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ON THE GLAUBERMAN CORRESPONDENT OF A BLOCK

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ABSTRACT. In this paper, we analyze the compatibility of Fong's reduction and the Glauberman correspondence of characters and then clarify that the p-solvable hypothesis in a paper of Harris and Linckelmann is not necessary.

Let $\mathcal{O}$ be a complete discrete valuation ring with an algebraically closed residue field k of characteristic p and a quotient field $\mathcal{K}$ of characteristic 0. In addition, $\mathcal{K}$ is also assumed to be big enough for all finite groups that we consider below. Let G be a finite group. We denote by $\mathrm{Irr}(G)$ the set of all irreducible characters of G. Let A be another finite group acting on G. Then clearly A acts on $\mathrm{Irr}(G)$. The following theorem is due to Glauberman [4] and relates the A-fixed set $(\mathrm{Irr}_{\mathcal{K}}(G))^A$ with the set $\mathrm{Irr}_{\mathcal{K}}(C_G(A))$.

Theorem 1. *For any A-group G with A solvable and $|G|$ and $|A|$ coprime, there is a bijection $\pi(G, A) : \mathrm{Irr}_{\mathcal{K}}(G)^A \to \mathrm{Irr}_{\mathcal{K}}(C_G(A))$ such that*

1.1. For any normal subgroup B of A, the bijection $\pi(G, B)$ maps $(\mathrm{Irr}_{\mathcal{K}}(G))^A$ to $(\mathrm{Irr}_{\mathcal{K}}(C_G(B)))^A$, and we have $\pi(G, A) = \pi(C_G(B), A/B) \circ \pi(G, B)$ on $(\mathrm{Irr}_{\mathcal{K}}(G))^A$.

1.2. If A is a q-group for some prime q, for any $\chi \in (\mathrm{Irr}_{\mathcal{K}}(G))^A$ the corresponding irreducible character $\pi(G, A)(\chi)$ of $C_G(A)$ is the unique irreducible constituent of $\mathrm{Res}^G_{C_G(A)}(\chi)$ occurring with a multiplicity prime to q.

2. Continue to keep the notation in Theorem 1. Let b be a block idempotent of $\mathcal{O}G$. We denote by $\mathrm{Irr}(G, b)$ the set of all irreducible characters of G provided by some $\mathcal{K}Gb$-module. If A centralizes a defect group P of b, by [9, Prop. 1 and Th. 1], A stabilizes all characters of $\mathrm{Irr}(G, b)$ and there is a unique block idempotent c of $\mathcal{O}C_G(A)$ such that $\mathrm{Irr}(C_G(A), c) = \pi(G, A)(\mathrm{Irr}(G, b))$; moreover b and c are perfectly isometric (refer to [1]). Such a block idempotent c is called the Glauberman correspondent of b (see [9]). A perfect isometry between blocks is considered to be the character-theoretic 'shadow' of a derived equivalence. So it is very interesting to ask whether there is such a derived equivalence between a block and its Glauberman correspondent. More strongly, with the p-solvable hypothesis, Harris and Linckelmann proved that a block and its Glauberman correspondent are Morita equivalent. In this paper, we analyze the compatibility of Fong's reduction and the Glauberman correspondence of characters and then clarify that the p-solvable hypothesis in [5] is not necessary.

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Theorem 3. *Let G be an A-group with A solvable and $|G|$ and $|A|$ be co-prime. Let b be a block idempotent of $\mathcal{O}G$ with a defect group P centralized by A and let c be the Glauberman correspondent of b (refer to [9]). If $G = O_{p'}(G)C_G(A)$, then there is an indecomposable $\mathcal{O}(G \times C_G(A))$-module M having the following properties:*

3.1. The p-subgroup $\Delta(P) = \{(x, x)|x \in P\}$ of $G \times C_G(A)$ is a vertex of M.

3.2. M induces a Morita equivalence between $\mathcal{O}Gb$ and $\mathcal{O}C_G(A)c$, and the bijection between $\mathrm{Irr}(G, b)$ and $\mathrm{Irr}(C_G(A), c)$ induced by this Morita equivalence coincides with the Glauberman correspondence from $\mathrm{Irr}(G, b)$ to $\mathrm{Irr}(C_G(A), c)$.

