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SEMIPRIME SMASH PRODUCTS AND H-STABLE PRIME RADICALS FOR PI-ALGEBRAS

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ABSTRACT. Assume that H is a finite-dimensional Hopf algebra over a field k and that A is an H-module algebra satisfying a polynomial identity (PI). We prove that if H is semisimple and A is H-semiprime, then $A\#H$ is semiprime. If H is cosemisimple, we show that the prime radical of A is H-stable.

1. INTRODUCTION

A long-standing open question on actions of Hopf algebras, raised in 1986 by Cohen and Fischman [CF], asks whether a smash product $A\#H$ is semiprime, when H is semisimple and the algebra A is semiprime. Here H is a finite-dimensional Hopf algebra and A is an H-module algebra. Previous results on this problem have usually involved additional assumptions about the Hopf algebra H as well as the kind of action of H on A. There has been renewed interest in this problem recently, in papers by the second author and Schneider [MS], Lomp [Lo], Braun [Br], Skryabin and van Oystaeyen [SvO], and a paper by the present authors together with Small [LMS]. Most of these newer results involve assumptions on the algebra A, rather than on H. In [LMS] we proved that the CF-conjecture was true if A was an affine PI-algebra which is H-semiprime of characteristic 0; in characteristic $p > 0$ additional assumptions were needed. In the new paper of [SvO], the CF conjecture is proved for Noetherian rings A; this result includes the main semiprimeness results of [Lo] and [Br].

In this paper we prove that the CF-conjecture is true for any PI-algebra A which is H-semiprime.

A second open question is whether, if H is a finite-dimensional cosemisimple Hopf algebra acting on an algebra A, the prime radical $P(A)$ is H-stable. In fact the two questions are "dual" in the following sense: $P(A)$ is H-stable for all actions of $H \iff$ the CF-semiprimeness conjecture is true for all actions of H^* [MS], [LMS]; this fact will be stated precisely in Section 2. The first recent result on the second problem was in [L], where it was shown to be true when A is finite-dimensional, with suitable assumptions on the characteristic of k. Here we prove that when H is cosemisimple, the prime radical of any H-module PI-algebra A is H-stable.

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Finally we give an application to the "invariant theory" of Hopf algebra actions and show that if H is semisimple and A is an H-semiprime PI-algebra, with the usual assumptions on characteristic, then every non-zero H-stable left or right ideal of A intersects the invariant ring A^H non-trivially.

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2. Preliminaries

Throughout H denotes a finite-dimensional Hopf algebra over the field k and A is an H-module algebra. We first discuss some elementary facts which are true for any finite-dimensional Hopf algebra H and for any H-module algebra A.

First, recall that the standard left action of H on its dual H^* is given by $h \rightharpoonup f = \sum f_2(h) f_1$.

It is also well known that A is a left H-module algebra if and only if A is a right H^*-comodule algebra [M, Lemma 1.6.4], when H is finite-dimensional. This correspondence is given as follows:

First, assume that A is a left H-module algebra, and choose $a \in A$. Since $H \cdot a$ is finite dimensional, we may choose a basis $\{a_1, \ldots, a_m\}$ for $H \cdot a$. Now for any $h \in H$, we may write $h \cdot a = \sum_{i=1}^m \gamma_{a,h,i} a_i$, for $\gamma_{a,h,i} \in k$. Setting $f_{a,i}(h) := \gamma_{a,h,i}$, one sees that $f_{a,i} \in H^*$ and that A becomes a right H^*-comodule algebra via $\delta : A \to A \otimes H^*$, $a \mapsto \sum_i a_i \otimes f_{a,i}$.

Conversely if A is a right H^*-comodule algebra via $\delta : A \to A \otimes H^*$, $a \mapsto \sum a_0 \otimes a_1$, then A becomes a left H-module algebra by defining $h \cdot a = \sum \langle a_1, h \rangle a_0$ for all $a \in A, h \in H$.

Our first lemma is well known; we include it for completeness.

