ON UNBOUNDEDNESS OF MAXIMAL OPERATORS
FOR DIRECTIONAL HILBERT TRANSFORMS

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Abstract. We show that for any infinite set of unit vectors $U$ in $\mathbb{R}^2$ the maximal operator defined by
$$H_U f(x) = \sup_{u \in U} \left| \frac{1}{pv} \int_{-\infty}^{\infty} f(x - tu) \frac{t}{t} dt \right|, \quad x \in \mathbb{R}^2,$$
is not bounded in $L^2(\mathbb{R}^2)$.

1. Introduction

For a rapidly decreasing function $f$ and a unit vector $u = (\cos \theta, \sin \theta), \theta \in [0, 2\pi]$, we define
$$H_u f(x, y) = \frac{1}{pv} \int_{-\infty}^{\infty} f(x - t \cos \theta, y - t \sin \theta) \frac{t}{t} dt,$$
which is the one-dimensional Hilbert transform along the direction $u$. It is well known that this operator can be extended to a bounded operator from $L^p(\mathbb{R}^2)$ to itself when $1 < p < \infty$. In this paper we study operators
$$H_U f(x, y) = \sup_{u \in U} |H_u f(x, y)|,$$
where $U$ is a set of unit vectors $u$ in $\mathbb{R}^2$. Analogous operators for the maximal functions are properly investigated. The case of lacunary $U$ was first considered in the papers [18], [7], [15]. A final result was obtained by A. Nagel, E. M. Stein and S. Wainger in [15]. They proved the boundedness of the norms of these operators in $L^p(\mathbb{R}^2), 1 < p < \infty$, for a lacunary $U$. Upper bounds of such operators depending on the cardinality $\# U$ of the set $U$ are considered in the papers [1], [2], [3], [10], [17]. The definitive estimates are due to N. Katz ([13], [14]), where he obtained a logarithmic order for the norms of two different maximal operators depending on $\# U$. Various generalizations of these results were considered in series of papers ([1], [2], [3], [10], [17]).

As for the operator (1), there were no results except for the bound
$$\|H_U f(x)\|_{L^2} \lesssim \log \# U \|f\|_{L^2}.$$
This is an immediate consequence of the Menshov-Rademacher theorem (see [9] or [12]), in spite of the fact that in Katz’s theorems a subtle range of ideas are used.
It was not even known whether or not $H_U$ is bounded in $L^2$ for an infinite lacunary set $U$. The main result of the paper is unboundedness of $H_U$ in $L^2$ for any infinite $U$.

**Theorem 1.** For any infinite set of unit vectors $U$ the operator $H_U$ cannot be extended to a bounded operator from $L^2(\mathbb{R}^2)$ to $L^p(\mathbb{R}^2)$ for all $1 \leq p < \infty$.

This theorem is an immediate consequence of the following estimate.

**Theorem 2.** If $U$ is a finite set and $1 \leq p < \infty$, then $\|H_U\|_{L^2 \to L^p} \geq c \sqrt{\log \#U}$, where $c > 0$ is an absolute constant.

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### 2. Proof of the Theorems

Let $f_n(x)$, $n = 1, 2, \cdots, 2^m - 1$ ($f_n \neq 0$), be a system of functions defined on the square

$$Q = [-\pi, \pi] \times [-\pi, \pi].$$

In some places we shall use for $f_n(x)$ double numbering, defined by

$$f_j^{(k)}(x) = f_n(x), \quad n = 2^k + j - 1, \quad 1 \leq j \leq 2^k, \quad k = 0, 1, \cdots, m - 1.$$

We shall say the sequence $f_n(x) = f_j^{(k)}(x)$ is a tree-system, if

$$\text{(2) } \text{supp } f_{2^j-1}^{(k+1)} \subset \{x \in Q : f_j^{(k)}(x) > 0\}, \quad \text{supp } f_{2^j}^{(k+1)} \subset \{x \in Q : f_j^{(k)}(x) < 0\}.$$  

