightarrow U_i^{-1}$. If σ_{k+1} has type (T3), that is, $W_i \rightarrow SW_iS^{-1}$, then no change is needed and $\mathcal{U}(k + 1) = \mathcal{U}(k)$.

Now consider the main case when σ_{k+1} has type (T2) and $W_i \rightarrow W_iW_j, j \neq i, 1 \leq i, j \leq m + s$. Note that for some reduced word C in the alphabet $(\mathcal{A} \cup \mathcal{B})^{\pm 1} = \{a_1^{\pm 1}, \dots, a_m^{\pm 1}, b_1^{\pm 1}, \dots, b_s^{\pm 1}\}$, the word $U_iCU_jC^{-1}$ is freely conjugate to W_iW_j .

Let P_i denote a cyclically reduced word which is freely conjugate to $U_iCU_jC^{-1}$. We will construct a sequence of CEN-operations that turn the word U_i into P_i and preserve all other components of $\mathcal{U}(k)$. This new $(2m + 2s + 1)$ -tuple will be the desired $\mathcal{U}(k + 1)$.

Let $C \equiv c_1 \dots c_d$, where c_1, \dots, c_d are letters in $(\mathcal{A} \cup \mathcal{B})^{\pm 1}$, and let Q_1, \dots, Q_d be cyclic permutations of some words among $U_{m+s+1}^{\pm 1}, \dots, U_{2m+2s}^{\pm 1}$ (see (4)) such that $Q_1 \equiv c_1T_1, \dots, Q_d \equiv c_dT_d$.

Consider a sequence of CEN-operations (CT1)–(CT3) so that

$$\begin{aligned} U_i &\rightarrow yU_iy \rightarrow yU_iyc_1T_1 \rightarrow T_1yU_iyc_1 \rightarrow yT_1yU_iyc_1c_2T_2 \\ &\rightarrow yT_2yT_1yU_iyc_1c_2 \rightarrow \dots \rightarrow yT_dyT_{d-1} \dots yT_1yU_iyc_1 \dots c_dyU_j \\ &\rightarrow U_iyc_1 \dots c_dyU_jyT_dyT_{d-1} \dots yT_1y \equiv U_{i,1} \end{aligned}$$

and all other components of $\mathcal{U}(k)$ are preserved. Now we will use more CEN-operations to turn the subwords T_d, \dots, T_1 of $U_{i,1}$ into $c_d^{-1}, \dots, c_1^{-1}$, respectively.

Doing this transforms the word $U_{i,1}$ into

$$U_{i,2} \equiv U_i y c_1 \dots c_d y U_j y c_d^{-1} y c_{d-1}^{-1} \dots y c_1^{-1} y.$$

Using more CEN-operations of types (CT1)–(CT3) (the j index in CEN-operations of type (CT2) is now $2m + 2s + 1$), we delete all occurrences of letters $y^{\pm 1}$ in $U_{i,2}$. Doing this turns the word $U_{i,2}$ into a cyclic permutation of the word P_i and an application of (CT3) finally yields the desired $(2m + 2s + 1)$ -tuple $\mathcal{U}(k + 1)$ whose i th component is P_i and all other components are those of $\mathcal{U}(k)$.

Thus, by induction, it is proved that for every $k \geq 0$ there is a sequence of CEN-operations that turn the tuple $\mathcal{U}(0)$ (see (3)) into a $(2m + 2s + 1)$ -tuple $\mathcal{U}(k) = (U_1, \dots, U_{2m+2s+1})$ such that $U_{m+s+1}, \dots, U_{2m+2s+1}$ are given by formulas (4) and U_1, \dots, U_{m+s} are cyclically reduced words freely conjugate to W_1, \dots, W_{m+s} , respectively, where $\mathcal{W}(k) = (W_1, \dots, W_{m+s})$ is obtained from $\mathcal{W}(0)$ by the first k EN-operations $\sigma_1, \dots, \sigma_k$. Since $\mathcal{W}(\ell) = (a_1, \dots, a_m, b_1, \dots, b_s)$, it follows that

$$\mathcal{U}(\ell) = (a_1, \dots, a_m, b_1, \dots, b_s, U_{m+s+1}, \dots, U_{2m+2s+1}).$$

Now we can use obvious CEN-operations and $m + 2s + 1$ negative stabilizations to change $\mathcal{U}(\ell)$ into (a_1, \dots, a_m) . Theorem 2 is proved. \square

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