Lemma 2.1. *Let A be a left H-module algebra, and consider A as a right H-comodule algebra via $\delta : A \to A \otimes H^*$ as above. Then $\delta(h \cdot a) = (id \otimes (h \rightharpoonup\ ))(\delta(a))$, or equivalently $\sum (h \cdot a)_0 \otimes (h \cdot a)_1 = \sum a_0 \otimes \langle a_2, h \rangle a_1 = \sum a_0 \otimes (h \rightharpoonup a_1)$.*

Proof. $\delta(h \cdot a) = \sum \langle a_1, h \rangle \delta(a_0) = \sum \langle a_1, h \rangle a_{00} \otimes a_{01} = \sum \langle a_2, h \rangle a_0 \otimes a_1 = \sum a_0 \otimes \langle a_2, h \rangle a_1 = \sum a_0 \otimes (h \rightharpoonup a_1) = (id \otimes (h \rightharpoonup\ ))(\delta(a))$. □

Lemma 2.2. *Let W be a left A-module and consider the vector space $M = W \otimes_k H^*$. Consider A as a right H^*-comodule algebra as in the previous lemma. Then M becomes a left $A \# H$-module by defining, for all $a \in A$, $h \in H$, $w \in W$, and $f \in H^*$,*

$$(a \# h) \cdot (w \otimes f) := \sum a_0 w \otimes a_1 (h \rightharpoonup f) = \delta(a)(id \otimes (h \rightharpoonup\ ))(w \otimes f).$$

Proof. Also choose $b \in A$ and $g \in H$. Then

$$\begin{aligned}
(b \# g) \cdot \big[(a \# h) \cdot (w \otimes f)\big] &= \delta(b)(id \otimes (g \rightharpoonup\ ))\big(\delta(a)(id \otimes (h \rightharpoonup\ ))(w \otimes f)\big) \\
&= \delta(b)\big[(id \otimes (g_1 \rightharpoonup\ ))\delta(a)\big]\big[(id \otimes (g_2 h \rightharpoonup\ ))(w \otimes f)\big] \\
&= \delta(b)\delta(g_1 \cdot a)(id \otimes (g_2 h \rightharpoonup\ ))(w \otimes f) \\
&= (b(g_1 \cdot a) \# g_2 h) \cdot (w \otimes f) \\
&= \big((b \# g)(a \# h)\big) \cdot (w \otimes f).
\end{aligned}$$

Here we have used Lemma 2.1, replacing h with g_1, in the third equality. Thus $W \otimes H^*$ is a left $A \# H$-module. □

Lemma 2.3. *Let B be another k-algebra and let $\Phi : A\#H \to B$ be an algebra map. Then there is a well-defined action of H on $\Phi(A\#1)$, given by*

$$h \cdot \Phi(a\#1) := \Phi((h \cdot a)\#1),$$

for any $a \in A$, $h \in H$.

Proof. Let J be the kernel of Φ; it suffices to show that J is H-stable. Choose $a = a\#1 \in J$ and $h \in H$. Then $(h \cdot a)\#1 = (1\#h_1)(a\#1)(1\#S(h_2)) \in (1\#H)J(1\#H) \subseteq J$. Thus J is H-stable and the action is well defined. □

We now review some definitions and facts concerning radicals and Hopf actions. Throughout we let $J(A)$ denote the Jacobson radical of A, and $P(A)$ denote the prime radical of A. For any ideal I of A, $(I : H)$ denotes the largest H-stable ideal of A contained in I. It is always true that $H \cdot I$ is an H-stable ideal of A [L, Lemma 2.2].

As usual, we say that A is *H-semiprime* if A contains no non-zero H-stable nilpotent ideals. It is shown in [MS, Section 8] that A is H-semiprime if and only if $(P(A) : H) = 0$.

Following [LMS], we say that A is *H-semiprimitive* if (0) is an intersection of H-primitive ideals, where an ideal I of A is *H-primitive* if there exists a primitive ideal P of $A\#H$ such that $I = P \cap A$. It is proved in [LMS, Corollary 2.6] that A is H-semiprimitive if and only if $(J(A) : H) = 0$.

Recall that the standard left action of H^* on H extends to the smash product $A\#H$ as follows: for $f \in H^*$, $h \in H$, and $x \in A$,

$$f \cdot (x\#h) := x\#(f \rightharpoonup h) = x\# \sum f(h_2)h_1. \tag{2.4}$$

It is then easy to see that every H^*-stable ideal of $A\#H$ is extended from an H-stable ideal of A. Using this fact one can show:

Lemma 2.5. (1) [MS, 8.9-8.14] *If A is H-semiprime, then $A\#H$ is H^*-semiprime.*
(2) [LMS, Lemma 2.7] *If A is H-semiprimitive, then $A\#H$ is H^*-semiprimitive.*

We next give a precise statement of the duality between the two conjectures, mentioned in the introduction. It is essentially in [MS, 8.9-8.14], although not stated exactly in this form. The present statement is from [LMS].

