ON THE LOCALIZATION PRINCIPLE
FOR THE AUTOMORPHISMS OF PSEUDOELLIPSOIDS

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ABSTRACT. We show that Alexander’s extendibility theorem for a local automorphism of the unit ball is valid also for a local automorphism \( f \) of a pseudoellipsoid \( E_{(p_1,\ldots,p_k)}^n = \{ z \in \mathbb{C}^n : \sum_{j=1}^{n-k} |z_j|^2 + |z_{n-k+1}|^{2p_1} + \cdots + |z_n|^{2p_k} < 1 \} \), provided that \( f \) is defined on a region \( U \subset E_{(p)}^n \) such that: i) \( \partial U \cap \partial E_{(p)}^n \) contains an open set of strongly pseudoconvex points; ii) \( U \cap \{ z_i = 0 \} \neq \emptyset \) for any \( n - k + 1 \leq i \leq n \). By the counterexamples we exhibit, such hypotheses can be considered as optimal.

1. Introduction

For a given \( k \)-tuple of integers \( p = (p_1,\ldots,p_k) \), with each \( p_i \geq 2 \), let us denote by \( E_{(p_1,\ldots,p_k)}^n \) (or, more simply, \( E_{(p)}^n \)) the pseudoellipsoid in \( \mathbb{C}^n \) defined by

\[
E_{(p_1,\ldots,p_k)}^n = \{ z \in \mathbb{C}^n : \sum_{j=1}^{n-k} |z_j|^2 + |z_{n-k+1}|^{2p_1} + \cdots + |z_n|^{2p_k} < 1 \}.
\]

When \( k = 0 \), we assume \( E_{(p)}^n \) to be the unit ball \( B^n = \{ z \in \mathbb{C}^n : |z| < 1 \} \). Now, let us consider the following definition.

Definition 1.1. We define a local automorphism of \( E_{(p)}^n \) to be any biholomorphic map \( f : U_1 \subset E_{(p)}^n \to U_2 \subset E_{(p)}^n \) between two connected open subsets of \( E_{(p)}^n \) such that:

a) each of the intersections \( \partial U_i \cap \partial E_{(p)}^n, i = 1,2 \), contains a boundary open set \( \Gamma_i \subset \partial E_{(p)}^n \);

b) there exists at least one sequence \( \{ x_k \} \subset U_1 \) which converges to a point \( x_\alpha \in \Gamma_1 \), which is not a limit point of \( \partial U_1 \cap E_{(p)}^n \), and so that \( \{ f(x_k) \} \) converges to a point \( \hat{x}_\alpha \in \Gamma_2 \), which is not a limit point of \( \partial U_2 \cap E_{(p)}^n \).

We say that a local automorphism \( f : U_1 \subset E_{(p)}^n \to U_2 \subset E_{(p)}^n \) extends to a global automorphism of \( E_{(p)}^n \) if there exists some \( F \in \text{Aut}(E_{(p)}^n) \) such that \( F|_{U_1 \cap E_{(p)}^n} = f|_{U_1 \cap E_{(p)}^n} \).

By a celebrated theorem of Alexander and its generalization obtained by Rudin ([Al, Ru]), when \( E_{(p)}^n = B^n \), any local automorphism extends to a global one.

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This crucial extendibility result is often referred to as the *localization principle* for the automorphisms of $B^n$, and it has been extended or established under different but similar hypotheses for a wide class of domains besides the unit balls (see e.g. [DS, PI, PI1]). On the other hand, even if it is known that the pseudoellipsoids $E_n^p$ share many useful properties with $B^n$ for what concerns the global automorphisms and the proper holomorphic maps (see for instance [We, La, LS, DS]), some simple examples show that Alexander’s theorem cannot be true in full generality for a pseudoellipsoid $E_n^p$ different from $B^n$ (see e.g. Example 3.4 below).

Nonetheless, for each $E_n^p$, it is possible to determine, precisely and in an efficient way, the class of local automorphisms that can be extended to global ones. In this short note we give a characterization of such local automorphisms by means of the following generalization of Alexander’s theorem.

**Theorem 1.2.** Let $f : U_1 \subset E_n^p \to U_2 \subset E_n^p$ be a local automorphism of a pseudoellipsoid $E_n^p$, with $p = (p_1, \ldots, p_k)$, and satisfying the following two conditions:

i) there exists a sequence $\{x_i\}$ as in (b) of Definition 1.1, whose limit point $x_0 \in \partial E_n^p$ is Levi non-degenerate;

ii) for any $n - k + 1 \leq i \leq n$, the intersection $U_1 \cap \{z_i = 0\}$ is not empty.

