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RIGID CANTOR SETS IN R^3 WITH SIMPLY CONNECTED COMPLEMENT

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Abstract. We prove that there exist uncountably many inequivalent rigid wild Cantor sets in R^3 with simply connected complement. Previous constructions of wild Cantor sets in R^3 with simply connected complement, in particular the Bing-Whitehead Cantor sets, had strong homogeneity properties. This suggested it might not be possible to construct such sets that were rigid. The examples in this paper are constructed using a generalization of a construction of Skora together with a careful analysis of the local genus of points in the Cantor sets.

1. Introduction

A subset $A \subset R^n$ is *rigid* if whenever $f\colon R^n \to R^n$ is a homeomorphism with $f(A) = A$ it follows that $f|_A = id_A$. There are known examples in R^3 of wild Cantor sets that are either rigid or have simply connected complement. However, until now, no examples were known having both properties.

The class of wild Cantor sets having simply connected complement known as Bing-Whitehead Cantor sets seemed to suggest that no such example exists because every one-to-one mapping between two finite subsets of a Bing-Whitehead Cantor set $X \subset R^3$ is extendable to a homeomorphism of R^3 which takes X to X (see [Wr4] for details). In fact, any Cantor set in R^3 with simply connected complement has the property that any 2 points in the Cantor set can be separated by a 2-sphere missing the Cantor set (see [Sk]). This allows the components of the stages of a defining sequence to be separated and again suggests some type of homogeneity might exist which would prevent rigidity.

See Kirkor [Ki], DeGryse and Osborne [DO], Ancel and Starbird [AS], and Wright [Wr4] for further discussion of wild Cantor sets with simply connected complement.

Two Cantor sets X and Y in R^3 are said to be *topologically distinct* or *inequivalent* if there is no homeomorphism of R^3 to itself taking X to Y. Sher proved in [Sh] that there exist uncountably many inequivalent Cantor sets in R^3. He showed that varying the number of components in the Antoine construction leads to these inequivalent Cantor sets.

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Shilepsky used Sher's result and constructed a rigid Cantor set in R^3 (see [Sl]). Using a slightly different approach, Wright constructed a rigid Cantor set in R^3 as well (see [Wr2]), and using the Blankinship construction [Bl] Wright later extended this result to R^n, $n \geq 4$ (see [Wr3]). All these results rely heavily on the linking of the components of defining sequences for the Cantor sets. This linking yields non-simply connected complements of the Cantor sets, so these constructions cannot be modified to give examples of rigid Cantor sets with simply connected complement.

Martin [Ma] gave an example of a rigid sphere in R^3. The proof of the rigidity of the sphere used a clever idea of constructing a specific countable dense set with special properties. A similar idea will be used in our paper (see Lemma 3.1). The proof of the wildness of our examples is based on a modification of the proof of the wildness of the Antoine construction as detailed in Daverman [Da]. We will show that in fact uncountably many inequivalent examples of rigid Cantor sets with simply connected complement exist. The key technique used is that of local genus, introduced in [Ze].

2. Local genus of points in a Cantor set

Let us review the definition and some basic facts from [Ze] about the genus of a Cantor set and the local genus of points in a Cantor set.

A *defining sequence* for a Cantor set $X \subset R^3$ is a sequence (M_i) of compact 3-manifolds with boundary such that

(a) each M_i consists of pairwise disjoint cubes with handles;
(b) $M_{i+1} \subset \operatorname{Int} M_i$ for each i; and
(c) $X = \bigcap_i M_i$.

Let $\mathcal{D}(X)$ be the set of all defining sequences for X.

It is known (see [Ar]) that every Cantor set has a defining sequence, but the sequence is not uniquely determined. In fact, every Cantor set has many nonequivalent (see [Sh] for the definition) defining sequences.

Let M be a handlebody. We denote the genus of M by $g(M)$. For a disjoint union of handlebodies $M = \bigsqcup_{\lambda \in \Lambda} M_\lambda$, we define $g(M) = \sup\{g(M_\lambda);\ \lambda \in \Lambda\}$.

