ON SUMS INVOLVING COEFFICIENTS OF AUTOMORPHIC $L$-FUNCTIONS

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ABSTRACT. Let $L(s, \pi)$ be the automorphic $L$-function associated to an automorphic irreducible cuspidal representation $\pi$ of $GL_m$ over $\mathbb{Q}$, and let $a_\pi(n)$ be the $n$th coefficient in its Dirichlet series expansion. In this paper we prove that if at every finite place $p$, $\pi_p$ is unramified, then for any $\varepsilon > 0$,

$$A_\pi(x) = \sum_{n \leq x} a_\pi(n) \ll_{\varepsilon, \pi} \begin{cases} \frac{x^{m/2} + \varepsilon}{x^{m/2} - m + 1} & \text{if } m = 2, \\ \frac{x^{m/2} + \varepsilon}{x^{m/2} - m + 1} & \text{if } m \geq 3. \end{cases}$$

1. INTRODUCTION AND MAIN RESULTS

Let $a(n)$ be an arithmetic function. It is an important problem in number theory to establish the asymptotic formula for the summatory function

$$A(x) = \sum_{n \leq x} a(n).$$

The asymptotic behavior of $A(x)$ is often closely linked with the analytic properties of the Dirichlet series

$$A(s) = \sum_{n=1}^{\infty} a(n)n^{-s}.$$ 

The Langlands program predicts that the most general $L$-functions arise from automorphic representations of $GL_m$ over a number field and that such $L$-functions can be decomposed into products of primitive automorphic $L$-functions arising from irreducible cuspidal representations of $GL_m$ over $\mathbb{Q}$. Therefore in this paper we focus our attention on primitive automorphic $L$-functions of $GL_m$ over $\mathbb{Q}$.

To be precise, let us recall some basic facts about primitive automorphic $L$-functions of $GL_m$ over $\mathbb{Q}$ (see Godement and Jacquet [4], Jacquet and Shalika [8], or Rudnick and Sarnak [11]). Let $\pi$ be an automorphic irreducible cuspidal representation of $GL_m$ over $\mathbb{Q}$. Then there exists a finite set $\mathfrak{F}$ of places of $\mathbb{Q}$ such that $\pi$ is unramified at all finite places $p \notin \mathfrak{F}$.
representation of $\text{GL}_m$ over $\mathbb{Q}$ with unitary central character. Then $\pi$ is a restricted tensor product:

$$\pi = \otimes_p \pi_p.$$ 

To $\pi$ one associates an Euler product

$$(1.1) \quad L(s, \pi) = \prod_p L(s, \pi_p)$$

given by a product of local factors. Outside of a finite set of primes, $\pi_p$ is unramified. To every finite place $p$ where $\pi_p$ is unramified we associate a semisimple conjugacy class

$$A_\pi(p) = \begin{pmatrix} \alpha_{\pi, p}(1) \\ \vdots \\ \alpha_{\pi, p}(m) \end{pmatrix},$$

and we define the local $L$-function for the finite place $p$ as

$$(1.2) \quad L(s, \pi_p) = \det(I - p^{-s} A_\pi(p))^{-1} = \prod_{j=1}^{m} (1 - \alpha_{\pi, p}(j)p^{-s})^{-1}.$$ 

It is possible to write the local factors at ramified primes $p$ in the form of (1.2) with the convention that some of the $\alpha_{\pi, p}(j)$’s may be zero. In fact, the local factors at the ramified primes can best be described by the Langlands parameters of $\pi_p$.

The general Ramanujan conjectures for cuspidal automorphic representations $\pi$ of $\text{GL}_m$ over $\mathbb{Q}$ assert that for $p$ unramified, $|\alpha_{\pi, p}(j)| = 1$. For certain $\pi$, this conjecture has been proved. But in general it is still open. In this direction, Serre [12] first observed that the analytic properties of the Rankin-Selberg $L$-function, in conjunction with Landau’s lemma, can lead to

$$(1.3) \quad |\alpha_{\pi, p}(j)| \leq p^{1/2 - 1/(m^2 + 1)}.$$ 

For $m = 2$, this has been refined in [9] to

$$(1.4) \quad |\alpha_{\pi, p}(j)| \leq p^{7/64}.$$ 

The product (1.1) over primes gives a Dirichlet series representation: for $\Re s > 1$,

$$(1.5) \quad L(s, \pi) = \sum_{n=1}^{\infty} \frac{a_{\pi}(n)}{n^s}.$$ 

The aim of this paper is to study the summatory function for the coefficients $a_{\pi}(n)$ of automorphic $L$-functions attached to automorphic irreducible cuspidal representations of $\text{GL}_m$ over $\mathbb{Q}$, i.e.

$$A_\pi(x) = \sum_{n \leq x} a_{\pi}(n).$$

Our main result is the following.