Applying (2) several times we get

$$\text{supp } f_{2^j}^{(k+r)} \subset \{x \in Q : f_j^{(k)}(x) > 0\} \iff i \in (2^r j - 2^r, 2^r j - 2^r - 1],$$

$$\text{supp } f_{2^j}^{(k+r)} \subset \{x \in Q : f_j^{(k)}(x) < 0\} \iff i \in (2^r j - 2^r - 1, 2^r j],$$

and then

$$\text{(3) } \text{supp } f_{2^j}^{(k+r)} \cap \{x \in Q : f_j^{(k)}(x) > 0\} = \emptyset \iff i \notin (2^r j - 2^r, 2^r j - 2^r - 1],$$

$$\text{(4) } \text{supp } f_{2^j}^{(k+r)} \cap \{x \in Q : f_j^{(k)}(x) < 0\} = \emptyset \iff i \notin (2^r j - 2^r - 1, 2^r j].$$

The following lemma for the Haar system has been proved by E. M. Nikishin and P. L. Ul’yanov [12], and we use the same idea to prove a general one (see also [12]).

**Lemma 1.** If $f_n(x)$, $x \in Q$, $n = 1, 2, \cdots, 2^m - 1$, is tree-system, then there exists a permutation $\sigma$ of the numbers $\{1, 2, \cdots, 2^m - 1\}$ such that

$$\sup_{1 \leq l < 2^m} \left| \sum_{n=1}^{l} f_{\sigma(n)}(x) \right| \geq \frac{1}{3} \sum_{n=1}^{2^m - 1} |f_n(x)|.$$  

**Proof.** We connect with each $f_n(x)$, $n = 2^k + j - 1$, a number

$$t_n = \frac{2j - 1}{2^{k+1}} \in [0, 1].$$

Note they are not equal for different $n$’s. Define the permutation $\sigma$ so that

$$t_{\sigma(1)} < t_{\sigma(2)} < \cdots < t_{\sigma(2^m-1)}.$$
We shall prove that for any \( x \in Q \) there exists a number \( l = l(x) \) with

\[
(5) \quad f_{\sigma(n)}(x) \geq 0, \text{ if } n > l(x), \\
(6) \quad f_{\sigma(n)}(x) \leq 0, \text{ if } n \leq l(x).
\]

Defining

\[
l = l(x) = \sup\{n : 1 \leq n < 2^m, f_{\sigma(n)}(x) \leq 0\},
\]

we shall have (5) immediately and if \( \nu = \sigma(l(x) + 1) \), then

\[
(7) \quad f_{\nu}(x) > 0.
\]

To prove (5) it is enough to show that if (7) holds and \( t_n > t_{\nu} \), then \( f_n(x) \geq 0 \).

Suppose

\[
\nu = 2^k + j - 1, \quad n = 2^r + i - 1.
\]

According to the assumption

\[
(8) \quad t_{\nu} = \frac{2j - 1}{2^{k+1}} < \frac{2i - 1}{2^{s+1}} = t_n.
\]

If \( s > k \), then \( s = k + r \quad (r > 0) \). From (8) we get

\[
i > 2^r j - 2^{r-1}.
\]

Therefore by (3) we obtain

\[
supp f_n \cap \{f_{\nu} > 0\} = \emptyset.
\]

Since \( f_{\nu}(x) > 0 \) we get \( f_n(x) = 0 \).

If \( s < k \), then \( k = s + r \quad (r > 0) \). Applying (8) we get

\[
j \leq 2^r i - 2^{r-1}.
\]

Hence by (3) we have

\[
supp f_{\nu} \cap \{f_n < 0\} = \emptyset.
\]

By (7) we conclude \( f_n(x) \geq 0 \). So (5) and (6) are proved. Using them, we obtain

\[
\max_{1 \leq l < 2^m} \left| \sum_{n=1}^{l} f_{\sigma(n)}(x) \right| = \max_{1 \leq l(x)} \left| \sum_{n=1}^{l(x)} f_{\sigma(n)}(x) \right| > \frac{1}{3} \sum_{n=1}^{2^m-1} |f_{\sigma(n)}(x)|.
\]

