LOW FOR RANDOM REALS
AND POSITIVE-MEASURE DOMINATION

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(Communicated by Julia Knight)

Abstract. The low for random reals are characterized topologically, as well as in terms of domination of Turing functionals on a set of positive measure.

1. Introduction

A function \( f : \omega \to \omega \) is \textit{uniformly almost everywhere (a.e.) dominating} if for measure-one many \( X \), and all \( g \) computable from \( X \), \( f \) dominates \( g \). Such functions were first studied by Kurtz \cite{Kurtz} who showed that uniformly a.e. dominating functions exist and that in fact \( 0' \), the Turing degree of the halting problem, computes one of them. If we replace measure by category, there are no such functions, as is not hard to see. A few decades later Dobrinen and Simpson \cite{DobrinenSimpson} made use of a.e. domination in Reverse Mathematics. They made a couple of fundamental conjectures that were promptly refuted in \cite{Hirschfeldt} and \cite{HirschfeldtSimpson}. In this article we strengthen the results of \cite{Hirschfeldt} to provide a characterization of a related concept, positive-measure domination, in terms of lowness for randomness. Conversely, we characterize low for random reals in terms of such domination. The following characterizations are already known.

(We assume the reader is familiar with the definition of Martin-Löf random reals and of prefix-free Kolmogorov complexity \( K \).)

Theorem 1.1 (Nies, Hirschfeldt, Stephan, Terwijn \cite{NiesHirschfeldtStephanTerwijn}, \cite{StephanTerwijn}, \cite{Terwijn}). The following are equivalent for \( A \in 2^\omega \):

- \( A \) is low for random: each Martin-Löf random real is Martin-Löf random relative to \( A \).
- \( A \) is \( K \)-trivial: \( \exists \forall n \ K(A \upharpoonright n) \leq K(\emptyset \upharpoonright n) + c \).
- \( A \) is low for \( K \): \( \exists \forall n \ K(n) \leq K^A(n) + c \).
- \( \exists Z \geq_T A \), \( Z \) is ML-random relative to \( A \).
- \( A \leq_T 0' \) and \( \Omega \) is ML-random relative to \( A \).

The low for random reals induce a \( \Sigma^0_3 \) nonprincipal ideal in the Turing degrees bounded above by a \( \Delta^0_2 \) degree \( 10 \), and have already found application to long-standing open problems in computability theory. Our characterizations in...
this paper are distinguished by not being couched in the language of randomness
and Kolmogorov complexity. They do however refer to measure; it remains open
whether a characterization purely in terms of domination or traces can be given
such as that found for low for Schnorr random reals [1], [13].

The first main result of Section 2 is Theorem 2.10 which is a characterization
of the low for random reals in terms of effectively closed sets of
positive measure. Building on this result, Theorem 2.12 is a characterization of low
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To obtain our topological characterization, we will pass first from a certain uni-
versal Martin-Löf test (given in terms of $K$) to an arbitrary Martin-Löf test, and
then to an arbitrary open set of measure < 1.

**Theorem 2.1** (Kraft-Chaitin Theorem [3]). Suppose $\langle n_k, \sigma_k \rangle$, $k \in \omega$, is a recursive
sequence, with $\sum_k 2^{-n_k} \leq 1$. Then there exists a prefix-free machine $M$ and a
collection of strings $\tau_k$ with $|\tau_k| = n_k$ and $M(\tau_k) = \sigma_k$.

**Definition 2.2** (Chaitin). Let $A \in 2^\omega$. An information content measure relative
to $A$ is a partial function $\hat{K} : 2^{<\omega} \to \omega$ such that
$$\sum_{\sigma \in 2^{<\omega}} 2^{-\hat{K}(\sigma)} \leq 1$$
and $\{ (\sigma, k) : \hat{K}(\sigma) \leq k \}$ is r.e. in $A$.

**Proof.** Let $\sigma_k$ be the $k$th string to enter $V_n^A$, and let $n_k = |\sigma_k| - n$. Note that
$\sum_k 2^{-n_k} = 2^n \sum_k 2^{-|\sigma_k|} \leq 1$, so we can let $M$ be as in Theorem 2.1. In other
words, if $\sigma$ enters $V_n^A$, then produce an $M$-description of $\sigma$ of length $\leq |\sigma| - n$. Let
$\hat{K}(\sigma)$ be the length of the shortest description of $\sigma$ so produced. \(\Box\)

**Lemma 2.3** (Chaitin). If $\hat{K}$ is an information content measure relative to a real
$A$, then for all $n$, $K^A(n) \leq \hat{K}(n) + O(1)$.