**Theorem 1.1.** Let $L(s, \pi)$ be the automorphic $L$-function associated to an automorphic irreducible cuspidal representation $\pi$ of $\text{GL}_m$ over $\mathbb{Q}$, and let $a_{\pi}(n)$ be its $n$th coefficient in (1.5). If at every finite place $p$, $\pi_p$ is unramified, then we have that for any $\varepsilon > 0$,

$$A_\pi(x) = \sum_{n \leq x} a_{\pi}(n) \ll_{\varepsilon, \pi} \begin{cases} \\ x^{\frac{71}{192} + \varepsilon} & \text{if } m = 2, \\
x^{\frac{m^2 - m + \varepsilon}{m^2 + 1}} & \text{if } m \geq 3, \\ \end{cases}$$
where throughout this paper the notation \( \ll_{\varepsilon, \pi} \) means that the implied constant depends on \( \varepsilon \) and \( \pi \).

Our Theorem 1.1, for which the Ramanujan-Petersson conjecture is not known to hold, can be compared with the results of Iwaniec and Friedlander [3]: if the Ramanujan-Petersson conjecture is assumed, then the coefficients \( a(n) \) of a general \( L \)-function of degree \( m \) with a functional equation and suitable analytic properties satisfy

\[
\sum_{n \leq x} a(n) = \text{main term} + O_L \left( x^{\frac{m-1}{2}} \varepsilon \right).
\]

Our result can also be compared with one result of Miller [10], which states that for any \( \varepsilon > 0 \) and any real number \( \alpha \),

\[
\sum_{n \leq x} a(m, n)e(n\alpha) \ll_{\varepsilon, m, \Phi} x^{\frac{4}{3}\varepsilon},
\]

where \( a(m, n) \) are the Fourier coefficients of a cusp form \( \Phi \) for \( GL(3, \mathbb{Z}) \setminus GL(3, \mathbb{R}) \).

As an application of our Theorem 1.1, we shall consider the sum

\[
\sum_{n \leq x} t(n^2),
\]

where \( t(n) \) is the \( n \)th normalized Fourier coefficient of a Hecke-Maass cusp form \( \varphi \) corresponding to the eigenvalue \( l = \kappa^2 + \frac{1}{4} \) with respect to the full modular group \( SL(2, \mathbb{Z}) \), which coincides with the eigenvalue of the \( n \)th Hecke operator \( T_n \).

**Corollary 1.2.** Let \( t(n) \) be the \( n \)th normalized Fourier coefficient of a Hecke-Maass cusp form \( \varphi \) with respect to the full modular group \( SL(2, \mathbb{Z}) \). Then for any \( \varepsilon > 0 \), we have

\[
S(x) = \sum_{n \leq x} t(n^2) \ll_{\varepsilon, \varphi} x^{\frac{3}{4} + \varepsilon},
\]

where throughout this paper the notation \( \ll_{\varepsilon, \varphi} \) means that the implied constant depends on \( \varepsilon \) and the Maass cusp form \( \varphi \).

Our result improves a previous result given by Ivić [6]:

\[
S(x) \ll_{\varphi} x \exp \left( -A(\log x)^{\frac{1}{2}} (\log \log x)^{-\frac{1}{2}} \right),
\]

where \( A > 0 \) is a suitable constant.

**2. Three lemmas**

To prove Theorem 1.1, we need the following three lemmas.