4. Firstly we introduce some notation and terminology used in this paper. Let S be a set and H be a group acting on S. For any $h \in H$ and $s \in S$, we write the action of h on s as $h \cdot s$. Let A be an $\mathcal{O}$-algebra with the identity element. We denote by A^* the multiplicative group of all invertible elements of A. Let K be a finite group. A group $\hat{K}$ is an $\mathcal{O}^*$-group with the $\mathcal{O}^*$-quotient K if there is an injective group homomorphism $\rho : \mathcal{O}^* \to Z(\hat{K})$ such that $\hat{K}/\rho(\mathcal{O}^*) \cong K$. Then $\hat{K}$ can be regarded as a central extension of K by $\mathcal{O}$, and, in this case, we denote by $\mathcal{O}_*\hat{K}$ the twisted group algebra corresponding to this central extension (see [10, 10.4]). For any $\lambda \in \mathcal{O}^*$ and $x \in \hat{K}$, we write the product $\rho(\lambda)x$ as $\lambda \cdot x$ for convenience. For any subgroup L of K, we denote by $\hat{L}$ the inverse image of L through the canonical surjective homomorphism $\hat{K} \to K$; then $\hat{L}$ is an $\mathcal{O}^*$-group with the $\mathcal{O}^*$-quotient L. The group $\hat{K}$ with another injective homomorphism $\mathcal{O}^* \to Z(\hat{K})$, $\lambda \mapsto \rho(\lambda^{-1})$ is also an $\mathcal{O}^*$-group with the $\mathcal{O}^*$-quotient K. We call this $\mathcal{O}^*$-group the opposite $\mathcal{O}^*$-group of $\hat{K}$ and denote it by $\hat{K}^\circ$. Let $\hat{H}$ be an $\mathcal{O}^*$-group with the $\mathcal{O}^*$-quotient H. A homomorphism $\theta : \hat{K} \to \hat{H}$ is called a homomorphism of $\mathcal{O}^*$-groups if for any $\lambda \in \mathcal{O}^*$ and $x \in \hat{K}$, we have $\theta(\lambda \cdot x) = \lambda \cdot \theta(x)$. Below, we will occasionally use the induction of interior K-algebras, the Brauer quotients and the Brauer homomorphisms; for their definitions and corresponding notation, readers can refer to [10]; by the word "character", we sometimes indicate a character on a group and sometimes indicate a character on an algebra, but that does not cause any confusion.

Next we begin to analyze the compatibility of Fong's reduction and the Glauberman correspondence of characters. Let G be a finite group, b be a block idempotent of $\mathcal{O}G$ and P be a defect group of b. Let A be another finite group acting on G. We assume that $|G|$ and $|A|$ are coprime, $O_{p'}(G) \cap C_G(A) = O_{p'}(C_G(A))$, A is cyclic with the order a power of some prime q and A centralizes P. Let c be the Glauberman correspondent of b. Set $C = C_G(A)$ and let R be a Sylow p-subgroup of C containing P.

5. Let I be the set of all block idempotents f of $\mathcal{O}O_{p'}(G)$ such that $bf \neq 0$. The action of A on $O_{p'}(G)$ induces an action of A on $\mathcal{O}O_{p'}(G)$, which again induces an action of A on I. We consider the action of the semidirect $G \rtimes A$ defined by the equality $g \cdot f = gfg^{-1}$, where $g \in G$ and $f \in I$, together with the action of A on I. By this action, it is well known that G acts transitively on I; moreover since $|G|$ and $|A|$ are co-prime, by [3, Lemma 13.8 and Cor. 13.9], the set I^A of A-fixed points in I is non-empty and $C_G(A)$ acts transitively on I^A. We denote by K the stabilizer of f in G and set $\mathrm{Tr}_K^G(f) = \sum_{x \in G/K} xfx^{-1}$, where G/K is a set of representatives of right cosets of K in G. Clearly $ff^x = 0$ for any $x \in G - K$;

therefore by [3, 2.6.3], we have an isomorphism

$$\operatorname{Ind}_K^G(\mathcal{O}Kf) \cong \mathcal{O}G\operatorname{Tr}_K^G(f) \tag{5.1}$$

which maps $1 \otimes y \otimes 1$ onto y for any $y \in \mathcal{O}Kf$.

6. Since any element of $Z(\operatorname{Ind}_K^G(\mathcal{O}Kf))$ can be written as the sum $\sum_{x\in G/K} x \otimes a \otimes x^{-1}$ for some $a \in Z(\mathcal{O}Kf)$, by the isomorphism (5.1), the map

$$Z(\mathcal{O}Kf) \to Z(\mathcal{O}G\operatorname{Tr}_K^G(f)),\ a \mapsto \sum_{x\in G/K} xax^{-1} \tag{6.1}$$

is an isomorphism. Obviously $Z(\mathcal{O}Kf)$ and $Z(\mathcal{O}G\operatorname{Tr}_K^G(f))$ are both A-stable and the isomorphism (6.1) is compatible with the A-actions on $Z(\mathcal{O}Kf)$ and $Z(\mathcal{O}G\operatorname{Tr}_K^G(f))$. Since $b\operatorname{Tr}_K^G(f) = b$, $b \in Z(\mathcal{O}G\operatorname{Tr}_K^G(f))$ and we denote by d the inverse image of b through (6.1), which is A-stable. Then $b = \sum_{x\in G/K} xdx^{-1}$ and $dxdx^{-1} = 0$ for any $x \in G - K$. Again by [3, 2.6.3], we have an isomorphism

$$\operatorname{Ind}_K^G(\mathcal{O}Kd) \cong \mathcal{O}Gb \tag{6.2}$$

mapping $1 \otimes y \otimes 1$ onto yb for any $y \in \mathcal{O}Kd$.