**Definition 2.4.** For any real $X$, let $S^A = \{ S^A_n \}_{n \in \omega}$, where $S^A_n = \{ X : \exists m
K^A(X \upharpoonright m) \leq m - n \}$.

**Lemma 2.5.** If $V_p^A$ is a Martin-Löf test relative to $A$, then for each $n$ there exists
$p$ such that $V_p^A \subseteq S^A_n$.

**Proof.** For each $m$, write $V^A_{2m} = \bigcup \{ [\sigma_{m,k}] : k \in \omega \}$ where the function $f$ given by
$f(m, k) = \sigma_{m,k}$ is computable and the sets $\{ \sigma_{m,k} : k \geq 1 \}$ are prefix-free. Define
numbers $n_{m,k} = |\sigma_{m,k}| - m + 1$ for $m, k \in \omega$. We have
$$\sum_{m,k} 2^{-|n_{m,k}|} = \sum_{m} 2^{m-1} \sum_k 2^{-|\sigma_{m,k}|} = \sum_{m} 2^{m-1} \mu V_{2m} \leq \sum_{m} 2^{m-1} 2^{-2m} = 1.$$ 
Hence by Theorem 2.1 we have a partial $A$-recursive prefix-free machine $M$ and
strings $\tau_{m,k}$ with $|\tau_{m,k}| = n_{m,k}$ and $M(\tau_{m,k}) = \sigma_{m,k}$. Thus $K_M$, complexity based
on the machine $M$, satisfies $K_M(\sigma_{m,k}) \leq n_{m,k}$, and so by Lemma 2.3 there is a
constant $c$ such that $K^A(\sigma_{m,k}) \leq n_{m,k} + c = |\sigma_{m,k}| - m + 1 + c$. This means that $V^A_{2m} \subseteq S^A_{m-c-1}$ for each $m$.

Thus, given $n$, let $m = n + c + 1$ and $p = 2m$. Then $V^A_p = V^A_{2m} \subseteq S^A_{m-c-1} = S^A_n$, as desired. \hfill \Box

Schnorr \cite{12} showed that $S^A$ is a universal Martin-Löf test relative to $A$.

**Proof.** In Lemma 2.3 we can conclude that $\bigcap_n V^A_n \subseteq \bigcap_n S^A_n$. Moreover $\mu S^A_n = \sum\{2^{-|\sigma|} : K(\sigma) \leq |\sigma| - n\} < 2^{-n}$. \hfill \Box

**Definition 2.6.** Let $n \geq 1$. Let $\Sigma^n_k$ denote the collection of all $\Sigma^n_k$ classes of measure $< 1$. The complement of a $\Sigma^n_k$ class is a $\Pi^n_k$ class. The complement of $U$ is denoted $\overline{U}$. The clopen subset of $2^\omega$ generated by $\sigma \in 2^\omega$ is denoted $[\sigma]$, and concatenation of strings is denoted by juxtaposition.

If $U$, $V$ are open subsets of $2^\omega$ given by $U = \bigcup\{[\sigma] : \sigma \in \hat{U}\}$ and $V = \bigcup\{[\sigma] : \sigma \in \hat{V}\}$, where $U$ and $V$ are prefix-free sets of strings, then we define

$$UV = \bigcup\{[\sigma\tau] : \sigma \in \hat{U}, \tau \in \hat{V}\}.$$ 

This product depends on $\hat{U}$ and $\hat{V}$, not just on $U$ and $V$. So when considering a $\Sigma^n_k$ class $U$, we implicitly fix a suitable recursively enumerable set $\hat{U}$ for $U$.

We define $U^n = U^{n-1}$ where $U^1 = U$. We can also think of this exponentiation as acting on a closed set $Q$, defining $Q^n$ via the equation $\overline{Q^n} = \overline{Q^n}$. It will be clear whether we are considering a set as open or closed.