Let $(M_i) \in \mathcal{D}(X)$ be a defining sequence for a Cantor set $X \subset R^3$. For any subset $A \subset X$ we denote by M_i^A the union of those components of M_i which intersect A. Define

$$\begin{aligned} g_A(X;(M_i)) &= \sup\{g(M_i^A);\ i \geq 0\} \quad \text{and} \\ g_A(X) &= \inf\{g_A(X;(M_i));\ (M_i) \in \mathcal{D}(X)\}. \end{aligned}$$

The number $g_A(X)$ is called *the genus of the Cantor set X with respect to the subset A*. For $A = \{x\}$ we call the number $g_{\{x\}}(X)$ *the local genus of the Cantor set X at the point x* and denote it by $g_x(X)$. For $A = X$ we call the number $g_X(X)$ *the genus of the Cantor set X* and denote it by $g(X)$.

Let x be an arbitrary point of a Cantor set X and let $h\colon R^3 \to R^3$ be a homeomorphism. Then any defining sequence for X is mapped by h onto a defining sequence for $h(X)$. Hence the local genus $g_x(X)$ is the same as the local genus $g_{h(x)}(h(X))$.

Determining the (local) genus of a given Cantor set using the definition is not easy. If a Cantor set is given by a defining sequence, one can easily determine an upper bound. The idea of slicing discs introduced in [Ba] can be used to derive the following addition theorem for local genus. This can then be used for establishing the exact local genus. See [Ze, Theorem 14] for details.

Theorem 2.1. *Let $X, Y \subset S^3$ be Cantor sets and let p be a point in $X \cap Y$. Suppose there exists a 3-ball B and a 2-disc $D \subset B$ such that*

(1) *$p \in \operatorname{Int} B$, $\operatorname{Fr} D = D \cap \operatorname{Fr} B$, $D \cap (X \cup Y) = \{p\}$; and*
(2) *$X \cap B \subset B_X \cup \{p\}$ and $Y \cap B \subset B_Y \cup \{p\}$ where B_X and B_Y are the components of $B \setminus D$.*

Then $g_p(X \cup Y) = g_p(X) + g_p(Y)$.

The 2-disc D in the above theorem is called a slicing disc for the Cantor set $X \cup Y$.

3. Main results

Lemma 3.1. *Let $X \subset R^3$ be a Cantor set and let $A \subset X$ be a countable dense subset such that*

(1) *$g_x(X) \leq 2$ for every $x \in X \setminus A$,*
(2) *$g_a(X) > 2$ for every $a \in A$, and*
(3) *$g_a(X) = g_b(X)$ for $a, b \in A$ if and only if $a = b$.*

Then X is a rigid Cantor set in R^3.

Proof. Let $h \colon R^3 \to R^3$ be a homeomorphism such that $h(X) = X$. We will prove that $h(x) = x$ for every $x \in X$. Since A is dense in X it suffices to prove that $h(a) = a$ for every $a \in A$.

Let $b = h(a)$. As in Section 2, $g_a(X) = g_{h(a)}(h(X)) = g_{h(a)}(X) = g_b(X)$. If $b \notin A$, then $g_b(X) \leq 2$, but $g_a(X) > 2$. Hence $b \in A$ and then it follows from $g_a(X) = g_b(X)$ that $a = b$. □

Remark 3.2. In the lemma above one can replace the function g (i.e. the local genus) by an arbitrary real valued embedding invariant function satisfying conditions (1), (2) and (3). In this setting the set A need not be countable.

The main theorem, which we will prove after detailing the construction, is the following.

Theorem 3.3. *For each increasing sequence $S = (n_1, n_2, \ldots)$ of integers such that $n_1 > 2$, there exist a wild Cantor set in R^3, $X = C(S)$, and a countable dense set $A = \{a_1, a_2, \ldots\} \subset X$ such that the following conditions hold.*

(1) *$g_x(X) \leq 2$ for every $x \in X \setminus A$,*
(2) *$g_{a_i}(X) = n_i$ for every $a_i \in A$, and*
(3) *$R^3 \setminus X$ is simply connected.*

An immediate consequence of this theorem is the following.