**Lemma 2.1.** Let \( L(f, s) \) be a Dirichlet series with Euler product of degree \( m \geq 1 \), which is defined by

\[
L(f, s) = \sum_{n=1}^{\infty} \lambda_f(n)n^{-s} = \prod_{p < \infty} \prod_{j=1}^{m} \left( 1 - \frac{\alpha_f(p, j)}{p^s} \right)^{-1},
\]

where \( \alpha_f(p, j), j = 1, \cdots, m \), are the local parameters of \( L(f, s) \) at prime \( p \). This series and Euler product are absolutely convergent for \( \text{Re} s > 1 \). Let the gamma
factor be given by

\[ L_\infty(f, s) = \prod_{j=1}^{m} \pi^{-\frac{s+\mu_f(j)}{2}} \Gamma\left(\frac{s+\mu_f(j)}{2}\right), \]

where \( \mu_f(j), j = 1, \ldots, m, \) are the local parameters of \( L(f, s) \) at \( \infty \). We also define the completed \( L \)-function \( \Lambda(f, s) \) by

\[ \Lambda(f, s) = q(f)^{\frac{1}{2}} L_\infty(f, s) L(f, s), \]

where \( q(f) \) is the conductor of \( L(f, s) \). We assume that \( \Lambda(f, s) \) admits an analytic continuation to the whole complex plane \( \mathbb{C} \) and is an entire function. Assume that it also satisfies a functional equation

\[ \Lambda(f, s) = \epsilon_f \Lambda(\bar{f}, 1 - s) \]

where \( \epsilon_f \) is the root number with \( |\epsilon_f| = 1 \) and \( \bar{f} \) is the dual of \( f \) such that \( \lambda_f(n) = \bar{\lambda}_f(n), \mu_f(j) = \bar{\mu}_f(j), \) and \( q(\bar{f}) = q(f) \).

Then for every \( \eta \geq 0 \) we have

\[ \sum_{n \leq x} \lambda_f(n) \ll_f x^{\frac{1}{2} - \frac{\beta}{m} + \left(\frac{m}{2} - \frac{1}{2}\right)\eta} + \sum_{x < n \leq x^{1 + \frac{1}{2m} - \eta}} |\lambda_f(n)|. \]

Proof. This is a special case of Theorem 4.1 in Chandrasekharan and Narasimhan [2] with

\[ \delta = 1, \quad A = \frac{m}{2}, \quad \beta = 1, \quad u = \frac{1}{2} - \frac{1}{2m} \quad \text{and} \quad q = -\infty. \]

We reformulate it in the language used in Chapter 5 of Iwaniec and Kowalski [7].

**Lemma 2.2.** With the same notation as in Lemma 2.1, we assume that the Dirichlet series \( L(f, s) \) with Euler product of degree \( m \geq 1 \) has non-negative coefficients, i.e. \( \lambda_f(n) \geq 0 \), and converges for \( \text{Re} s \) sufficiently large. Suppose further that \( L(f, s) \) has a meromorphic continuation to \( \mathbb{C} \) with, at most, poles of finite order at \( s = 0, 1 \). Assume also that \( L(f, s) \) is of finite order and satisfies a functional equation

\[ \Lambda(f, s) = \epsilon_f \Lambda(f, 1 - s). \]

Then we have that for any \( \varepsilon > 0 \),

\[ \sum_{n \leq x} \lambda_f(n) = P(\log x)x + O_{\varepsilon, f}\left(x^{\frac{m+1}{m+\varepsilon}}\right), \]

where \( P \) is a polynomial depending only on \( L \), whose degree equals the order of the pole of \( L(f, s) \) at \( s = 1 \).

Proof. This is a refined version of Landau’s lemma; see Barthel and Ramakrishnan [1].

**Lemma 2.3.** Let \( b(1), b(2), \ldots \) be a sequence of complex numbers. Define the sequence \( a(0) = 1, a(1), a(2), \ldots \) by means of the formal identity

\[ \exp\left(\sum_{k=1}^{\infty} \frac{b(k)}{k} x^k\right) = \sum_{n=0}^{\infty} a(n)x^n. \]
For \( j = 1 \) or \( 2 \), define the sequence \( A_j(0) = 1, A_j(1), A_j(2), \ldots \) by means of the formal identity
\[
\exp \left( \sum_{k=1}^{\infty} \frac{|b(k)|^j}{k} x^k \right) = \sum_{n=0}^{\infty} A_j(n)x^n.
\]
Then \( A_j(n) \geq |a(n)|^j \).

Proof. See Lemma 3.1 in Soundararajan [13].