**Lemma 2.7** (Kučera \cite{7}). For each $\Pi^n_1(A)$ class $Q$ there is a computable function $f$ such that $\{\overline{Q^n}\}_{n \in \omega}$, is a Martin-Löf test relative to $A$.

**Proof.** Let $q > 0$ be a rational number such that $\mu Q \geq q$. Let $P = \overline{Q}$. Then $\mu P^n = (\mu P)^n \leq (1 - q)^n$. Let $f$ be a computable function such that for all $k \in \omega$, $\mu P^f(k) \leq 2^{-k}$. Let $V^A_k = P^{f(k)}$. Then $V^A_k$ is a Martin-Löf test relative to $A$. \hfill \Box

**Lemma 2.8.** If $P$ is an open set such that $P^n$ is contained in a $\Sigma^n_1$ class for some $n \geq 2$, then $P$ itself is contained in a $\Sigma^n_1$ class.

**Proof.** We write $U[\sigma] = \bigcup\{[\tau] : [\sigma\tau] \subseteq U\}$. Note that if $P$ is open, then so is $P^2$. Hence by iteration, it suffices to consider the case $n = 2$. So suppose $(\exists U) P^2 \subseteq U \in \Sigma^n_1$. Case 1: $\exists \sigma$, $\mu(U[\sigma]) < 1$, $\sigma \in \hat{P}$. Then $P^2 \cap [\sigma] = [\sigma]P$, the product of $[\sigma]$ and $P$. Then $P = ([\sigma]P)[\sigma] = (P^2 \cap [\sigma])[\sigma] = \overline{P^2}[\sigma] \subseteq \overline{U} \subseteq \sigma \in \Sigma^n_1$. Case 2: Otherwise; so $\hat{P} \subseteq \{\sigma : \mu(U[\sigma]) = 1\}$. Fix $\epsilon > 0$ such that $\mu U < 1 - \epsilon$, and let $V = \bigcup\{[\sigma] : \mu(U[\sigma]) \geq 1 - \epsilon\}$. Note that $V$ is $\Sigma^n_1$, contains $P$, and $\mu V < 1$ because $(1 - \epsilon)\mu \leq \mu U < 1 - \epsilon$. \hfill \Box

As usual, an $A$-random is a real that is Martin-Löf random relative to $A$. If $A, B \in 2^\omega$, then $A$ is a tail of $B$ if there exists $n$ such that $A(k) = B(n + k)$ for all $k \in \omega$.

**Lemma 2.9** (Kučera \cite{7}). For each $A \in 2^\omega$, each $\Pi^n_1(A)$ class contains a tail of each $A$-random real.

**Proof.** Let $Q$ be a $\Pi^n_1(A)$ class and suppose $X$ is $A$-random. Then by Lemma 2.7, there is an $m$ such that $X \in Q^m$. If $m = 2$, then clearly, as $Q$ is closed, some tail of $X$ is an element of $Q$. If $m > 2$, the result follows by iteration since each $Q^m$ is closed. \hfill \Box
Theorem 2.10. Let \( A \in 2^\omega \). The following are equivalent:

1. Each 1-random real is \( A \)-random (\( A \) is low for random [10]).
2. For each \( \Pi_1^\mu \) class \( Q \) consisting entirely of 1-random reals, there exist \( \sigma, n \) such that \( Q \cap [\sigma] \neq \emptyset \) but \( Q \cap S_n^A \cap [\sigma] = \emptyset \).
3. For some \( n, S_n^A \) has a \( \Pi_1^\mu \) subclass.
4. For each \( A \)-Martin-Löf test \( V_n^A \), there exists an \( n \) such that \( V_n^A \) has a \( \Pi_1^\mu \) subclass.
5. For each \( \Pi_1^\mu (A) \) class \( Q \) there exists an \( n \) such that \( Q^n \) has a \( \Pi_1^\mu \) subclass.
6. Each \( \Pi_1^\mu (A) \) class has a \( \Pi_1^\mu \) subclass.
7. Some \( \Pi_1^\mu (A) \) class consisting entirely of \( A \)-random reals has a \( \Pi_1^\mu \) subclass.
8. The class of \( A \)-random reals has a \( \Pi_1^\mu \) subclass.