Theorem 3.4. *There exist uncountably many inequivalent rigid wild Cantor sets in R^3 with simply connected complement.*

4. The construction

Let us fix an increasing sequence $S = (n_1, n_2, \ldots)$ of integers with $n_1 > 2$. We will construct inductively a defining sequence M_1, M_2, … for a Cantor set $X = C(S)$. The components of M_{2k+1} will be handlebodies of genus higher than

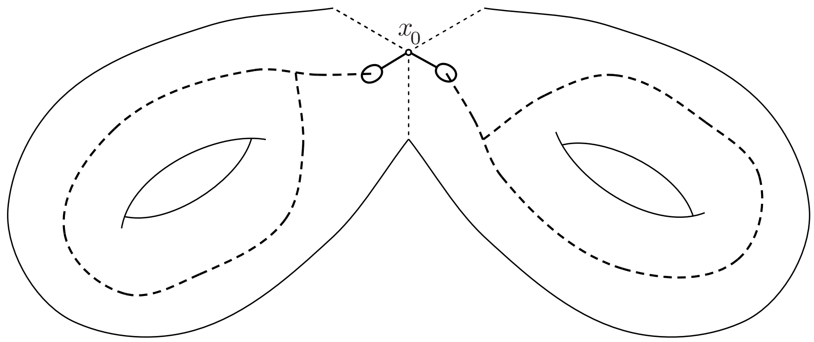

Figure 1. Manifold N

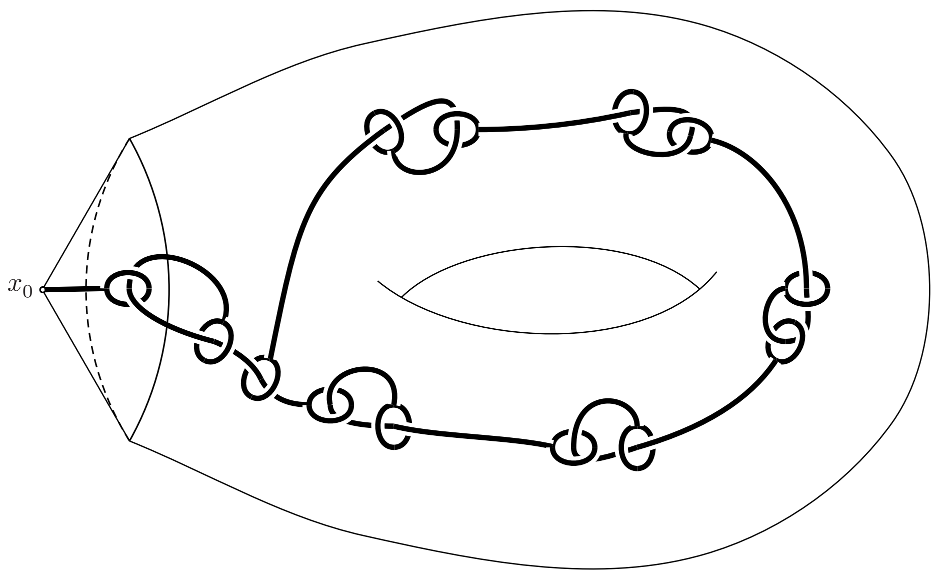

Figure 2. Linking along the spine of some handle of N

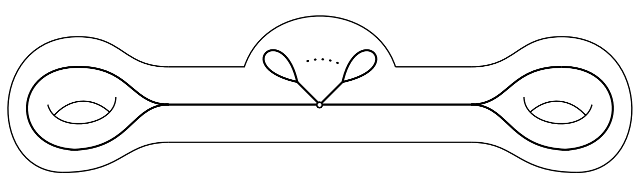

Figure 3. Modification in defining sequence

2 and these components will be obtained from M_{2k} by suitably replacing all genus 2 handlebodies. The components of M_{2k} will be obtained by replacing each component of M_{2k-1} by an appropriate chain of linked handlebodies. All except one handlebody in the chain will have genus 2.

To begin the construction, let M_1 be an unknotted genus n_1 handlebody in R^3.

4.1. **Stage $n+1$ if n is odd.** If n is odd, then by the inductive hypothesis every component of M_n is a handlebody of genus higher than 2. Let N be a genus r component of M_n.