Then $f$ extends to a global automorphism $f \in \text{Aut}(E_n^p)$.

We point out that the set $\partial E_n^p \cap \bigcup_{i=n-k+1}^{n} \{z_i = 0\}$ coincides with the set of points of Levi degeneracy of $\partial E_n^p$. So, Theorem 1.2 can be roughly stated by saying that $f$ is globally extendible as soon as it admits a holomorphic extension to some open subset $U \subset E_n^p$, which intersects each of the hyperplanes containing the Levi degeneracy set of $\partial E_n^p$ and, at the same time, the boundary $\partial U$ contains an open set of strongly pseudoconvex points of $\partial E_n^p$.

From Example 3.4 it will be clear that such hypotheses can be considered as optimal.

The properties of the pseudoellipsoid used in the proof are basically just two: (1) It admits a finite ramified covering over the unit ball; (2) Its automorphisms are “lifts” of the automorphisms of the unit ball that preserve the singular values of the covering. Since (2) is a consequence of (1), it is reasonable to expect that a similar result should be true for any arbitrary ramified covering of the unit ball.

About this more general problem, we refer to [KLS, KS] for what concerns the classification of the domains in $\mathbb{C}^2$ that admit a ramified holomorphic covering over $B^2$.

2. **On the Automorphisms of the Unit Ball**

First of all, we need to recall some basic facts on the automorphisms of the unit ball. Let us denote by $i : \mathbb{C}^n \to \mathbb{CP}^n$ the canonical embedding

$$i : \mathbb{C}^n \to \mathbb{CP}^n, \quad i(z) = \begin{bmatrix} z_1 \\ \vdots \\ z_n \\ 1 \end{bmatrix}$$

and let $\mathbb{C}^n = i(\mathbb{C}^n) = \mathbb{CP}^n \setminus \{[w] : w_{n+1} = 0\}$. We recall that, via the embedding, $B^n$ corresponds to the projective open set $\hat{B}^n = \{[w] \in \mathbb{CP}^n : \langle w, w \rangle < 0 \}$,
where we denote by $\langle \cdot , \cdot \rangle$ the pseudo-Hermitian inner product on $\mathbb{C}^{n+1}$ defined by

$$
\langle w, z \rangle = \bar{w}^t \cdot I_{n,1} \cdot z, \quad \text{where } I_{n,1} = \begin{pmatrix} I_n & 0 \\ 0 & -1 \end{pmatrix}.
$$

It is also known that a holomorphic map $F : B^n \to B^n$ is an automorphism of $B^n$ if and only if the corresponding map $\hat{F} = \hat{i} \circ F \circ \hat{i}^{-1} : \hat{B}^n \to \hat{B}^n$ is a projective linear transformation which preserves the quadric $\partial \hat{B}^n = \{ \ [w] : \langle w, w \rangle = 0 \}$ (see e.g. [Ve]). This means that $\hat{F}$ is of the form

$$
\hat{F}(\bar{z}) = [\hat{A} \cdot z],
$$

where $\hat{A}$ is a matrix in $SU_{n,1}$, i.e. such that $\hat{A}^t I_{n,1} \hat{A} = I_{n,1}$ and with $\det \hat{A} = 1$.

The correspondence $F \mapsto \hat{F} = \hat{i} \circ F \circ \hat{i}^{-1}$ gives an isomorphism between $Aut(B_n)$ and $SU_{n,1}/K$, where $K = \{ e^{\frac{2\pi}{n+1}} I_{n+1}, \ 0 \leq k \leq n \}$.

The identification of the elements of $Aut(B^n)$ with the corresponding projective linear transformations is often quite useful, for instance in order to establish the following fact (see also [Ve], §6).

**Lemma 2.1.** Let $F = (F_1, \ldots, F_n) \in Aut(B^n)$ be an automorphism such that

$$
F(B^n \cap \{ z_i = 0 \}) \subset \{ z_i = 0 \}
$$

for all $n - k + 1 \leq i \leq n$. Then the components $F_i$ are of the following form:

$$
F_j(z) = \sum_{\ell=1}^{n-k} A_j^\ell z_\ell + b_j \sum_{\ell=1}^{n-k} c_\ell z_\ell + d, \quad \text{for } 1 \leq j \leq n-k,
$$

$$
F_\ell(z) = e^{i\theta_j} z_\ell \sum_{\ell=1}^{n-k} c_\ell z_\ell + d, \quad \text{for } n-k+1 \leq \ell \leq n,
$$

for some $\theta_j \in \mathbb{R}$ and where $A = (A_j^\ell)$, $b = (b_j)$, $c = (c_\ell)$ and $d$ are such that

$$
\begin{pmatrix} A & b \\ c & d \end{pmatrix} \in SU_{n-k,1}.
$$

In particular, the maps $F_j$, $1 \leq j \leq n-k$, coincide with the components of an element of $Aut(B^{n-k})$, while $\sum_{\ell=1}^{n-k} c_\ell z_\ell + d \neq 0$ for any $z \in B^n$.