If

\[
- \sum_{n=1}^{l(x)} f_{\sigma(n)}(x) = \sum_{n=1}^{l(x)} |f_{\sigma(n)}(x)| < \frac{1}{3} \sum_{n=1}^{2^m-1} |f_{\sigma(n)}(x)|,
\]

then we get

\[
\sum_{n=1}^{2^m-1} |f_{\sigma(n)}(x)| > 2 \sum_{n=1}^{l(x)} |f_{\sigma(n)}(x)|
\]

and therefore

\[
\left| \sum_{n=1}^{l(x)+1} f_{\sigma(n)}(x) \right| \geq \left| \sum_{n=1}^{l(x)+1} f_{\sigma(n)}(x) \right| - \left| \sum_{n=1}^{l(x)} f_{\sigma(n)}(x) \right| > \frac{1}{2} \sum_{n=1}^{2^m-1} |f_{\sigma(n)}(x)| > \frac{1}{3} \sum_{n=1}^{2^m-1} |f_{\sigma(n)}(x)|.
\]
Thus we conclude
\[
\sup_{1 \leq l < 2^m} \left| \sum_{n=1}^{l} f_{\sigma(n)}(x) \right| > \frac{1}{3} \sum_{n=1}^{2^m-1} \left| f_{\sigma(n)}(x) \right| = \frac{1}{3} \sum_{n=1}^{2^m-1} |f_n(x)|, \quad \square
\]

Fix a Schwartz function \( \phi(x) \) with
\[
(9) \quad \phi(x) > 0, \quad \int_{\mathbb{R}} \phi(x) dx = 1, \quad \text{supp} \widehat{\phi} \subset [-1, 1].
\]
We consider operators
\[
(10) \quad \Phi_n(f) = \Phi_n(x, y, f) = n^2 \int_{\mathbb{R}} f(x - t, y - s) \phi(nt) \phi(ns) dt ds, \quad n = 1, 2, \ldots.
\]
Applying (10), for any \( f \in L^\infty(\mathbb{R}^2) \) we get
\[
(11) \quad \inf_{(x, y) \in \mathbb{R}^2} f(x, y) \leq \Phi_n(x, y, f) \leq \sup_{(x, y) \in \mathbb{R}^2} f(x, y),
\]
\[
(12) \quad \text{supp} \widehat{\Phi_n(f)} \subset [-n, n] \times [-n, n].
\]
If in addition \( f \) is compactly supported, using a standard argument, we conclude
\[
(13) \quad \| \Phi_n(f) - f \|_{L^2} \to 0, \quad \text{as} \ n \to \infty.
\]
If
\[
(14) \quad n = 2^k + j - 1, \quad 1 \leq j \leq 2^k, \quad k = 0, 1, \ldots, m - 1,
\]
then we denote
\[
\bar{n} = 2^{k-1} + \left[ \frac{j+1}{2} \right],
\]
where \( \lfloor \cdot \rfloor \) means an integer part of a number. Using this notation we may write the conditions (2) by
\[
(15) \quad \text{supp} f_{\bar{n}} \subset \{(-1)^{j+1} \cdot f_{\bar{n}} > 0\}.
\]
We shall consider sectors defined by
\[
\{ (x, y) \in \mathbb{R}^2 : x + iy = re^{i\theta}, r \geq 0, \alpha \leq \theta \leq \beta \}
\]
where \( 0 \leq \alpha < \beta \leq 2\pi \). Some arguments in the proof of following lemma are derived from the paper [11].

**Lemma 2.** Let \( S_n, n = 1, 2, \ldots, n = 2^m - 1, \) be sectors on the plane. Then there exist functions \( f_n \in L^2(\mathbb{R}^2), n = 1, 2, \ldots, \nu, \) such that
\[
(16) \quad \text{supp} \widehat{f_n} \subset S_n,
\]
\[
(17) \quad \sum_{j=1}^{\nu} \|f_j\|_{L^2} \leq c_1,
\]
\[
(18) \quad |\{(x, y) \in Q : \max_{1 \leq \nu} |\sum_{j=1}^{\nu} f_{\sigma(j)}(x, y)| > c_3 \sqrt{\log \nu} \}| > c_2,
\]
where \( \sigma \) is the permutation from Lemma [1] and all the constants are absolute.