The manifold N can be viewed as a union of r handlebodies of genus 1, $T_1 \cup \ldots \cup T_r$, identified along some 2-discs in their boundaries as shown in Figure 1.

We replace the component N of genus r by a single smaller central genus r handlebody and a linked chain of genus 2 handlebodies. We use 6 genus 2 handlebodies for each handle of N. See Figure 2 for the linking pattern in one of the genus 1 handlebodies whose union is N.

Note that the new components in N are actually unlinked if we regard them as handlebodies in R^3. Stage $n+1$ consists of all the new components constructed as above. The construction can be done so that each new component at stage $n+1$ has diameter less than half of the diameter of the component that contains it at stage n.

4.2. **Stage $n+1$ if n is even.** If n is even, we replace every genus r torus in M_n, $r > 2$, by a parallel interior copy of itself and every genus 2 torus by an embedded higher genus handlebody as shown in Figure 3.

More precisely, let us assume inductively that there exist handlebodies of genus $n_1, n_2, \ldots, n_N$ among the components of M_n. There are also K genus 2 components for some K and we replace one of these genus 2 handlebodies by a genus n_{N+1} handlebody, one by a genus n_{N+2} handlebody, $\ldots$, and one by a genus n_{N+K} handlebody. The components of M_{n+1} then consist of handlebodies of genus $n_1, \ldots, n_{N+K}$.

This completes the inductive description of the defining sequence. Define the Cantor set associated with the sequence S, $X = C(S)$ to be

$$X = \bigcap_i M_i \ .$$

In the next section we will derive some results needed for computing the local genus of points of X. In the following section we will prove that X has simply connected complement and is rigidly embedded in R^3. From the construction it is clear that X is a Cantor set.

5. Results needed for local genus computations

The following technical results will be needed in the next section in the proof of the main results. Let N be a component of M_{2i+1}. Then N is a union of genus 1 handlebodies as in the previous section. Let T be one of these genus 1 handlebodies. By construction we have that $Bd(T) \cap X$ is a singleton $\{x_0\}$. Let W be a loop in $Bd(T)$ that bounds a disc in $Bd(T)$ containing x_0 in its interior as in Figure 4.

Lemma 5.1. *If there exists a 2-disc $D \subset T$ such that $D \cap M_{r+1} = \emptyset$ for some $r > 2i+1$, and $Bd(D) = W$, then there exists a 2-disc $D' \subset T$ such that $Bd(D') = Bd(D)$ and $D' \cap M_r = \emptyset$.*

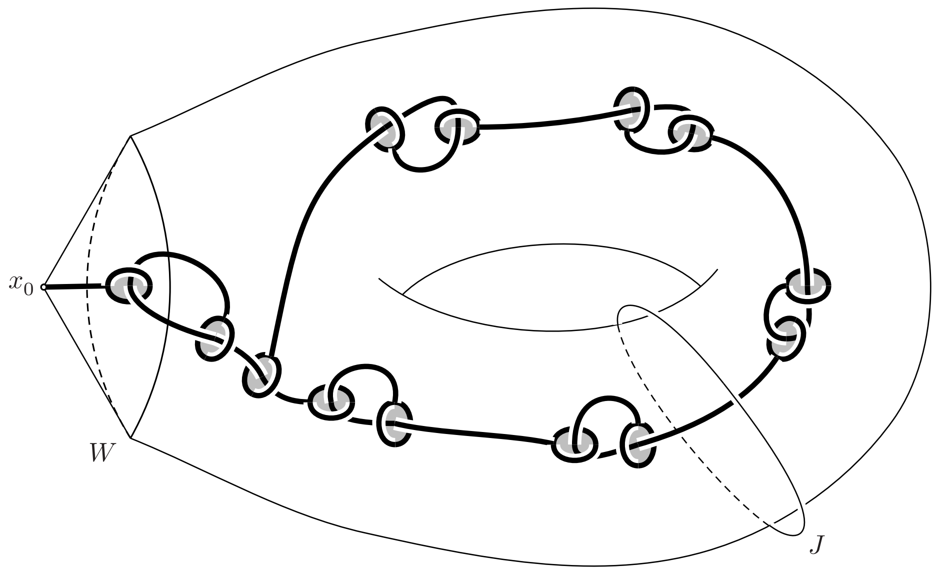

FIGURE 4. Added annuli

Proof. We consider separately the cases where r is even and where r is odd.