7. Let J be the set of all defect groups of d. It is well known that K acts transitively on J. Since A stabilizes d, A also acts on J. We consider the action of $K \rtimes A$ on J defined by these two actions of A and K on J. Since $|A|$ and $|K|$ are co-prime, by [3, Lemma 13.8 and Cor. 13.9], the J^A of all A-stable elements of J is non-empty and $C_K(A)$ acts transitively on J^A. Similarly b also has A-stable defect groups and $C_G(A)$ acts transitively on all these defect groups. By the isomorphism (6.2), all defect groups of d are also defect groups of b. Since P is a defect group centralized by A, adjusting K and d by suitable $C_G(A)$-conjugation, we can assume that $P \le K$ and P is a defect group of d. Let e and g be the Glauberman correspondents of d and f respectively. Since we assume $O_{p'}(G) \cap C_G(A) = O_{p'}(C_G(A))$, g is actually a block idempotent of $\mathcal{O}O_{p'}(C_G(A))$.

Proposition 8. *Keep the notation and hypotheses as above. Then the stabilizer of g under the $C_G(A)$-conjugation is equal to $C_K(A)$, $ge = e$ and there is an isomorphism*

$$\operatorname{Ind}_{C_K(A)}^{C_G(A)}(\mathcal{O}C_K(A)e) \cong \mathcal{O}C_G(A)c. \tag{8.1}$$

Proof. It is trivial to see that [5, Th. 5.1] still holds without the p-solvable condition.

Now we assume that the block idempotent f of $\mathcal{O}O_{p'}(G)$ is stabilized by $G \rtimes A$; then $K = G$, $d = b$, $e = c$ and $C_K(A) = C_G(A)$. □

9. Obviously G acts on the full matrix algebra $\mathcal{O}O_{p'}(G)f$ over $\mathcal{O}$, and by the Skolem-Noether theorem, there exists a group homomorphism

$$\rho : G \to \operatorname{Aut}(\mathcal{O}O_{p'}(G)f) \cong (\mathcal{O}O_{p'}(G)f)^*/\mathcal{O}^*.$$

We denote by $\hat{G}$ the set of all elements (x, s) such that $\rho(x)$ is the image of s in $(\mathcal{O}O_{p'}(G)f)^*/\mathcal{O}^*$, where $s \in (\mathcal{O}O_{p'}(G)f)^*$ and $x \in G$. $\hat{G}$ is a group with respect to the multiplication $(x, s)(x', s') = (xx', ss')$. Endowed with the homomorphism $\mathcal{O}^* \to \hat{G}$ mapping λ onto $(1, \lambda)$, $\hat{G}$ becomes an $\mathcal{O}^*$-group with the $\mathcal{O}^*$-quotient G. There is an injective group homomorphism $O_{p'}(G) \to \hat{G}$, $x \mapsto (x, xb)$, whose image is normal in $\hat{G}$ and intersects $\mathcal{O}^*$ trivially. Later we always identify $O_{p'}(G)$ with this image through the injective homomorphism.

10. We set

$$\bar{G} = G/O_{p'}(G) \quad \text{and} \quad \hat{\bar{G}} = \hat{G}/O_{p'}(G).$$

For any $(x,\, s) \in \hat{G}$, we denote by $\overline{(x,\, s)}$ the image of $(x,\, s)$ in $\bar{G}$. With the obvious injective homomorphism $\mathcal{O}^* \to \hat{G}/O_{p'}(G)$ mapping λ onto $\overline{(1,\, \lambda)}$, $\hat{\bar{G}}$ is an $\mathcal{O}^*$-group with the $\mathcal{O}^*$-quotient $\bar{G}$. The actions of A on $\mathcal{O}O_{p'}(G)g$ and G induce an action of A on $\hat{G}$ defined by the equality

$$a \cdot (x, s) = (a \cdot x, a \cdot s)$$

for any $a \in A$ and $(x, s) \in \hat{G}$. Obviously $O_{p'}(G)$ is an A-stable normal subgroup of $\hat{G}$ and thus the action of A on $\hat{G}$ induces an action of A on $\hat{\bar{G}}$. Set $m = |G||O_{p'}(G)|$ and denote by μ_m the subgroup of $\mathcal{O}^*$ of all m-th roots of unity. For any $x \in G$, choose $s_x \in (\mathcal{O}O_{p'}(G)f)^*$ such that $(x, s_x) \in \hat{G}$ and $\det(s_x) = 1$. Set $G' = \{(x, \lambda s_x) | \lambda \in \mu_m, x \in G\}$.