**Proof.** We will assume (14) everywhere below. For a given \( \varepsilon > 0 \) define sets \( E_n = E_1^{(k)} \subset Q \) \( (E_1 = E_1^{(0)} = Q), g_n \in L^\infty(\mathbb{R}^2) \) and \( p_n, q_n \in \mathbb{Z} \) with conditions
a) \( E_n = \{ (x, y) \in \mathbb{R}^2 :(-1)^{j+1} \cos(p_n x + q_n y) > 0 \}, n = 2, 3, \ldots, \nu, \)
b) \( 0 \leq g_n \leq 1, \|g_n - I_{E_n}\|_{L^2} \leq \varepsilon, n = 1, 2, \ldots, \nu, \)
c) \( \text{supp} \widehat{g_n} \subset S_n - (p_n, q_n), n = 1, 2, \ldots, \nu, \)
d) \( \int_{E_n} |\cos(p_n x + q_n y)|\ dx\ dy > \frac{|E_n|}{3}, \)
We do it by induction. Take \( E_1 = E_1^{(0)} = Q \). According to (13) there exists \( l > 0 \) with
\[
\|\Phi_l(\mathbb{I}_{E_1}) - \mathbb{I}_{E_1}\|_{L^2} < \varepsilon.
\]
Define \( g_1 = \Phi_l(\mathbb{I}_{E_1}) \) and then applying (11) we get b) for \( n = 1 \). We note that if \( E \) is a measurable set then
\[
(19) \quad \int_E |\cos(px + qy)|dxdy \geq \int_E \cos^2(px + qy)dxdy = \frac{|E|}{2} + \int_E \cos(2(px + qy))dxdy \to \frac{|E|}{2} \text{ as } |p|, |q| \to \infty.
\]
This observation shows that for sufficiently large \( p_1 = p \) and \( q_1 = q \) we shall have condition d) for \( n = 1 \). On the other hand by (12) \( \text{supp} \hat{g}_1 \) is bounded. Thus for an appropriate \( p_1, q_1 \) we will have also c) (with \( n = 1 \)). Certainly we can choose \( p_1 \) and \( q_1 \) common for both conditions c) and d). Now we suppose that the conditions a)–d) hold for any \( k < n \), in particular for \( \bar{n} \). We define \( E_n \) by the equality in a). Then we choose the positive integer \( l \) with
\[
\|\Phi_l(\mathbb{I}_{E_n}) - \mathbb{I}_{E_n}\|_{L^2} < \varepsilon.
\]
and denote \( g_n = \Phi_l(\mathbb{I}_{E_n}) \). Again applying (19) and using the boundedness of \( \text{supp} \hat{g}_n \) we may choose integers \( p_n, q_n \) satisfying c) and d) together. Using condition a), it is easy to check that the sets \( E_n \) satisfies the conditions
\[
E_j^{(k)} \cap E_j^{(k')} = \emptyset, \text{ if } j \neq j',
\]
\[
E_{2j-1}^{(k+1)} \cup E_{2j}^{(k+1)} \subset E_j^{(k)}, \quad |E_j^{(k)} \setminus (E_{2j-1}^{(k+1)} \cup E_{2j}^{(k+1)})| = 0.
\]
Using this properties we conclude
\[
(20) \quad \sum_{n=1}^\nu \mathbb{I}_{E_n}(x, y) = m, \text{ almost everywhere on } Q.
\]
Now we define
\[
(21) \quad f_n(x, y) = \frac{e^{i(p_n x + q_n y)} g_n(x, y)}{\sqrt{m}}.
\]
Condition (10) immediately follows from c), because
\[
\text{supp} \hat{f}_n = \text{supp} \hat{g}_n + (p_n, q_n).
\]
On the other hand taking a small \( \varepsilon \) by b) and (20) we obtain
\[
\sum_{n=1}^\nu \int_Q |f_n|^2 = \frac{1}{m} \sum_{n=1}^\nu \int_Q |g_n|^2 \leq \frac{2\nu}{m} \varepsilon^2 + \frac{2}{m} \sum_{n=1}^\nu \int_Q \mathbb{I}_{E_n} \leq c_1,
\]
which gives (17). Now consider functions
\[
(22) \quad \hat{f}_n = \text{Re } f_n \cdot \mathbb{I}_{E_n} = \frac{\cos(p_n x + q_n y) \cdot g_n(x, y) \cdot \mathbb{I}_{E_n}(x, y)}{\sqrt{m}}.
\]
Applying b) and (21) we get
\[
(23) \quad \|\hat{f}_n - \text{Re } f_n\|^2_{L^2} = \int_{\mathbb{R}^2 \setminus E_n} |\text{Re } f_n|^2 \leq \int_{\mathbb{R}^2 \setminus E_n} |f_n|^2 = \frac{1}{m} \int_{\mathbb{R}^2 \setminus E_n} |g_n|^2 \leq \frac{\varepsilon^2}{m}.
\]
On the other hand we have
\[
\max_{1 \leq n \leq N} \left| \sum_{j=1}^{n} f_{\sigma(j)} \right| \geq \max_{1 \leq n \leq N} \left| \sum_{j=1}^{n} \text{Re} f_{\sigma(j)} \right| \\
\geq \max_{1 \leq n \leq N} \left| \sum_{j=1}^{n} \tilde{f}_{\sigma(j)} \right| - \sum_{j=1}^{\nu} |\tilde{f}_j - \text{Re} f_j|.
\]