3. PROOF OF THEOREM 1.1

Associated with \( \pi \), an automorphic representation of \( \text{GL}_m \) over \( \mathbb{Q} \), there is also an Archimedean \( L \)-factor defined as
\[
L(s, \pi_\infty) = \prod_{j=1}^{m} \pi^{-\frac{s+\mu_{\pi}(j)}{2}} \Gamma \left( \frac{s + \mu_{\pi}(j)}{2} \right),
\]
where \( \mu_{\pi}(j) \), \( j = 1, 2, 3, \ldots, m \), are local parameters at \( \infty \). In connection with (1.1), the completed \( L \)-function associated to \( \pi \) is defined by
\[
\Lambda(s, \pi) = L(s, \pi_\infty) L(s, \pi).
\]
This completed \( L \)-function has analytic continuation, is entire everywhere (note that in our case \( m \geq 2 \)), and satisfies the functional equation
\[
\Lambda(s, \pi) = \epsilon_\pi q_\pi^{-\frac{s}{2}} \Lambda(1 - s, \pi^\dagger),
\]
where \( \pi^\dagger \) is the contragredient of \( \pi \), \( \epsilon_\pi \) is a complex number of modulus 1, and \( q_\pi \) is a positive integer called the arithmetic conductor of \( \pi \). For any place \( p \leq \infty \), \( \pi_p \) is equivalent to the complex conjugate \( \pi^\dagger_p \), and we have
\[
\{\alpha_{\pi, p}(j)\} = \{\alpha_{\pi, p}(j^\dagger)\}, \quad \{\mu_{\pi}(j)\} = \{\mu_{\pi^\dagger}(j)\}.
\]
Therefore, from Lemma 2.1 and (3.1), we have
\[
A_{\pi}(x) = \sum_{n \leq x} a_{\pi}(n) \ll_{\pi} x^{\frac{1}{2} - \frac{3}{16m} + \frac{1}{16m} + \frac{1}{16m} \eta} + \sum_{x < n \leq x + x^{\frac{1}{2} - \frac{1}{16m} - \eta}} |a_{\pi}(n)|,
\]
for every \( \eta \geq 0 \).

For \( m = 2 \), from (1.4) we have
\[
|a_{\pi}(n)| \leq \tau(n)n^{\frac{3}{8m}},
\]
where \( \tau(n) \) is the divisor function. From (3.2) with \( m = 2 \), we have
\[
A_{\pi}(x) = \sum_{n \leq x} a_{\pi}(n) \ll_{\pi} x^{\frac{1}{2} + \frac{1}{8} \eta} + \sum_{x < n \leq x + x^{\frac{1}{2} - \frac{1}{8} - \eta}} |a_{\pi}(n)|.
\]

From (3.3), we obtain
\[
A_{\pi}(x) \ll_{\pi} x^{\frac{1}{2} + \frac{1}{8} \eta} + x^{\frac{39}{56} - \eta + \varepsilon}.
\]
On taking \( \eta = \frac{39}{56} \), we have
\[
A_{\pi}(x) \ll_{\pi} x^{\frac{23}{70} + \varepsilon}.
\]
In order to give the result for \( m \geq 3 \), we recall some basic facts about the Rankin-Selberg \( L \)-function \( L(s, \pi \times \tilde{\pi}) \) associated to \( \pi \) and its contragredient \( \tilde{\pi} \). It is defined as a product of local factors:

\[
L(s, \pi \times \tilde{\pi}) = \prod_p L(s, \pi_p \times \tilde{\pi}_p).
\]

For unramified primes \( p \), the local factor is given by

\[
L(s, \pi_p \times \tilde{\pi}_p) = \prod_{j=1}^{m} \prod_{k=1}^{m} (1 - \alpha_{\pi, p}(j)\alpha_{\pi, p}(k)p^{-s})^{-1}.
\]

It can be defined similarly at primes \( p \) where \( \pi_p \) is ramified. By (1.3), the product \( \prod_p L(s, \pi_p \times \tilde{\pi}_p) \) converges absolutely on \( \text{Res} > 2 - \frac{2}{m+1} \) (in fact on \( \text{Res} > 1 \); see e.g. Jacquet and Shalika [8] or Rudnick and Sarnak [11]). We write this product as a Dirichlet series:

\[
L(s, \pi \times \tilde{\pi}) = \sum_{n=1}^{\infty} \frac{a_{\pi \times \tilde{\pi}}(n)}{n^s}.
\]