Proof. (1)\( \Rightarrow \) (2): For this implication we use an argument of Nies and Stephan [9]. Suppose \( A \) is low for random but (2) fails. So there is a \( \Pi_1^\mu \) class \( Q \) consisting entirely of 1-random reals, such that for all \( \sigma, n \), if \( Q \cap [\sigma] \neq \emptyset \), then \( Q \cap S_n^A \cap [\sigma] \neq \emptyset \). Let \( \sigma_0 = \lambda \), and \( \sigma_{n+1} \supseteq \sigma_n \), with \( [\sigma_{n+1}] \subseteq S_n^A \) but \( [\sigma_n] \cap Q \neq \emptyset \). Then \( Y = \bigcup_{n \in \omega} \sigma_n \) is not \( A \)-random, but is \( 1 \)-random, since \( Y \subseteq Q \).

(2)\( \Rightarrow \) (3): Let \( Q \) be as in (2), and let \( n, \sigma \) be as guaranteed by (2) for \( Q \). Then \( Q \cap [\sigma] \) is the desired subclass. It has positive measure because no 1-random belongs to a \( \Pi_1^\mu \) class of measure zero.

(3)\( \Rightarrow \) (4): Lemma 2.9.

(4)\( \Rightarrow \) (5): Let \( Q \) be a \( \Pi_1^\mu \) class. By Lemma 2.7, \( V_f^A = Q^f(k) \) is a Martin-Löf test relative to \( A \) for some computable \( f \). By (4), \( Q^f(m) = V^A_m \supseteq F \) for some \( F \in \Pi_1^\mu \) and \( m \); let \( n = f(m) \).

(5)\( \Rightarrow \) (6): Lemma 2.9.

(6)\( \Rightarrow \) (7): If \( U^A \) is a universal Martin-Löf test for \( A \)-randomness, then we can let \( Q = \{ \}^1 \).

(7)\( \Rightarrow \) (8): Since any class consisting entirely of \( A \)-randoms is contained in the class of all \( A \)-randoms.

(8)\( \Rightarrow \) (1): Suppose \( X \) is 1-random; we need to show \( X \) is \( A \)-random. Let \( F \) be a \( \Pi_1^\mu \) subclass of the class of \( A \)-randoms. By Lemma 2.9, some tail of \( X \) is an element of \( F \). Hence a tail of \( X \) is \( A \)-random, and thus \( X \) itself is \( A \)-random.

To characterize the low for random reals in terms of domination we first introduce some notation. We write

\[ \text{Tot}(\Phi) = \{ X : \Phi^X \text{ is total} \} \quad \text{and} \quad \varphi^X(n) = (\forall m < n)(\Phi^X_s(m) \leq s). \]

Note that \( \text{Tot}(\Phi) \) is a \( \Pi_2^\mu \) class for each \( \Phi \), and \( \text{Tot}(\Phi) = \text{Tot}(\varphi) \). The function \( \varphi \) is the running time of \( \Phi \), explicitly satisfying \( \varphi^X(n) \leq \varphi^X(n) \) for all \( n \). Let \( \Phi \) be a Turing functional and \( B \in 2^\omega \). If there exists \( f \leq_T B \) such that for positive-measure many \( X \), \( \Phi^X \) is dominated by \( f \), then we write \( \Phi < B \). By \( \sigma \)-additivity this is equivalent to the statement that there exists \( f \leq_T B \) such that for positive-measure many \( X \), \( \Phi^X \) is majorized by \( f \). We also write \( \Phi < B \) in the case that \( \text{Tot}(\Phi) \) has measure zero.

Lemma 2.11 (implicit in [5]). Let \( B \in 2^\omega \) and let \( \Phi \) be a Turing functional. Then \( \varphi < B \) iff \( \text{Tot}(\Phi) \) has a \( \Pi_1^\mu (B) \) subclass.
Proof. First suppose \( \varphi < B \), as witnessed by \( f \). Then \( \{X : \forall n \Phi_{\langle n \rangle}^X(n) \downarrow \} \) is a \( \Pi_1^\mu(B) \) subclass of \( \text{Tot}(\Phi) \). Conversely, let \( F \) be a \( \Pi_1^\mu(B) \) subclass of \( \text{Tot}(\Phi) \). By compactness, \( \{\varphi^X(n) : X \in F\} \) is finite for each \( n \), and \( \{(n,m) : \forall X (X \in F \rightarrow \varphi^X(n) < m)\} \) is a \( \Sigma_1^0(B) \) class. Hence by \( \Sigma_1^0(B) \) uniformization there is a function \( f \leq_T B \) such that \( \forall n \forall X (X \in F \rightarrow \varphi^X(n) < f(n)) \); i.e., \( f \) witnesses that \( \varphi < B \).