Lemma 11. *Keep the notation and hypotheses as above. Then G' is an A-stable finite subgroup of $\hat{G}$ such that $\hat{G} = \mathcal{O}^* G'$ and $O_{p'}(G) \leq G'$; moreover, the exponent of G' divides m.*

Proof. This lemma easily follows from the proof of [5, 4.5 (i)]. □

12. Since $\mathcal{O}O_{p'}(G)f$ is a full matrix algebra over $\mathcal{O}$ and its rank over $\mathcal{O}$ is prime to p, the restriction to R of the homomorphism ρ can be lifted to a unique homomorphism $\sigma : R \to (\mathcal{O}O_{p'}(G)f)^*$ such that $\det(\sigma(u)) = 1$ for any $u \in R$ (refer to [7, para. 6.2]). By this homomorphism, we define a new homomorphism $R \to \hat{G}$, $u \mapsto (u, \sigma(u))$, which is injective and whose image intersects $\mathcal{O}^*$ trivially. We identity R with its image through this homomorphism. We claim that

$$R \subset C_{G'}(A). \tag{12.1}$$

Firstly, since $\mathcal{O}O_{p'}(G)f$ is a full matrix algebra over $\mathcal{O}$, by the Skolem-Noether theorem, for any $a \in A$ and $u \in R$, $a(\sigma(u))$ is equal to some conjugate of $\sigma(u)$ in $\mathcal{O}O_{p'}(G)f$ and thus $\det(a(\sigma(u))) = 1$; then it follows from the uniqueness of the homomorphism σ that $a(\sigma(u)) = \sigma(u)$ for any $a \in A$ and $u \in R$. This shows that A centralizes the subgroup R of $\hat{G}$. Secondly it is clear that $s_u\sigma(u)^{-1}$ is an invertible element of $\mathcal{O}$ with order dividing m. Therefore there is some $\lambda \in \mu_m$ such that $s_u = \lambda\sigma(u)$ and then $(u, \sigma(u)) \in G'$. The claim is proved.

13. By [5, Th. 4.4], there is an algebra isomorphism

$$\mathcal{O}Gf \cong \mathcal{O}O_{p'}(G)f \otimes_{\mathcal{O}} \mathcal{O}_* \hat{\bar{G}}^\circ \tag{13.1}$$

mapping xf onto $s \otimes \overline{(x, s)}$ for any $x \in G$. Since $fb = b$, this isomorphism determines a central primitive idempotent $\bar{b}$ of $\mathcal{O}_* \hat{\bar{G}}^\circ$ such that

$$\mathcal{O}Gb \cong \mathcal{O}O_{p'}(G)f \otimes_{\mathcal{O}} \mathcal{O}_* \hat{\bar{G}}^\circ \bar{b}. \tag{13.2}$$

Moreover the image of P in $\bar{G}$ is a defect group of $\bar{b}$ (refer to [8, Cor. 6.6]). Since P and the image of P in $\bar{G}$ are isomorphic, we identify them. We let α be the unique character of $\mathcal{O}O_{p'}(G)f$ and denote by $\mathrm{Irr}(\hat{\bar{G}}^\circ, \bar{b})$ the set of characters of all irreducible $\mathcal{K} \otimes_{\mathcal{O}} \mathcal{O}_* \hat{\bar{G}}^\circ \bar{b}$-modules. Then by the isomorphism (13.2), we obtain a bijection

$$\mathrm{Irr}(G, b) \to \mathrm{Irr}(\hat{\bar{G}}^\circ, \bar{b}),\ \chi \mapsto \bar{\chi} \tag{13.3}$$

such that for any $x \in G$, $\chi(x) = \alpha(s)\bar{\chi}(x, s)$.

14. By paragraph 9 applied to C, $\mathcal{O}O_{p'}(C)$ and g, we obtain a group homomorphism

$$\varrho : C \to \mathrm{Aut}(\mathcal{O}O_{p'}(C)g) \cong (\mathcal{O}O_{p'}(C)g)^*/\mathcal{O}^*.$$