(24)

From (24) we obtain
\[
\left\| \sum_{j=1}^{\nu} |\tilde{f}_j - \text{Re} f_j| \right\|_{L^2} \leq \frac{\nu \varepsilon}{\sqrt{m}}.
\]

Therefore taking a small \( \varepsilon > 0 \) we can say that
\[
\left| \left\{ (x, y) \in Q : \max_{1 \leq n \leq \nu} \left| \sum_{j=1}^{n} \tilde{f}_{\sigma(j)}(x, y) \right| > c_1 \sqrt{\log \nu} \right\} \right| \leq \delta,
\]

(25)

for any given \( \delta > 0 \). From (24) and (25) we conclude that to prove (18) and also the lemma, it is enough to prove
\[
\left| \left\{ (x, y) \in Q : \max_{1 \leq n \leq \nu} \left| \sum_{j=1}^{n} \tilde{f}_{\sigma(j)}(x, y) \right| > c_1 \sqrt{\log \nu} \right\} \right| > c_2.
\]

(26)

Let us show that \( \tilde{f}_n \) is a tree-system, i.e. it satisfies (15). Since \( g_n > 0 \) from (22) we get that \( \tilde{f}_n(x, y) \) and \( \cos(p_n x + q_n y) \) have same sign in the set \( E_n \). Therefore by a) we obtain
\[
supp \tilde{f}_n \subset E_n = \{(x, y) \in E_n : (-1)^{j+1} \cos(p_n x + q_n y) > 0 \}
\]
\[
= \{(x, y) \in E_n : (-1)^{j+1} \tilde{f}_n(x, y) > 0 \}
\]
\[
= \{(x, y) \in E_n : (-1)^{j+1} \tilde{f}_n(x, y) > 0 \}
\]
\[
= \{(x, y) \in Q : (-1)^{j+1} \tilde{f}_n(x, y) > 0 \}.
\]

Hence \( \tilde{f}_n \) is a tree-system. So according to Lemma 1 we have
\[
\max_{1 \leq n \leq \nu} \left| \sum_{j=1}^{n} \tilde{f}_{\sigma(j)}(x, y) \right| \geq \frac{1}{3} \sum_{j=1}^{\nu} |\tilde{f}_j(x, y)|.
\]

(27)

From (21) and the conditions b) and d) we get
\[
\int_Q |\tilde{f}_n| = \frac{1}{\sqrt{m}} \int_{E_n} |g_n \cos(p_n x + q_n y)| dx dy \geq \frac{1}{\sqrt{m}} \int_{E_n} |\cos(p_n x + q_n y)| dx dy
\]
\[
- \frac{1}{\sqrt{m}} \int_{E_n} |(1 - g_n) \cos(p_n x + q_n y)| dx dy \geq \frac{|E_n|}{3\sqrt{m}} - \frac{\varepsilon}{\sqrt{m}}.
\]

If we take \( \varepsilon > 0 \) to be small, then from (20) we obtain
\[
\int_Q \sum_{j=1}^{\nu} |\tilde{f}_j| \geq \frac{1}{3\sqrt{m}} \sum_{n=1}^{\nu} |E_n| - \frac{\nu \varepsilon}{\sqrt{m}} \geq \sqrt{m}.
\]

Combining this and (27) we get
\[
\int_Q \max_{1 \leq n \leq \nu} \left| \sum_{j=1}^{n} \tilde{f}_{\sigma(j)}(x, y) \right| \geq \sqrt{m}.
\]