If r is even, each component C of $M_r \cap T$ is either a genus 2 handlebody that is also a component of M_r or a genus 1 handlebody containing x_0 that is one of the genus 1 handlebodies whose union is a component of M_r as in Figure 1. In both cases, $C \cap M_{r+1}$ consists of a single component that contains a spine of C. So D misses a spine of C and so $D \cap C$ can be isotoped to be near $Bd(C)$. Using a bicollar of $Bd(C)$ the disc D can be further pushed outside C.

We repeat the same procedure for every component C of $M_r \cap T$ and finally obtain the disc D'.

If r is odd, each component C of $M_r \cap T$ is either a genus g handlebody for $g \geq 3$ that is also a component of M_r or a genus 1 handlebody containing x_0 that is one of the genus 1 handlebodies whose union is a component of M_r as in Figure 1.

Let C be any component of $M_r \cap T$. Then C is either a genus 1 handlebody or a union of genus 1 handlebodies. Let C' be one of these genus 1 handlebodies as in Figure 4. The manifold $C' \cap M_{r+1}$ together with added discs $B_1, B_2, \ldots, B_s$ as in Figure 4 contains a spine of C'. Adjust D so that it is transverse to each B_k. Then $D \cap (B_1 \cup \ldots \cup B_k)$ is a finite collection of single closed curves. Pick an innermost one, say L, with respect to D.

If L bounds a disc on some annulus $B_l \setminus M_{r+1}$, we replace the disc on D bounded by L by a disc on $B_l \setminus M_{r+1}$ and then push the new D off $B_l \setminus M_{r+1}$. Repeating the same procedure one can modify D to obtain a disc D' so that there are no simple closed curves in $D \cap (B_1 \cup \ldots \cup B_k)$ which bound a disc on $(B_1 \cup \ldots \cup B_k) \setminus M_{r+1}$.

Now assume that L is a loop in $D \cap (B_1 \cup \ldots \cup B_k)$ which does not bound a disc on $(B_1 \cup \ldots \cup B_k) \setminus M_{r+1}$. L certainly bounds a disc, say E_l, on some B_l. By construction we have (consider Figure 4) that for every disc B_l there exists a loop J_0 in some small neighborhood of $M_{r+1} \cup B_l$ which transversally intersects E_l in one point. Now we attach to E_l a disc on D bounded by L to get a 2-sphere which transversally intersects a loop J_0 in one point. But this is impossible, so there are no essential loops in $D \cap (B_1 \cup \ldots \cup B_k) \setminus M_{r+1}$.

Hence D can be modified not to intersect the added discs $B_1 \cup \ldots \cup B_k$ and one can use the same idea as in the case where r is even to push $D \cap C'$ outside C'. □

Lemma 5.2. *Let T be one of the genus 1 handlebodies making up a component N of some M_i of genus ≥ 3. Let W be a loop on $Bd(T)$ as in Figure 4 and let x_0 be the point in $X \cap Bd(T)$. If $g_{x_0}(X \cap T) = 0$, then W bounds a disc D in T missing X.*

Proof. If $g_{x_0}(X \cap T) = 0$, there exists an arbitrary small 2- sphere S having x_0 in its interior and not intersecting $X \cap T$. Let B be the disc bounded by W in $Bd(T)$. We may assume that S intersects the disc B transversally. Then $S \cap B$ is a finite collection of simple closed curves. By cutting and pasting one can easily modify S to remove all simple closed curves in $S \cap B$ which do not encircle x_0. Because x_0 lies in the interior of S, there are an odd number of simple closed curves in $S \cap B$ encircling x_0. If there is more than one such curve, one can pick two consecutive ones (starting from the outer one), say J_1 and J_2, and modify the sphere S by replacing the annulus on S, bounded by J_1 and J_2, by the annulus on B, bounded by J_1 and J_2. Hence the sphere S can be modified to some small 2-sphere which contains x_0 in its interior and intersects B in only one simple closed curve L. The disc D is then formed as a union of the annulus on B bounded by L and W and a disc on $S \cap T$ bounded by L. □

Remark 5.3. By a small move, D can be adjusted to intersect the boundary of T only in its boundary, i.e. $D \cap Bd(T) = Bd(D)$.