**Proof.** By hypothesis, the corresponding automorphism $\hat{F} = \hat{i} \circ F \circ \hat{i}^{-1} \in Aut(\hat{B}^n)$ maps all hyperplanes $H_i = \{ \ [w] \in \mathbb{C}P^n : w_i = 0 \}$ into themselves and hence fixes their poles relative to the quadric $\partial \hat{B}^n$, i.e. fixes all the points

$$
[e_\ell] = [0 : \ldots : 0 : 1 : 0 : \ldots : 0], \quad n-k+1 \leq \ell \leq n.
$$

This implies that the matrix $\hat{A}$ which determines the projective transformation $\hat{F}$ is of the form

$$
\hat{A} = \begin{pmatrix} A & 0 & \ldots & 0 & b \\ 0 & e^{\theta_{n-k+1}} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & e^{\theta_{n}} \\ c & 0 & \ldots & 0 & d \end{pmatrix},
$$

where $A$, $b$, $c$ and $d$ are such that $A' \defeq \begin{pmatrix} A & b \\ c & d \end{pmatrix}$ belongs to $SU_{n-k,1}$. From this, (2.4) and (2.5) follow immediately. The last claim follows from the fact that the value $\sum_{\ell=1}^{n-k} c_\ell z_\ell + d$ is the last homogeneous coordinate of the element
\[ \{ z_1 : \ldots : z_{n-k} : 1 \} \in \mathbb{CP}^{n-k} \text{ and it is clearly different from 0, since the map } \]
\[ [w] \mapsto [\mathcal{A} \cdot w] \text{ is an automorphism of } \mathbb{B}^{n-k} \subset \mathbb{CP}^{n-k} \setminus \{ w_{n-k+1} \neq 0 \}. \]

3. Proof of Theorem 1.2

First of all, we need to introduce the following notation. For any \( p = (p_1, \ldots, p_k) \), we will use the symbol \( \pi^{(p)} \) to denote the map
\[ \pi^{(p)} : \mathbb{C}^n \to \mathbb{C}^n, \quad \pi^{(p)}(z) = (z_1, \ldots, z_{n-k}, z_{n-k+1}^{p_1}, \ldots, z_{n}^{p_k}) . \]
We recall that the restriction \( \pi^{(p)}|_{\mathcal{E}^{(p)}} \) gives a proper holomorphic map \( \pi^{(p)} : \mathcal{E}^{(p)} \to \mathbb{B}^n. \)

Secondly, we need to recall a useful theorem by Forstneric and Rosay (\cite{FR}).

Given a domain \( D \subset \mathbb{C}^n \), we say that a boundary point \( z_0 \in \partial D \) satisfies the condition (P) if:

- \( \partial D \) is of class \( C^{1+\varepsilon} \) near \( z_0 \) for some \( \varepsilon > 0; \)
- there exist a continuous negative plurisubharmonic function \( \rho \) on \( D \) and a neighborhood \( U \) of \( z_0 \) so that \( \rho(z) \geq -c d(z, \partial D) \) at all points of \( U \cap D \) for some constant \( c > 0 \).

Theorem 1.1 and some related remarks of \cite{FR} can be summarized as follows.

**Theorem 3.1.** Let \( h : D \to D' \) be a proper holomorphic map between two domains of \( \mathbb{C}^n \) and let \( z_0 \in \partial D \) be a point that satisfies the condition (P).

If there exists a sequence \( \{ z_j \} \subset D \) so that \( \lim_{j \to 0} z_j = z_0 \) and \( \lim_{j \to \infty} h(z_j) = z_o \) for some \( z_o \in \partial D' \) at which \( \partial D' \) is \( C^2 \) and strictly pseudoconvex, then \( h \) extends continuously to all points of a neighborhood \( V \) of \( z_0 \) in \( \overline{D} \).

We may now prove the following lemma.