Lemma 2.6 ([LMS, Lemma 2.9])**.** *Let H be a finite-dimensional Hopf algebra. Then the following are equivalent:*
(1) *for every H-module algebra A, $P(A)$ is H-stable;*
(1)′ *for every H-semiprime H-module algebra A, A is semiprime;*
(2) *for every H^*-semiprime H^*-module algebra A, $A\#H^*$ is semiprime.*

Note that the proof that (1)′ implies (1) is elementary ring theory: pass to the quotient $A/(P(A) : H)$, which is necessarily H-semiprime by the fact from [MS] mentioned above.

The proof of Lemma 2.6 holds for any class of algebras which is closed under taking quotients and under finite extensions. Thus (in particular) it applies to PI algebras and to Noetherian algebras. A fairly complete proof is included here as the second proof of Theorem 3.5.

Next we give the precise statement of Linchenko's main theorem.

Theorem 2.7 ([L, Theorem 3.1])**.** *Let H be a Hopf algebra such that $S^2 = id$. Let A be a finite-dimensional H-module algebra and assume that the characteristic of k is either 0 or greater than* $\dim A$*. Then $J(A)$ is H-stable.*

The theorem actually says that the prime radical $P(A)$ is H-stable since in a finite-dimensional algebra, $P(A) = J(A)$. We also use the following special case of [SvO, Theorem 8.3]:

Theorem 2.8 ([SvO, Theorem 8.3])**.** *Let H be a finite-dimensional Hopf algebra, and let A be a Noetherian H-module algebra. Then*
(1) *if H is semisimple and A is H-semiprime, then $A\#H$ is semiprime;*
(2) *if H is cosemisimple and A is H-semiprime, then A is semiprime.*

We summarize what is known when a Hopf algebra is involutive (that is, $S^2 = id$). If H is involutive and the characteristic of k is either 0 or $p > 0$ and p does not divide $\dim H$, then H is semisimple and cosemisimple [La]. In characteristic 0, H is semisimple $\iff$ H is cosemisimple, and in either case H is involutive [LaR], [LaR2].

In characteristic p, if H is both semisimple and cosemisimple, then $S^2 = id$ [EG]. However if H is only semisimple or (cosemisimple), then it is open as to whether $S^2 = id$; the best result so far is that it is true if $p > n^{\phi(n)}$ [EG].

3. The prime radical and smash products

We recall some basic facts about PI algebras. First, every nil PI algebra is locally nilpotent (a theorem of Amitsur), and second, every nil ideal of a PI algebra A is contained in $P(A)$ (this follows from a theorem of Levitzki). For a reference, see [J, p. 232].

Proposition 3.1. *Let H be a finite-dimensional Hopf algebra over k. Let A be an H-module PI algebra and let I be a nil ideal of A. Then $H \cdot I \subset J(A)$ if either:*
(1) *H is involutive and the characteristic of k is zero, or*
(2) *H is cosemisimple and the characteristic of k is $p > 0$.*

Proof. Let A be as in the statement of the theorem. Let V be an irreducible left A-module, and let $\rho_V : A \longrightarrow End_k(V)$ be the corresponding representation map. If the polynomial identity satisfied by A has degree d, then by Kaplansky's theorem [H, p. 157], $\rho_V(A)$ is a simple algebra with center C, with $\dim_C \rho_V(A) \leq [d/2]^2$.

Let $G = H \otimes_k C$ and let $B = A \otimes_k C$. Now G is a Hopf algebra over the field C, and B becomes a G-module algebra over C in the natural way via $(h \otimes c) \cdot (a \otimes d) = h \cdot a \otimes cd$.

V becomes a left B-module under the action $(a \otimes c) \cdot v = (\rho_V(a)c) \cdot v$; moreover V is irreducible as a B-module. Next let $W = V \otimes_C G^*$; by Lemma 2.2 W is a $B\#_C G$-module, and $\dim_C W = (\dim_C V)(\dim_C G^*) \leq [d/2]^2 \dim_k H$.

Let $\rho_W : B\#G \longrightarrow End_C(W)$ be the representation map of the $B\#G$-module W. Thus $\rho_W(B\#1)$ is a G-module algebra, on which G acts via $g \cdot \rho_W(b\#1) = \rho_W((g \cdot b)\#1)$; this action is well defined by Lemma 2.3. Moreover $\dim_C \rho_W(B\#1) \leq m := [d/2]^4 (\dim H)^2$.