6. Proof of the main results

Let $S = (n_1, n_2, \ldots)$ be an increasing sequence of integers and let $X = C(S)$ be the Cantor set constructed as in Section 4. We prove that X has the properties listed in Theorem 3.3.

6.1. **The countable dense subset A.** Each point p in X can be associated with a nondecreasing sequence of positive integers greater than 2 as follows. At stage $2n-1$, p is in a unique component. Let m_n be the genus of this component. The sequence we are looking for is $m_1, m_2, \ldots$. By construction, each m_{n+1} is either equal to m_n or is greater than m_n. It is greater than m_n precisely when the component of stage $2n$ containing p is a genus 2 torus. Let A be the set of points in X for which the associated sequence is bounded. Then A is countable and each point in A is associated with a sequence that is eventually constant. A is dense because each component of each M_i contains a point of A.

6.2. **Local genus at points of A.** Given a point x_0 in A, the associated sequence is eventually constant at an integer $K \geq 3$. We can replace the original defining sequence in the construction of X by the defining sequence consisting of only the odd stages in the original sequence past the point where the component containing x_0 is always a handlebody of genus K. Let $M_1', M_2', \ldots$ be this new defining sequence, and let N_i be the component of M_i' containing x_0. Then each N_i is a genus K handlebody and this new defining sequence for X shows that $g_{x_0}(X) \leq K$ by definition.

Note that $N_1 \supset N_2 \supset \ldots$ and that $\bigcap_i N_i = x_0$. Any two successive stages N_i and N_{i+1} are positioned like the manifold N and the smaller central copy of N in Figure 1. As in the description of the construction, the manifold N_1 can be viewed as a union of K handlebodies of genus 1, $T_1 \cup \ldots \cup T_K$, identified along some 2-discs in their boundaries. These 2-discs can be viewed as slicing discs that satisfy Theorem 2.1. So $g_{x_0}(X) = g_{x_0}(X_1) + g_{x_0}(X_2) + \ldots + g_{x_0}(X_K)$.

Here $X_j = X \cap T_j$ and $X \cap N_1$ is a wedge of Cantor sets $X_1, \ldots, X_K$, wedged at x_0. We will prove that for each j, $1 \leq j \leq K$, $g_{x_0}(X_j) \geq 1$. It will follow that $g_{x_0}(X \cap N_1) \geq K$ and therefore $g_{x_0}(X) = K$.

Assume to the contrary that for some j, $g_{x_0}(X_j) = 0$. Then By Lemma 5.2 the loop W in $Bd(T_j)$ (see Figure 4) bounds a disc D in T_j missing X_j.

Then by construction, there is a stage M'_{r+1} in the defining sequence such that D misses M'_{r+1}. Among all such discs bounded by W missing X_j, choose one for which r is minimal. That is, all discs in T_j bounded by W necessarily intersect M'_r. By Lemma 5.1, D may now be adjusted so as to miss M'_r. This contradicts the minimality of r. Hence D does not intersect M'_2. Using the same idea as in the proof of 5.1 we may adjust D to miss the spine of T_j. Hence D can be pushed onto $Bd(T_j)\backslash\{x_0\}$ but this is impossible since $Bd(D)$ is not contractible in $Bd(T_j)\backslash\{x_0\}$.

As a consequence, $g_{x_0}(X_j)$ cannot be 0. So $g_{x_0}(X_j) \geq 1$. This completes the proof that $g_{x_0}(X) = K$.

6.3. **Local genus at points of $X \setminus A$.** Let x_0 be a point of $X \setminus A$. Then the non-decreasing sequence of integers associated with x_0 is unbounded. Suppose this sequence is $(m_1, m_2, m_3, \ldots)$. Choose a subsequence of this sequence as follows. Keep only the terms in the sequence that represent the first time that an integer appears. That is, if $m_i = m_{i-1}$, discard the term m_i. The subsequence $m_1, m_{n_2}, m_{n_3}, \ldots$ obtained has the property that it is strictly increasing.