**Lemma 3.2.** Let \( f : U_1 \subset \mathcal{E}^{(p)} \to U_2 \subset \mathcal{E}^{(p)} \) be a local automorphism of a pseudoellipsoid \( \mathcal{E}^{(p)} \) with \( p = (p_1, \ldots, p_k) \) and assume that

i) there exists a sequence \( \{ x_i \} \) as in (b) of Definition 1.1 whose limit point \( x_o \in \partial \mathcal{E}^{(p)} \) is Levi non-degenerate;

ii) for any \( n-k+1 \leq i \leq n \), the intersection \( U_1 \cap \{ z_i = 0 \} \) is not empty.

Then, up to composition with a coordinate permutation,
\[ (z_1, \ldots, z_n) \mapsto (z_{\sigma(1)}, \ldots, z_{\sigma(n)}), \]
the map \( f \) sends the points of the hyperplane \( \{ z_i = 0 \} \) into the same hyperplane for any \( n-k+1 \leq i \leq n \).

**Proof.** In all the following we will use the symbols \( \Gamma_i \), \( x_o \), and \( \hat{x}_o \) with the same meaning as in Definition 1.1.

First of all, notice that \( \hat{x}_o \in \Gamma_2 \subset \partial U_2 \) satisfies the condition (P) and hence, by Theorem 1.1 for any sufficiently small ball \( \mathcal{B}_\varepsilon(\hat{x}_o) \), centered at \( \hat{x}_o \) and of radius \( \varepsilon \), the holomorphic map \( f^{-1} : U_2 \to U_1 \) extends continuously to all points of \( \overline{\mathcal{B}_\varepsilon(\hat{x}_o)} \cap \Gamma_2 \). In particular, we may assume that \( f^{-1}(\overline{\mathcal{B}_\varepsilon(\hat{x}_o)} \cap \Gamma_2) \) is contained in a neighborhood of \( x_o = f^{-1}(\hat{x}_o) \) in \( \Gamma_1 \) in which there are no Levi degenerate points.

Pick a Levi non-degenerate point \( x_o' \in \overline{\mathcal{B}_\varepsilon(\hat{x}_o)} \cap \Gamma_2 \) and consider a sequence \( \{ x_i' \} \subset \overline{\mathcal{B}_\varepsilon(\hat{x}_o)} \cap U_2 \) which converges to \( \hat{x}_o' \). By construction, the sequence \( \{ x_i' = f^{-1}(x_i') \} \subset U_1 \) converges to the Levi non-degenerate point \( x_o' = f^{-1}(\hat{x}_o') \in \Gamma_1 \).

It follows that, replacing \( x_o \) by \( x_o' \) and \( \hat{x}_o \) by \( \hat{x}_o' \) and by Theorem 3.1 applied to
$f$ and $f^{-1}$, there is no loss of generality if we assume that $x_o$ and $\hat{x}_o$ are both Levi non-degenerate and that, for any sufficiently small $\varepsilon_1 > 0$, the map $f$ extends continuously to a map

$$f : U_1 \cup \left( \overline{B_{\varepsilon_1}(x_o) \cap \Gamma_1} \right) \to U_2 \cup (B_{\varepsilon}(\hat{x}_o) \cap \Gamma_2),$$

which is a homeomorphism onto its image.

Since the complex Jacobian matrices $J\pi(o)x_o$ and $J\pi(o)\hat{x}_o$ are of maximal rank (recall that $x_o$ and $\hat{x}_o \in \partial E^n$, are both Levi non-degenerate), from the fact that $x_o$ is not a limit point of $\partial U_1 \cap E^n$ and by the continuity of $f$ and $f^{-1}$ around $x_o$ and $\hat{x}_o$, respectively, we may choose $\varepsilon_1$ and $\varepsilon_2$ so that:

a) $\pi(o)|_{B_{\varepsilon_1}(x_o)}$ and $\pi(o)|_{B_{\varepsilon_2}(\hat{x}_o)}$ are both biholomorphisms onto their images;
b) $f(\overline{B_{\varepsilon_1}(x_o) \cap U_1}) \subset B_{\varepsilon_2}(\hat{x}_o)$ and $f|_{|B_{\varepsilon_1}(x_o) \cap U_1}$ extends to a homeomorphism between $\overline{B_{\varepsilon_1}(x_o) \cap U_1}$ and $f(B_{\varepsilon_1}(x_o) \cap U_1)$ which induces a homeomorphism between $B_{\varepsilon_2}(\hat{x}_o) \cap \Gamma_1$ and $f(B_{\varepsilon_2}(\hat{x}_o) \cap \Gamma_1) \subset \Gamma_2$.