The completed Rankin-Selberg \( L \)-function is defined by

\[
\Lambda(s, \pi \times \tilde{\pi}) = L(s, \pi_\infty \times \tilde{\pi}_\infty)L(s, \pi \times \tilde{\pi})
\]

with

\[
L(s, \pi_\infty \times \tilde{\pi}_\infty) = \prod_{j=1}^{m^2} \pi^{-\frac{s+\mu_{\pi \times \tilde{\pi}}(j)}{2}} \Gamma\left(\frac{s + \mu_{\pi \times \tilde{\pi}}(j)}{2}\right).
\]

When \( \pi_\infty \) is unramified,

\[
\{\mu_{\pi \times \tilde{\pi}}(j)\}_{1 \leq j \leq m^2} = \{\mu_{\pi}(j) + \mu_{\tilde{\pi}}(k)\}_{1 \leq j \leq m, 1 \leq k \leq m}.
\]

It is known that \( a_{\pi \times \tilde{\pi}}(n) \geq 0 \) and \( L(s, \pi \times \tilde{\pi}) \) has a simple pole at \( s = 1 \). The completed Rankin-Selberg \( L \)-function \( \Lambda(s, \pi \times \tilde{\pi}) \) has a meromorphic continuation to the entire complex plane and satisfies a functional equation

\[
\Lambda(s, \pi \times \tilde{\pi}) = \epsilon_{\pi \times \tilde{\pi}} \frac{s-1}{s} \Lambda(1 - s, \pi \times \tilde{\pi}),
\]

where \( |\epsilon_{\pi \times \tilde{\pi}}| = 1 \) and \( q_{\pi \times \tilde{\pi}} > 0 \).

Therefore by applying Lemma 2.2 to \( L(s, \pi \times \tilde{\pi}) \) with degree \( m^2 \), we have

\[
\sum_{n \leq x} a_{\pi \times \tilde{\pi}}(n) = c_\pi x + O_{\epsilon, \pi}(x^{\frac{m^2-1}{m^2+1} + \epsilon}),
\]

where \( c_\pi \) is a positive constant.

From (3.10), we find that for any \( \eta \geq 0 \),

\[
\sum_{x < n \leq x + x^{1-\frac{1}{m^2-1} - \eta}} a_{\pi \times \tilde{\pi}}(n) \ll_{\epsilon, \pi} x^{\frac{m^2-1}{m^2+1} + \epsilon}.
\]

From (3.7), (3.8) and (3.9), we have that for \( \text{Res} > 2 - \frac{2}{m+1} \),

\[
\sum_{k=0}^{\infty} \frac{a_{\pi \times \tilde{\pi}}(p^k)}{p^{ks}} = \exp\left(\sum_{v=1}^{\infty} \frac{a_v(p^v)}{v} p^{-v s}\right),
\]
where
\[ \lambda_p(p^v) = \sum_{j=1}^{m} \alpha_{\pi,p}(j)^v. \]

From (1.1), (1.2) and (1.5), we have
\[ \sum_{k=0}^{\infty} \frac{a_{\pi}(p^k)}{p^{ks}} = \exp \left( \sum_{v=1}^{\infty} \frac{\lambda_p(p^v)}{v} p^{-vs} \right). \]

From (3.12), (3.13) and Lemma 2.3 with \( j = 2 \), we have that for an unramified prime \( p \),
\[ |a_{\pi}(p^k)|^2 \leq a_{\pi \times \tilde{\pi}}(p^k), \]
and thus in our case
\[ |a_{\pi}(n)|^2 \leq a_{\pi \times \tilde{\pi}}(n). \]

Therefore we have
\[ \sum_{x < n \leq x + x^{1 - \frac{1}{m} - \eta}} |a_{\pi}(n)|^2 \ll \sum_{x < n \leq x + x^{1 - \frac{1}{m} - \eta}} a_{\pi \times \tilde{\pi}}(n). \]

Now we begin to estimate (3.2). By Cauchy’s inequality, we find that the short-interval sum in (3.2) satisfies
\[ \sum_{x < n \leq x + x^{1 - \frac{1}{m} - \eta}} |a_{\pi}(n)| \leq \left( \sum_{x < n \leq x + x^{1 - \frac{1}{m} - \eta}} |a_{\pi}(n)|^2 \right)^{\frac{1}{2}} \left( \sum_{x < n \leq x + x^{1 - \frac{1}{m} - \eta}} 1 \right)^{\frac{1}{2}}. \]