Now consider the defining sequence for the Cantor set X obtained by only considering stages M_{2i} where $2i+1$ is equal to some n_j. Consider a specific stage M_{2i} in this new defining sequence. Let N_{2i} be the component of this stage containing x_0. This component must be a genus 2 handlebody because at the very next stage x_0 is contained in a genus n_j handlebody for the first time. So the new defining sequence for X has the property that at every stage the component containing x_0 is a genus 2 handlebody. This shows that $g_{x_0}(X) \leq 2$.

6.4. **Simple connectivity of the complement.** Let $\gamma\colon S^1 \to S^3 \setminus X$. The set $\gamma(S^1)$ is compact and misses X so there exists n large enough such that $\gamma(S^1) \cap M_n = \emptyset$. We may assume that n is odd so M_n consists of handlebodies of genus higher than 2.

It is clear from the construction that the components of M_n are not linked in R^3. In fact they lie in pairwise disjoint 3-cells. Since the components are cubes with unknotted handles, the fundamental group of the complement of the components is generated by the meridional curves on the components. It therefore suffices to show how one meridional loop (say J) of some component N can be shrunk to a point in the complement of the components. By construction it is clear that J can be moved in $N \setminus M_{n+1}$ to the waist loop W of N (see Figure 4) and then moved off N. Hence $[J] = 0 \in \pi_1(S^3 \setminus X)$.

This completes the proof of Theorem 3.3 and by Lemma 3.1 we can conclude that X is indeed rigidly embedded in R^3. $\square$

Comment on wildness of X. This follows from the fact that $g_x(X) > 0$ for every $x \in A$. By a theorem of Osborne [Os, Theorem 4] we know that the Cantor set $X \subset R^3$ is tame if and only if $g_x(X) = 0$ for every point $x \in X$.

6.5. **Proof of Theorem 3.4.** The above shows that for each increasing sequence $S = (n_1, n_2, \ldots)$ of integers, such that $n_1 > 2$, there is a wild Cantor set $C(S)$ with a countable dense subset of points $a_1, a_2, \ldots$ so that the local genus at a_i is n_i and the local genus at other points is less than or equal to 2. It is well known that there are uncountably many increasing sequences of integers $S = (n_1, n_2, \ldots)$ such that $n_1 > 2$.

To complete the proof, it suffices to show that the Cantor sets associated with distinct sequences S and S' are embedded in an inequivalent way. Let $X = C(S)$ and $X' = C(S')$ where sequences $S = (n_1, n_2, \ldots)$ and $S' = (n'_1, n'_2, \ldots)$ are distinct. Without loss of generality there exists k, such that $n'_i \neq n_k$ for every i. Let $a_k \in X$ be the point where $g_{a_k}(X) = n_k$. Assume to the contrary that there exists a homeomorphism $h\colon R^3 \to R^3$ such that $h(X) = X'$. Then we have $g_{h(a_k)}(h(X)) = g_{a_k}(X) = n_k$. This is a contradiction as there is no point in $h(X) = X'$ at which the local genus of X' is equal to n_k. $\square$

7. Questions

As stated in the introduction Bing-Whitehead Cantor sets have some strong homogeneity properties and therefore are not rigid.

- Does varying the numbers of consecutive Bing links and Whitehead links yield inequivalent Cantor sets? (This number cannot be arbitrary. See [AS] and [Wr4] for details.)

The construction above gives a rigid Cantor set such that $g_x(X) \leq 2$ for $x \in X \setminus A$ and $g_{a_i}(X) = n_i$ for $a_i \in A$. Hence $g(X) = \infty$.

Let a positive integer r be given.

- Does there exist a rigid Cantor set X such that $g_x(X) = r$ for every $x \in X$? (For $r = 1$ the answer is affirmative. See [Sl], [Wr2].)
- Does there exist a rigid Cantor set X having simply connected complement such that $g_x(X) = r$ for every $x \in X$?

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