Notice that, by definition, $x_o$ is not a limit point of $\partial (B_{\varepsilon_1}(x_o) \cap U_1) \cap E^n$ and, by (b), $\hat{x}_o$ is not a limit point of $\partial f (B_{\varepsilon_2}(x_o) \cap U_1) \cap E^n$. So, if we set

$$U_1 \overset{\text{def}}{=} B_{\varepsilon_1}(x_o) \cap U_1, \quad U_2 \overset{\text{def}}{=} f(U_1) \subset B_{\varepsilon_2}(\hat{x}_o), \quad V_i \overset{\text{def}}{=} \pi(o)(U_i), \quad i = 1, 2,$$

then the maps

$$f|_{U_1} : U_1 \to U_2$$

and

$$\tilde{f} = \pi(o) \circ f \circ \pi(o)^{-1} \mid_{V_1} : V_1 \subset B^n \to V_2 \subset B^n$$

are local automorphisms of $E^n$ and of the unit ball, respectively.

By Rudin’s generalization of Alexander’s theorem ([Ru]), this implies that $\tilde{f}$ extends to a global automorphism of $B^n$, which we denote by $\tilde{f}$ as well. By construction, for any $z \in U_1 = \pi(o)^{-1}(V_1)$, we have

$$\tilde{f} \circ \pi(o)(z) = \pi(o) \circ f(z),$$

but since both sides have a holomorphic extension on $U_1$, we get that (3.2) must be true also for any $z$ in such a larger set.

In particular,

$$J(\tilde{f})|_{\pi(o)(z)} \cdot J(\pi(o))|_z = J(\pi(o))|_{f(z)} \cdot J(f)|_z, \quad \text{for any } z \in U_1.$$

Since for any $z \in U_1$, $\det J(f)|_z \neq 0$ and

$$\{ J(\pi(o))|_z = 0 \} = \bigcup_{i=n-k+1}^n \{ z_i = 0 \},$$

equality (3.3) implies that, for any $n-k+1 \leq i \leq n$ and $z \in U_1 \cap \{ z_i = 0 \}$, the value of $J(\pi(o))|_{f(z)}$ is 0. By (3.4), this means that $f(U_1 \cap \{ z_i = 0 \})$ is contained in the union $\bigcup_{i=n-k+1}^n \{ z_i = 0 \}$. Indeed, it is contained in exactly one of the hyperplanes $\{ z_i = 0 \}$, because $f$ is a biholomorphism and consequently $f(U_1 \cap \{ z_i = 0 \})$ is an irreducible analytic variety. From this the conclusion follows.

We proceed by defining a rule that associates an automorphism of $B^n$ with any local automorphism of a pseudoellipsoid (see also [We], §6). Given a local automorphism $f : U \to \mathbb{C}^n$ of $E^n_{(p)}$, pick a point $x_o \in U \cap \partial E^n_{(p)}$ for which (b) of
Definition 1.1 holds and determine a small ball $B_\varepsilon(x_o)$ centered in $x_o$, as in the proof of the previous lemma. Then, we denote by $\tilde{f} \in \text{Aut}(B^n)$ the global automorphism of the unit ball that extends $f \subseteq \pi^{(p)} \circ f \circ \pi^{(p)} = 1_{\pi^{(p)}(U)}$ with $\nu \subseteq B_\varepsilon(x_o) \cap \mathcal{E}^{n}_{(p)}$. By the identity principle of the holomorphic maps, such an automorphism $\tilde{f}$ depends only on $f$ and will be called the (global) automorphism of $B^n$ associated with $f$.

With the help of such a correspondence, we may state the following criterion for extendibility of local automorphisms.

**Proposition 3.3.** A local automorphism $f: U_1 \subset \mathcal{E}^n_{(p)} \to U_2 \subset \mathcal{E}^n_{(p)}$ of a pseudoellipsoid $\mathcal{E}^n_{(p)}$, $p = (p_1, \ldots, p_k)$, extends to a global automorphism $\tilde{f} \in \text{Aut}(\mathcal{E}^n_{(p)})$ if and only if its associated automorphism $\tilde{f} \in \text{Aut}(B^n)$ satisfies (3.2) for any $n - k + 1 \leq i \leq n$, up to composition with a permutation of those coordinates $z_{n - k + j}$, for which the integers $p_j$ are of the same value.