Now let I be the given nil ideal of A. Since I is nil, it is locally nilpotent by the theorem of Amitsur mentioned above; it follows that $I \otimes_k C$ is also a locally nilpotent ideal of B. Thus $\rho_W(I \otimes_k C)$ is a nil ideal of $\rho_W(B)$, a finite-dimensional C-algebra, and so is in $J(\rho_W(B)) = P(\rho_W(B))$.

In case (1), $G \cdot \rho_W(I \otimes_k C) \subset J(\rho_W(B)) = P(\rho_W(B))$ by Theorem 2.7 [L], since G is involutive and the characteristic of k is zero.

In case (2), note $\rho_W(B)$ is Noetherian since it is finite-dimensional over C. Moreover G is cosemisimple since H is cosemisimple and finite-dimensional. Thus $G \cdot \rho_W(I \otimes_k C) \subset P(\rho_W(B))$ by Theorem 2.8 [SvO].

Now let $U = V \otimes C\lambda \subset W$, where λ is a non-zero left integral in G^*. Then U is a $B\#1$-submodule of W, since (using Lemma 2.2)

$$\begin{aligned}(b\#1)(v \otimes \lambda) &= \sum b_0 v \otimes b_1(1 \rightharpoonup \lambda)\\ &= \sum b_0 v \otimes b_1\lambda = \sum b_0\varepsilon(b_1)v \otimes \lambda = bv \otimes \lambda.\end{aligned}$$

Moreover $U \cong V$ is an irreducible $\rho_W(B)$-module, since V is an irreducible B-module. Finally

$$G \cdot (\rho_W(I \otimes_k C))(V \otimes_C \lambda) = \rho_W(G \cdot (I \otimes_k C))(V \otimes_C \lambda) \subset J(\rho_W(B))(V \otimes_C \lambda) = 0.$$

Hence $(H \cdot I)V = 0$ for any irreducible A-module V, and thus $H \cdot I \subset J(A)$. □

Corollary 3.2. *Let H be a finite dimensional cosemisimple Hopf algebra over k. Let A be an H-module algebra satisfying a polynomial identity and let I be a nil ideal of A. Then $H \cdot I \subset J(A)$.*

Proof. This follows from the proposition, noting that in characteristic 0, cosemisimple implies involutive by the Larson-Radford results. □

Remark 3.3. The reader may ask why we did not just assume that H was cosemisimple to begin with in Proposition 3.1, and give a unified proof using [SvO], regardless of characteristic. The reason is that by assuming H was involutive in characteristic 0, we could use Linchenko's result Theorem 2.7 instead. Linchenko's result is very short and elementary, whereas the theorem of [SvO] is much longer and fairly difficult.

In fact our original proof of Proposition 3.1 (2) also used Linchenko's theorem in characteristic $p > 0$. However, to do this we replaced the cosemisimplicity in (2) by the assumption that H was involutive and that $p > m = [d/2]^4(\dim H)^2$. Then, it followed that $p > \dim_C \rho_W(B)$, and thus by Theorem 2.7 that $G \cdot \rho_W(I \otimes_k C) \subset J(\rho_W(B))$. The rest of the proof proceeds as above.

It would be nice to have a more elementary proof in characteristic p also.

Theorem 3.4. *Let H be a semisimple Hopf algebra over k, and let A be an H-semiprime H-module algebra satisfying a polynomial identity. Then $A\#H$ is semiprime.*

Proof. By the theorem of Levitzki mentioned at the beginning of the section, if I is a nil ideal of the PI-ring A, then I is contained in the prime radical of A. Thus $(I : H) = 0$ since A is H-semiprime. As in [LMS], the H-action extends to the polynomial ring $A[t]$ by letting H act trivially on t. By a theorem of Amitsur (see [H, p. 153]), $J(A[t]) = N[t]$ for some nil ideal N of A. But then $(J(A[t]) : H) = (N : H)[t] = 0$ by the above remark. Thus $A[t]$ is H-semiprimitive.

Now consider $B = A[t]\#H$. H^* acts on B as in equation (2.4). Thus since $A[t]$ is H-semiprimitive, $A[t]\#H$ is H^*-semiprimitive by Lemma 2.5(2). Moreover H^* is cosemisimple since H is semisimple. Thus if I is a nilpotent ideal of B, then by Corollary 3.2 we have $H^* \cdot I \subset J(A[t]\#H)$. This is a contradiction unless

$I = 0$. Thus $B = A[t]\#H = (A\#H)[t]$ is semiprime. It follows that $A\#H$ is semiprime. □

Theorem 3.5. *Let H be a cosemisimple Hopf algebra over k. Let A be an H-module algebra satisfying a PI. Then the prime radical $P(A)$ is H-stable.*

Proof. The theorem follows immediately from Theorem 3.4 and Lemma 2.6 for the class of PI-algebras. Alternatively, we may give a direct proof, which is essentially repeating the proof of Lemma 2.6 from [LMS].