By this homomorphism, we construct an $\mathcal{O}^*$-group $\tilde{C}$ with the $\mathcal{O}^*$-quotient C, which is the set of all elements (y, t) such that $\varrho(y)$ is the image of t in $(\mathcal{O}O_{p'}(C)g)^*/\mathcal{O}^*$, where $t \in (\mathcal{O}O_{p'}(C)g)^*$ and $y \in C$. Just as $\hat{G}$ in paragraph 9, $\hat{C}$ has three subgroups: the first is the normal group $\{(y, yg)|y \in O_{p'}(C)\}$ and we identity this normal subgroup with $O_{p'}(C)$; the second is the subgroup C' consisting of all elements $(y, \lambda t_y)$, where $\lambda \in \mu_m$, $y \in C$ and t_y belongs to $(\mathcal{O}O_{p'}(C)g)^*$ such that $(y, t_y) \in \tilde{C}$ and $\det(t_y) = 1$; the third is the p-group $R = \{(u, \varsigma(u))|u \in R\}$, where $\varsigma : R \to (\mathcal{O}O_{p'}(C)g)^*$ is the unique lifting of the restriction of ϱ to R such that $\det(\varsigma(u)) = 1$ for any $u \in R$. Note that the exponent of C' also divides m (refer to Lemma 11) and that $R \subset C'$ (refer to (12.1)). We set

$$\bar{C} = C/O_{p'}(C) \quad \text{and} \quad \bar{\tilde{C}} = \tilde{C}/O_{p'}(C)$$

and denote by $\overline{(x,\, t)}$ the image of (x, t) in $\bar{\tilde{C}}$ for any $(x, t) \in \tilde{C}$. Then $\bar{\tilde{C}}$ is an $\mathcal{O}^*$-group with $\mathcal{O}^*$-quotient $\bar{C}$.

15. By [5, Th. 4.4], there is an algebra isomorphism

$$\mathcal{O}Cg \cong \mathcal{O}O_{p'}(C)g \otimes_{\mathcal{O}} \mathcal{O}_* \bar{\tilde{C}}^\circ \tag{15.1}$$

mapping xg onto $t \otimes \overline{(x, t)}$ for any $x \in C$. Since $gc = c$, this isomorphism determines a central primitive idempotent $\bar{c}$ of $\mathcal{O}_* \bar{\tilde{C}}^\circ$ such that

$$\mathcal{O}Cc \cong \mathcal{O}O_{p'}(C)g \otimes_{\mathcal{O}} \mathcal{O}_* \bar{\tilde{C}}^\circ \bar{c}. \tag{15.2}$$

Let β be the unique character of $\mathcal{O}O_{p'}(C)g$ and denote by $\mathrm{Irr}(\bar{\tilde{C}}^\circ, \bar{c})$ the set of characters of all irreducible $\mathcal{K} \otimes_{\mathcal{O}} \mathcal{O}_* \bar{\tilde{C}}^\circ \bar{c}$. By the isomorphism (15.2), we obtain a bijection

$$\mathrm{Irr}(C, c) \to \mathrm{Irr}(\bar{\tilde{C}}^\circ, \bar{c}),\ \psi \mapsto \bar{\psi} \tag{15.3}$$

such that for any $x \in C$, $\psi(x) = \beta(t)\bar{\psi}(x, t)$.

Lemma 16. *Keep the notation and hypotheses as above. Then $C_{\hat{G}}(A) = \hat{C}$ and $C_{G'}(A)$ is a subgroup of $\hat{C}$ such that $\mathcal{O}^* C_{G'}(A) = \hat{C}$.*

Proof. Take $x \in C$ and let K be the set of all the inverse images of x through the canonical surjective homomorphism $G' \to G$. Then since it follows from Lemma 11 that G' is A-stable, A stabilizes K. Consider the action of $\mu_m \times A$ on K defined by the left multiplication of μ_m on K and the above A-action on K together. Since μ_m acts regularly on K and A and μ_m have co-prime order, by [3, Lemma 13.8 and Cor. 13.9], A has to stabilize some element of K and μ_m regularly acts on these A-stabilized elements; in particular, A stabilizes all elements of K. This shows that $C_{G'}(A)$ is a subgroup of $\hat{C}$ such that $\mathcal{O}^* C_{G'}(A) = \hat{C}$. Thus $C_{\hat{G}}(A) = \hat{C}$. $\square$

The following two lemmas will be used to analyze the local structure of the $\mathcal{O}(G \times C_G(A))$-module in Theorem 3 inducing Morita equivalence between $\mathcal{O}Gb$ and $\mathcal{O}C_G(A)c$. For the compatibility of Fong's reduction with the Glauberman correspondence of characters (see Proposition 23 below), they can be skipped for now.