\[ \square \]

**Theorem 2.12.** Let \( A \in 2^\omega \). The following are equivalent:

1. \( A \) is low for random.
2. Each \( \Pi_1^\mu(A) \) class has a \( \Pi_1^\mu \) subclass.
3. (i) \( A \leq_T 0' \), and (ii) for each \( \Phi \), if \( \text{Tot}(\Phi) \) has a \( \Pi_1^\mu(A) \) subclass, then \( \varphi < 0 \).
4. (i) \( A \leq_T 0' \), and (ii) for each \( \Phi \), if \( \varphi < A \), then \( \varphi < 0 \).

**Proof.** (1)\( \Rightarrow \) (2) was shown in Theorem 2.10.

(2)\( \Rightarrow \) (3): Nies [10] shows that if \( A \) is low for random, then \( A \leq_T 0' \). Suppose \( \text{Tot}(\Phi) \) has a \( \Pi_1^\mu(A) \) subclass \( Q \). By (2), \( \text{Tot}(\Phi) \) has a \( \Pi_1^\mu \) subclass \( F \). By Lemma 2.11, we are done.

(3)\( \Rightarrow \) (2): Suppose (3) holds and suppose \( Q \) is a \( \Pi_1^\mu(A) \) class. Pick \( \Psi \) such that \( Q = \{X : \Psi^{X \oplus A}(0) \downarrow\} \). Since \( A \leq_T 0' \), \( A = \lim_n A_n \), the limit of a computable approximation. Let \( \Phi^X(s) = \mu t > s(\Psi^{X \oplus A_t}(0) \downarrow) \). Then \( Q = \text{Tot}(\Phi) \). Applying (3) to this \( \Phi \), we have \( \varphi < 0 \), and so by Lemma 2.11 we are done.

(3)\( \Rightarrow \) (4) is immediate from Lemma 2.11.

\[ \square \]

### 3. Positive-measure domination

In [5] it was asked whether the Turing degrees \( A \) of uniformly a.e. dominating functions are characterized by either of the inequalities \( A \geq 0' \) and \( A' \geq_T 0'' \). The case \( A \geq 0' \) was refuted by a direct construction in [2]. The case \( A' \geq_T 0'' \) was refuted in [2] using precursors to the results presented here. Namely, the dual of property 4(ii) above is \( \forall \varphi(\varphi < A) \) or equivalently \( \forall \varphi(\varphi < 0' \rightarrow \varphi < A) \). Relativizing our proofs shows that this is equivalent to: \( 0' \) is low for random relative to \( A \). If we restrict ourselves to \( A \leq_T 0' \), then by [10] this implies \( A' \geq_{tt} 0'' \), which is strictly stronger than \( A' \geq_{tt} 0'' \). We do not know whether the assumption \( A \leq_T 0' \) is necessary for either of the conclusions \( A' \geq_{tt} 0'' \), \( A' \geq_{tt} 0'' \).

We say that \( A \) is positive-measure dominating if for each \( \Phi, \Phi < A \). If each \( B \)-random real is \( A \)-random, then we write \( A \leq_{LR} B \) (\( A \) is low for random relative to \( B \)) following [10]. We write \( \Phi^A \) for the functional \( X \mapsto \Phi^{A \oplus X} \).

**Lemma 3.1.** Let \( A \in 2^\omega \) and \( C \subseteq 2^\omega \). Then \( C \) is a \( \Pi_2^0(A) \) class iff \( C \) is \( \text{Tot}(\Phi^A) \) for some Turing functional \( \Phi \).