First assume that A is H-semiprime. We claim that A is semiprime. First, by Lemma 2.5(1) we have that $B = A\#H$ is H^*-semiprime. Next, H^* acts on $B = A\#H$ as in (2.4). Also H cosemisimple implies that H^* is semisimple. We may therefore apply Theorem 3.4 to conclude that $B\#H^*$ is semiprime.

Finally the Duality Theorem [M, 9.4.14] says that $(A\#H)\#H^* \cong M_n(A)$. Thus $M_n(A)$ is semiprime. It follows that A is also semiprime.

Now let $I = (P(A) : H)$; then A/I is H-semiprime by the remarks in Section 2. By the above argument, A/I is semiprime. Thus $(P(A) : H) = P(A)$ and so $P(A)$ is H-stable. □

Note that Theorem 3.5 improves Proposition 3.1. Now let A^H denote the algebra of invariants of A under the H-action.

Corollary 3.6. *Assume that H is semisimple and let A be an H-module PI algebra. Then*

(1) $P(A\#H) = (P(A) : H)\#H$;

(2) $P(A^H) = P(A) \cap A^H$;

(3) *if A is H-semiprime and I is any non-zero left or right H-stable ideal of A, then* $I \cap A^H \neq 0$.

Proof. (1) Consider H^* acting on $A\#H$. H^* is cosemisimple, and so by Theorem 3.5, $P(A\#H)$ is H^*-stable. Thus it is extended from A; that is, $P(A\#H) = I\#H$ for some H-stable ideal I of A. Since $A\#H/P(A\#H) \cong (A/I)\#H$ is semiprime, it follows that A/I is H-semiprime. Thus $I \subseteq (P(A) : H)$ and so $P(A\#H) = I\#H \subseteq (P(A) : H)\#H$.

Conversely, $A\#H/(P(A) : H)\#H \cong (A/(P(A) : H))\#H$ is semiprimitive by Lemma 2.5, and thus $P(A\#H) \subseteq (P(A) : H)\#H$.

(2) Since H is semisimple, it has an integral $e = e^2 \neq 0$. Then it is known that $e(A\#H)e = A^H e \cong A^H$ [M, 4.3.4]. Moreover for any ring A and idempotent $e \in A$, $P(eAe) = eP(A)e$ [J], [P, p. 171]. Thus $P(A^H) \cong P(A^H e) = P(e(A\#H)e) = eP(A\#H)e = e((P(A) : H)\#H)e = (P(A) : H)^H e = (P(A) \cap A^H)e$, where (1) was used for the third to last equality. Thus (2) holds.

(3) By Theorem 3.4, in this case $A\#H$ is semiprime. The result then follows from [M, Lemma 4.4.6]. □

Note that if H is also cosemisimple, then Corollary 3.6(1) can be improved to say $P(A\#H) = P(A)\#H$, since in that case, $(P(A) : H) = P(A)$ by Corollary 3.5.

One of the questions studied in [LMS] was when the Jacobson radical $J(A)$ of a PI-algebra is stable under the action of a Hopf algebra. Here we give a new proof for the case of affine PI-algebras of characteristic 0 [LMS, Theorem 3.8], as well as improve our characteristic p result.

Corollary 3.7. *Let H be a cosemisimple Hopf algebra over a field k, and let A be an H-module PI algebra which is either k-affine or algebraic over k. Then the Jacobson radical $J(A)$ is H-stable.*

Proof. When A is PI and either affine or algebraic, then $J(A) = P(A)$, since in those two cases, $J(A)$ is locally nilpotent by work of Amitsur and Kaplansky. We are now done by Theorem 3.5. □

We note that the general question, as to whether $J(A)$ is stable under the action of H when H is cosemisimple, remains open for an arbitrary PI-algebra A. In the case of arbitrary algebras, stability of $J(A)$ for all actions of H implies stability of $P(A)$. Moreover, there exists an action of an infinite-dimensional Hopf algebra on a commutative algebra A such that $P(A)$ is stable but $J(A)$ is not [L]. Thus the stability of $J(A)$ seems to be a more subtle problem.

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