Lemma 17. *Keep the notation and hypotheses as above. There is an isomorphism*

$$\Phi : C_{G'}(A) \cong C',$$

which preserves R and $O_{p'}(C)$ element-wise and makes the following diagram commutative:

$$(17.1)\qquad \begin{array}{ccccccccc} 1 & \to & \mu_m & \to & C_{G'}(A) & \to & C & \to & 1 \\ & & \| & & \downarrow & & \| & & \\ 1 & \to & \mu_m & \to & C' & \to & C & \to & 1. \end{array}$$

Proof. We choose a local subring $\mathcal{O}'$ of $\mathcal{K}$ obtained by adding a primitive $|O_{p'}(G)|$-th root ξ of unity to the ring of rational integers $\mathbb{Z}$ and localizing $\mathbb{Z}[\xi]$ at a maximal ideal containing q; then $|O_{p'}(G)|$ is invertible in $\mathcal{O}'$ and the residue field k' of $\mathcal{O}'$ is equal to $\mathbb{F}_q(\xi)$ and has characteristic q. Let α be the unique character of $\mathcal{O}O_{p'}(G)f$. Then for any $a \in \mathcal{O}O_{p'}(G)f$, by the Fourier inversion formula, we have

$$a = \frac{\alpha(1)}{|O_{p'}(G)|} \sum_{z \in O_{p'}(G)} \alpha(az^{-1})z.$$

Whenever a takes f or s_x for $x \in C$, az^{-1} has finite order dividing m. Therefore $\alpha(az^{-1}) \in \mathbb{Z}[\xi]$, $f \in (\mathcal{O}'O_{p'}(G))^A$ and $s_x \in (\mathcal{O}'O_{p'}(G))^A$ for any $x \in C$. Furthermore $s_x \in (\mathcal{O}'O_{p'}(G)f)^A$ for any $x \in C$ since $s_x f = s_x$. Similarly $g \in \mathcal{O}'\mathcal{O}_{p'}(C)$ and $t_y \in \mathcal{O}'\mathcal{O}_{p'}(C)g$ for any $y \in C$.

We denote by $\bar{g}$ the image of g in $k'\mathcal{O}_{p'}(C)$. Since $br_A(s_x)$ and $br_A(t_x)$ both act on $k'\mathcal{O}_{p'}(C)\bar{g}$ as x for any $x \in C$, there is a unique $\nu_x \in k'^*$ such that $br_A(s_x) = \nu_x br_A(t_x)$. Moreover since $br_A(s_x)$ and $br_A(t_x)$ both have order dividing m, ν_x has finite order dividing m. Therefore we can uniquely lift ν_x to an invertible element of $\mathcal{O}'$ with the same order as ν_x; we still denote this invertible element by ν_x. Then by the proof of [5, 4.5.8], we can prove that there is an isomorphism of $\mathcal{O}^*$-groups

$$(17.2)\qquad \tilde{C} \cong \hat{C}$$

which maps (x, t_x) onto $(x, \nu_x s_x)$ for any $x \in C$ and (x, xg) onto (x, xf) for any $x \in O_{p'}(C)$. In particular, the isomorphism (17.2) induces an isomorphism $C_{G'}(A) \cong C'$. The latter preserves $O_{p'}(C)$ element-wise and fulfilling (17.1), but unfortunately it does not preserve R element-wise. So we have to adjust the isomorphism (17.2).

Since $R \subset C_{G'}(A)$ and $R \subset C'$ (refer to (12.1) and paragraph 14), without loss of generality, we take t_u to be $\varsigma(u)$ and s_u to be $\sigma(u)$ for any $u \in R$. Then the isomorphism $\tilde{C} \cong \hat{C}$ above implies that for any $u, v \in R$, $\nu_u \nu_v = \nu_{uv}$. This shows that the function $\nu : R \to \mathbb{Z}[\xi]$, $u \mapsto \nu_u$ is a linear character of R. We claim that ν is C-stable. By the uniqueness of σ and ς, we have $x\sigma(u)x^{-1} = \sigma(xux^{-1})$ and $x\varsigma(u)x^{-1} = \varsigma(xux^{-1})$ for any $x \in C$ and $u \in R$ such that $xux^{-1} \in R$. Then we have

$$\nu(xux^{-1}) = br_A(\sigma(xux^{-1}))br_A(\varsigma(xux^{-1})) = br_A(x\sigma(u)x^{-1})br_A(x\varsigma(u)x^{-1}) = \nu(u).$$

The claim is proved. $\square$

Notice that R is a Sylow p-subgroup of C. Then by [5, Prop. 2.6], ν can be extended to a linear character of C and we still denote this character by ν. By using ν, we define a map

$$\tilde{C} \to \hat{C},\ (x, t_x) \mapsto \nu(x^{-1})\nu_x(x, s_x).$$

Then it is easily checked that this map is an isomorphism of $\mathcal{O}^*$-groups and its inverse induces the desired isomorphism $\Phi : C_{G'}(A) \cong C'$ in the lemma.

18. We define a function α' on G' by setting

$$\alpha'(x, s) = \alpha(s)$$

for any $(x, s) \in G'$. Then α' is an irreducible character of G' extending α through the inclusion $O_{p'}(G) \hookrightarrow G'$. Since $\mathcal{O}O_{p'}(G)f$ is a full matrix algebra over $\mathcal{O}$, by the Skolem-Noether theorem, for any $(x, s) \in G'$ and $a \in A$, $a(s)$ is equal to some conjugate of s in $\mathcal{O}O_{p'}(G)f$. Therefore $\alpha'(a(x, s)) = \alpha'(x, s)$ and α' is A-stable. Similarly a function β' on C' defined by

$$\beta'(y, t) = \beta(t)$$

for any $(y, t) \in C'$ is also an irreducible character of C', which extends β through the inclusion $O_{p'}(C) \hookrightarrow C'$.