**Proof.** Suppose \( C \) is a \( \Pi_2^0(A) \) class, i.e., \( C = \{X : \forall y \exists s R(y, s, A, X)\} \), where \( R \) is a formula in the language of second-order arithmetic all of whose quantifiers are first-order and bounded. Then we can let \( \Phi^{A \oplus X}(y) = \mu s(R(y, s, A, X)) \). Conversely, \( \text{Tot}(\Phi^A) = \{X : \forall y \exists s(\Phi^{A \oplus X}(y) \downarrow)\} \).

**Theorem 3.2.** Let \( B \in 2^\omega \). Then \( 0' \) is low for random relative to \( B \) iff \( B \) is positive-measure dominating.
Proof. This is the special case \( A = 0 \) of the fact that for each \( A, B \in 2^\omega \), the following are equivalent:

1. \( A' \leq_{LR} A \oplus B \).
2. Each \( \Pi_1^0 (A') \) class has a \( \Pi_1^0 (A \oplus B) \) subclass.
3. Each \( \Pi_2^0 (A) \) class has a \( \Pi_1^0 (A \oplus B) \) subclass.
4. \( \forall \Phi \), if \( \text{Tot}(\Phi^A) \) has positive measure, then it has a \( \Pi_1^0 (A \oplus B) \) subclass.
5. \( \forall \Phi \), if \( \phi^A < A \oplus B \).

The equivalences are proved as follows. (1)\( \iff \) (2): Relativization of Theorem 2.10 gives: \( A \leq_{LR} B \) iff each \( \Pi_1^0 (A) \) class has a \( \Pi_1^0 (B) \) subclass. (3)\( \iff \) (4): Lemma 3.1 (4)\( \iff \) (5): Relativization of Lemma 2.11 (2)\( \iff \) (3): Let \( A \in 2^\omega \). \( A' \) is uniformly a.e. dominating relative to \( A \), hence \( A' \) is positive-measure dominating relative to \( A \). Hence by putting \( B = A' \) in (3)\( \iff \) (5), each \( \Pi_2^0 (A) \) class has a \( \Pi_1^0 (A') \) subclass.

Universal functionals. Suppose \( \Phi_i, i \in \omega \), are all the Turing functionals. As observed in [4], the functional \( \Psi \) given by \( \Psi \circ X = \Phi_i^X \) is universal for uniform a.e. domination, in the sense that any function that dominates \( \Psi \) on almost every \( X \) is a uniformly a.e. dominating function. As \( \Psi < 0 \), \( \Psi \) is not universal for positive-measure domination; however, the following functional is universal.

Fix \( c \in \omega \). Let \( \Xi^X_c (s) \) be the least stage \( t > s \) at which \( X \) looks like it is 2-random, with constant \( c \).

That is,

\[
\Xi^X_c (s) = (\mu t > s)(\forall n \leq t K^0_t (X \upharpoonright n) \geq n - c).
\]

Here \( 0'_i \) is the approximation to \( 0' \) at stage \( t \), and \( K^A_t \) the approximation to \( K^A \) for any \( A \in 2^\omega \). Considering \( S^X_c \) (Definition 2.4), it is clear that \( \Xi^X_c \) is total for positive-measure many \( X \), all of which are 2-randoms. The running time \( \xi_c \) of \( \Xi^X_c \) is universal for positive-measure domination in the following sense.

Theorem 3.3. The class \( \{ A : A \text{ is positive-measure dominating} \} \) is \( \Sigma^0_1 \). In fact, for each \( A \in 2^\omega \) and \( c \in \omega \), \( A \) is positive-measure dominating iff \( \xi_c < A \).

Proof. The complement of \( \text{Tot}(\Xi^X_c) \) is \( \{ X : \exists n K^0_n (X \upharpoonright n) < n - c \} \), which is open.

Hence \( \text{Tot}(\Xi^X_c) \) is closed and is in fact a \( \Pi_1^0 (0') \) class. Suppose \( \xi_c < A \). By Lemma 2.11 \( \text{Tot}(\Xi^X_c) \) has a \( \Pi_1^0 (A) \) subclass. Thus: Some \( \Pi_1^0 (0') \) class consisting entirely of \( 0' \)-randoms has a \( \Pi_1^0 (A) \) subclass. By Theorem 2.10 (7) relativized, \( 0' \leq_{LR} A \), and so by Theorem 5.2 \( A \) is positive-measure dominating.

References


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