By (3.11) and (3.14), we have
\[ \sum_{x < n \leq x + x^{1 - \frac{1}{m} - \eta}} |a_{\pi}(n)|^2 \ll_{\epsilon, \pi} \frac{m^2 - 1}{m^2 + 1} \eta + \frac{m^2 - 1}{2m^2 + 2} + \epsilon. \]

From (3.15) and (3.16), we obtain
\[ \sum_{x < n \leq x + x^{1 - \frac{1}{m} - \eta}} |a_{\pi}(n)| \ll_{\epsilon, \pi} x^{\frac{1}{2} - \frac{1}{2m} - \frac{m^2 - 1}{2m^2 + 2} + \epsilon}. \]

Inserting (3.17) into (3.2), we have
\[ A_{\pi}(x) = \sum_{n \leq x} a_{\pi}(n) \ll_{\epsilon, \pi} x^{\frac{1}{2} - \frac{1}{2m} + \left( \frac{m}{2} - \frac{1}{2} \right) \eta + \frac{1}{2} - \frac{1}{2m} - \frac{m^2 - 1}{m^2 + 1} \eta + \frac{m^2 - 1}{m^2 + 1} + \epsilon}. \]

On taking \( \eta = \frac{m^2 - 1}{m(m^2 + 1)} \), we get
\[ A_{\pi}(x) \ll_{\epsilon, \pi} x^{\frac{m^2 - m}{m^2 + 1} + \epsilon}. \]

This completes the proof of Theorem 1.1.
4. Proof of Corollary 1.2

To prove Corollary 1.2, we recall some basic facts from the books of Iwaniec and Kowalski [7], and of Goldfeld [5]. Associated to each Hecke-Maass cusp form \( \varphi \) for the full modular group \( SL(2, \mathbb{Z}) \) there is an \( L \)-function \( L(\varphi, s) \), which is defined, for \( \text{Re} \, s > 1 \), by

\[
L(\varphi, s) = \sum_{n=1}^{\infty} t(n)n^{-s} = \prod_{p} (1 - t(p)p^{-s} + p^{-2s})^{-1}
\]

with \( \alpha_p + \alpha'_p = t(p) \) and \( \alpha_p\alpha'_p = 1 \). The symmetric square \( L \)-function \( L(\text{Sym}^2 \varphi, s) \) is defined, for \( \text{Re} \, s > 1 \), by

\[
L(\text{Sym}^2 \varphi, s) = \zeta(2s) \sum_{n=1}^{\infty} t(n^2)n^{-s} = \prod_{p} \left(1 - \frac{\alpha^2_p}{p^s}\right)^{-1} \left(1 - \frac{1}{p^s}\right)^{-1} \left(1 - \frac{\alpha'_2}{p^s}\right)^{-1},
\]

where \( \zeta(s) \) is the Riemann zeta-function. Then we have

\[
(4.1) \quad \sum_{n=1}^{\infty} t(n^2)n^{-s} = \frac{L(\text{Sym}^2 \varphi, s)}{\zeta(2s)}.
\]

This gives

\[
(4.2) \quad t(n^2) = \sum_{d^2|n} \mu(d) t^{(2)} \left( \frac{n}{d^2} \right),
\]

where \( t^{(2)}(n) \) is the \( n \)th coefficient of the symmetric square \( L \)-function \( L(\text{Sym}^2 \varphi, s) \) with \( \text{Re} \, s > 1 \).

It follows from the Gelbart-Jacquet lift that \( L(\text{Sym}^2 \varphi, s) \) is an automorphic \( L \)-function of \( GL_3 \). Then from Theorem 1.1 with \( m = 3 \), we have

\[
(4.3) \quad \sum_{n \leq x} t^{(2)}(n) \ll x^{\frac{3}{2} + \varepsilon}.
\]

From (4.2) and (4.3), we have

\[
S(x) = \sum_{n \leq x} t(n^2) \ll x^{\frac{3}{2} + \varepsilon}.
\]

This completes the proof of Corollary 1.2.

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