Lemma 19. *Keep the notation and hypotheses as above. Then there is a linear character $\gamma' : C' \to \mathcal{O}$ such that the following hold:*

19.1. $\mu_m \subset \mathrm{Ker}(\gamma')$ and $\mathrm{Im}(\gamma') \subset \mu_m$.

19.2. The map $\Theta : C_{G'}(A) \to C',\ (t, y) \mapsto \gamma'(\Phi(t, y))\Phi(t, y)$ is an isomorphism preserving $O_{p'}(C)$ elementwise.

19.3. $\pi(G', A)(\alpha') \circ \Theta^{-1} = \beta'$.

Proof. By [5, Th. 2.3], $\pi(G', A)(\alpha') \circ \Phi^{-1}$ is an irreducible character of C' and is an extension of β. Therefore there is a linear character $\gamma' : C' \to \mathcal{O}$ of C' such that $\pi(G', A)(\alpha') \circ \Phi^{-1} = \gamma'\beta'$ and $O_{p'}(C) \subset \mathrm{Ker}(\gamma)$. Take $\xi \in \mu_m$. Since

$$(\pi(G', A)(\alpha') \circ \Phi^{-1})(1, \xi g) = \xi(\pi(G', A)(\beta') \circ \Phi^{-1})(1, g) = \xi\beta(1) = \beta'(\xi g, 1),$$

$\gamma'(\xi) = 1$ and $\mu_m \subset \mathrm{Ker}(\gamma')$. The inclusion $\mathrm{Im}(\gamma') \subset \mu_m$ is obvious since the exponent of C' divides m. So up to now, 19.1 is proved. The latter two can be trivially verified. □

Lemma 20. *Keep the notation and hypotheses as above. Then $C_{\bar{G}}(A) = C_G(A)O_{p'}(G)/O_{p'}(G)$ and we have an isomorphism*

$$C_{\bar{G}}(A) \cong \bar{C},\ xO_{p'}(G) \mapsto xO_{p'}(C). \tag{20.1}$$

Proof. Obviously $C_G(A)O_{p'}(G)/O_{p'}(G) \subset C_{\bar{G}}(A)$. Let $xO_{p'}(G) \in C_{\bar{G}}(A)$. We consider the $O_{p'}(G) \rtimes A$ on $O_{p'}(G)xO_{p'}(G)$ defined by the left multiplication of $O_{p'}(G)$ on $xO_{p'}(G)$ and the obvious action of A on $xO_{p'}(G)$ induced by the action of A on G. Since A and $O_{p'}(G)$ have co-prime order and $O_{p'}(G)$ acts regularly on $xO_{p'}(G)$, by [3, Lemma 13.8 and Cor. 13.9], the set of all A-stable elements in $xO_{p'}(G)$ is non-empty and $C_{O_{p'}(G)}(A)$ acts transitively on it. This shows that $C_{\bar{G}}(A) \subset C_G(A)O_{p'}(G)/O_{p'}(G)$. Consequently $C_G(A)O_{p'}(G)/O_{p'}(G) = C_{\bar{G}}(A)$. By this equality and the assumption that $C_G(A) \cap O_{p'}(G) = O_{p'}(C)$, it is trivial to see that (20.1) is an isomorphism. □

Below, we always identify $C_{\bar{G}}(A)$ with $\bar{C}$ through the isomorphism (20.1) and thus $\bar{C}$ is a subgroup of $\bar{G}$.

Lemma 21. *Keep the notation and hypotheses as above. Then we have* $C_{\hat{\bar{G}}}(A) = \hat{\bar{C}}$.

Proof. Set $\bar{G}' = G'/O_{p'}(G)$. By Lemma 11, $\bar{G}'$ is a subgroup of $\hat{\bar{G}}$ such that $\mathcal{O}^*\bar{G}' = \hat{\bar{G}}$. Then replacing $\hat{G}$ by $\hat{\bar{G}}$ and G' by $\bar{G}'$ in the proof of Lemma 16, we can prove this lemma. □