**Proof.** Assume that the local automorphism $f: U \to \mathbb{C}^n$ extends to a global automorphism $\tilde{f} \in \text{Aut}(\mathcal{E}^n_{(p)})$ and recall that, by construction, the associated automorphism $\tilde{f} \in \text{Aut}(B^n)$ satisfies (3.2) at all points where $f$ is defined (in this case, at all points of $\mathcal{E}^n_{(p)}$). Then, by Lemma 3.2 and the fact that $\pi^{(p)}(\mathcal{E}^n_{(p)} \cap \{ z_i = 0 \}) = B^n \cap \{ z_i = 0 \}$, the equality (3.3) implies that, up to a suitable permutation of coordinates, $\tilde{f}$ satisfies (2.3) for any $n - k + 1 \leq i \leq n$.

Conversely, assume that $f = (f_1, \ldots, f_n): U_1 \subset \mathcal{E}^n_{(p)} \to U_2 \subset \mathcal{E}^n_{(p)}$ is a local automorphism of $\mathcal{E}^n_{(p)}$ such that (up to a suitable permutation of coordinates) the associated automorphism $\tilde{f} = (\tilde{f}_1, \ldots, \tilde{f}_n) \in \text{Aut}(B^n)$ satisfies (2.3) for any $n - k + 1 \leq i \leq n$. From (2.4), (2.5) and (3.2), it follows that the components $f_j$ of $f$ are of the form

$$f_j(z) = \frac{\sum_{\ell=1}^{n-k} A_{j\ell} z_\ell + b_j}{\sum_{\ell=1}^{n-k} c_{j\ell} z_\ell + d} , \quad \text{for } 1 \leq j \leq n - k,$$

and

$$f_{n-k+j}(z) = e^{i\theta_j} \frac{z_j}{\left(\sum_{\ell=1}^{n-k} c_{j\ell} z_\ell + d\right)^{1/p_j}} , \quad \text{for } 1 \leq j \leq k,$$

for some fixed definitions of the $p_j$-th roots $w \mapsto w^{1/p_j}$.

From (3.5) and (3.6) it follows immediately that $f$ coincides with a globally defined automorphism of $\mathcal{E}^n_{(p)}$ (for the general expressions of the elements in $\text{Aut}(\mathcal{E}^n_{(p)})$, see [We, La]).

Now, Theorem 1.2 follows almost immediately. In fact, if $f: U_1 \subset \mathcal{E}^n_{(p)} \to U_2 \subset \mathcal{E}^n_{(p)}$ is a local automorphism satisfying the hypotheses of the theorem, by Lemma 3.2 and (3.2), the associated automorphism $\tilde{f} \in \text{Aut}(B^n)$ satisfies the hypotheses of Proposition 3.3 and the claim follows.

We conclude with the following simple construction of non-extendible local automorphisms of pseudoellipsoids.

**Example 3.4.** Let $\tilde{f} \in \text{Aut}(B^n)$ be an automorphism which does not satisfy (2.3) for some $n - k + 1 \leq j \leq n$. Pick a point $w_o \in \partial B \cap \{ \prod_{j=n-k+1}^{n} z_j \neq 0 \}$ so that also its image $\tilde{f}(w_o)$ is in $\partial B \cap \{ \prod_{j=n-k+1}^{n} z_j \neq 0 \}$. Then, let $z_o \in \partial \mathcal{E}^n_{(p)}$ so that $\pi^{(p)}(z_o) = w_o$ and consider a connected neighborhood $U$ of $z_o$ with the
following two properties: a) \( \pi^{(p)}|_{U} \) is a biholomorphism between \( U \) and its image \( \pi^{(p)}(U) \); b) \( \tilde{f}(\pi^{(p)}(U)) \) does not intersect \( \{ \prod_{j=n-k+1}^{n} z_j = 0 \} \) (a sufficiently small neighborhood \( U \) surely satisfies both requirements). Then, we may consider the map

\[
f : U_1 = U \cap E^n_{(p)} \to U_2 = f(U) \cap E^n_{(p)} , \quad f \overset{\text{def}}{=} \pi^{(p)}^{-1} \circ \tilde{f} \circ \pi^{(p)} .
\]

By construction, \( f \) is a local automorphism of \( E^n_{(p)} \) and its associated automorphism of \( \text{Aut}(B^n) \) is \( \tilde{f} \). By the hypotheses on \( \tilde{f} \) and by Proposition 3.3, \( f \) cannot extend to a global automorphism of \( E^n_{(p)} \).

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