(28)
On the other hand by (20), (21) and b) for any \((x, y) \in Q\) we have
\[
\max_{1 \leq n \leq N} \left| \sum_{j=1}^{n} \hat{f}_{\sigma(j)}(x, y) \right| \leq \sum_{j=1}^{\nu} \left| \hat{f}_{j}(x, y) \right| \leq \frac{1}{\sqrt{m}} \sum_{j=1}^{\nu} \|E_n(x, y)\| \leq \sqrt{m}.
\]
From (28) and (29) follows (26). \(\square\)

Proof of Theorem 2. For any region \(S \subset \mathbb{R}^2\) we denote
\[
T_S f(x, y) = \int_{S} e^{i(x+\eta y)} \hat{f}(\xi, \eta) d\xi d\eta.
\]
Since the multiplier for the Hilbert transform is \( i \cdot \text{sign} x \), for any direction \( u = (\cos \theta, \sin \theta) \) we have
\[
\hat{H}_u f(x, y) = i \cdot \text{sign}(x \cos \theta + y \sin \theta) \hat{f}(x, y).
\]
Thus we conclude
\[
H_u f = i(2 \cdot T_{u_n} f - f)
\]
where
\[
\Gamma_u = \{(x, y) \in \mathbb{R}^2 : x \cos \theta + y \sin \theta \geq 0\}.
\]
Denote
\[
T_U f = \sup_{u \in U} |T_{u_n} f|.
\]
Let \(U = \{u_k = (\cos \theta_k, \sin \theta_k) : k = 1, 2, \cdots, N\}\) be the set of directions from Theorem 2. Without loss of generality we can assume \(\theta_k \in (0, \pi/2)\), \(\theta_1 < \theta_2 < \cdots < \theta_N\) and \(N = 2^m\). According to (30), to prove Theorem 2 it is enough to prove that
\[
\|T_U f\|_{L^1} \geq \sqrt{\log N} \|f\|_{L^2}
\]
for some function \(f \in L^2(\mathbb{R}^2)\). We denote by \(S_k, k = 1, 2, \cdots, \nu = 2^m - 1\), the sectors obtained by the vectors \((u_k)^\perp = (\cos \theta_k, -\sin \theta_k)\) and \((u_{k+1})^\perp = (\cos \theta_{k+1}, -\sin \theta_{k+1})\), i.e.
\[(32) \quad S_k = \{(x, y) \in \mathbb{R}^2 : x \geq 0, x \cos \theta_k + y \sin \theta_k \geq 0, x \cos \theta_{k+1} + y \sin \theta_{k+1} \leq 0\}.
\]
Hence if we suppose
\[
\text{supp} \hat{f} \subset \bigcup_{k=1}^{\nu} S_k,
\]
then we can write
\[
(33) \quad T_U f(x, y) = \sup_{1 \leq l \leq \nu} \left| \sum_{k=1}^{l} T_{S_k} f(x, y) \right|.
\]
We define functions \(f_n\) satisfying the conditions of Lemma 2 corresponding to the sectors \(S'_n = S_{\sigma^{-1}(n)}\) in (32). Denote
\[
f = \sum_{k=1}^{\nu} f_k.
\]
Since \(S_n\) are mutually disjoint the functions \(f_n\) are orthogonal. Thus by (17) we get \(\|f\|_{L^2} \leq c_1\). From (16) we have supp \(\hat{f}_n \subset S_{\sigma^{-1}(n)}\), \(n = 1, 2, \cdots, \nu\), i.e. supp \(f_{\sigma(n)} \subset S_n\) and therefore
\[
T_{S_n} f(x) = f_n(x).
\]
According to (33) we obtain
\[ T_U f(x, y) = \max_{1 \leq \ell \leq N} \left| \sum_{j=1}^{\ell} f_{\sigma(j)}(x, y) \right|. \]

Using (17) and (18), we get
\[ \left| \{(x, y) \in Q : T_U f(x, y) > c_3 \sqrt{\log \nu} \} \right| > c_2, \]
and therefore
\[ \| T_U f \|_{L^p} \gtrsim \sqrt{\log N} \| f \|_{L^2}. \]

\[ \Box \]

References


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