Lemma 22. *Keep the notation and hypotheses as above and denote by*

$$\Gamma : \hat{C} \cong \tilde{C}$$

the isomorphism induced by Θ *(see Lemma* 19*). Then* Γ *induces an isomorphism of* $\mathcal{O}^*$*-groups*

$$\bar{\Gamma} : \hat{\bar{C}} \cong \tilde{\bar{C}}. \tag{22.1}$$

Proof. Since Γ maps $\hat{C} \cap O_{p'}(G)$ onto $O_{p'}(C)$, it induces an isomorphism of $\mathcal{O}^*$-groups

$$\bar{\Gamma} : \hat{C}O_{p'}(G)/O_{p'}(G) \cong \hat{C}/\hat{C} \cap O_{p'}(G) \cong \tilde{C}/O_{p'}(C) = \tilde{\bar{C}}.$$

By Lemma 16, $\hat{C} = C_{\hat{G}}(A)$; therefore $\hat{C}O_{p'}(G)/O_{p'}(G) = C_{\hat{G}}(A)O_{p'}(G)/O_{p'}(G) \subset C_{\hat{\bar{G}}}(A)$. On the other hand, it follows from Lemmas 20 and 21 that $\hat{C}O_{p'}(G)/O_{p'}(G)$ and $C_{\hat{\bar{G}}}(A)$ both are $\mathcal{O}^*$-groups with the $\mathcal{O}^*$-quotient $\bar{C}$. Therefore the above inclusion $C_{\hat{G}}(A)O_{p'}(G)/O_{p'}(G) \subset C_{\hat{\bar{G}}}(A)$ should be an equality. Then $\bar{\Gamma}$ is the desired isomorphism. □

Obviously the group isomorphisms Γ and $\bar{\Gamma}$ induce isomorphisms

$$\mathcal{O}_*\hat{C} \cong \mathcal{O}_*\tilde{C} \text{ and } \mathcal{O}_*\hat{\bar{C}}^\circ \cong \mathcal{O}_*\tilde{\bar{C}}^\circ.$$

We still denote these isomorphisms by Γ and $\bar{\Gamma}$ respectively.

Proposition 23. *Keep the notation and hypotheses as above. Then there is a bijection* $\pi(\hat{\bar{G}}^\circ, A) : \mathrm{Irr}(\hat{\bar{G}}^\circ, \bar{b}) \to \mathrm{Irr}(\tilde{\bar{C}}^\circ, \bar{c})$ *such that*

23.1. For any $\bar{\chi} \in \mathrm{Irr}(\hat{\bar{G}}^\circ, \bar{b})$, $\pi(\hat{\bar{G}}^\circ, A)(\bar{\chi})\circ\bar{\Gamma}^{-1}$ is the unique irreducible constituent of the restriction of $\bar{\chi}$ to $\mathcal{K} \otimes_{\mathcal{O}} \mathcal{O}_*\hat{\bar{C}}^\circ$, which occurs with a multiplicity prime to q.

23.2. If $\chi \in \mathrm{Irr}(G, b)$ corresponds to $\bar{\chi} \in \mathrm{Irr}(\bar{b})$ through the bijection (13.3), then $\pi(G, A)(\chi)$ corresponds to $\pi(\hat{\bar{G}}^\circ, A)(\bar{\chi})$ through the bijection (15.3).

Proof. In this proof, we will identify $\hat{C}$ with $\tilde{C}$ through Γ and $\hat{\bar{C}}$ with $\tilde{\bar{C}}$ through $\bar{\Gamma}$.

By our hypothesis, the map $\pi(G, A, b) : \mathrm{Irr}(G, b) \to \mathrm{Irr}(C, c)$, $\chi \mapsto \pi(G, A)(\chi)$ is a bijection. We denote by $\pi(\hat{\bar{G}}^\circ, A)$ the composition of the inverse of the bijection (13.3), $\pi(G, A, b)$ and the bijection (15.3). Take $\chi \in \mathrm{Irr}(G, b)$ and set $\psi = \pi(G, A)(\chi)$; then $\pi(\hat{\bar{G}}^\circ, A)(\bar{\chi}) = \bar{\psi}$. Then in order to prove the proposition above, it suffices to show that $\bar{\psi}$ is the unique irreducible constituent of the restriction of $\bar{\chi}$ to $\mathcal{K} \otimes_{\mathcal{O}} \mathcal{O}_*\tilde{\bar{C}}^\circ$, which occurs with a multiplicity prime to q.

We inflate χ to χ' through the canonical surjective homomorphism $G' \to G$ and ψ to ψ' through the canonical surjective homomorphism $C' \to C$. Since $\pi(G, A)(\chi) = \psi$, $\pi(G', A)(\chi') = \psi'$. We define a function $\bar{\chi}'$ on G' by setting $\bar{\chi}'(x, s) = \bar{\chi}(\overline{(x, s)})$ for any $(x, s) \in G'$. Then $\bar{\chi}'$ is an irreducible character of G' and it follows from the isomorphism (13.3) that $\chi'(x,s) = \alpha'(x,s)\bar{\chi}'(x,s)$ for any $(x,s) \in G'$. Moreover since χ' and α' are A-stable, so is $\bar{\chi}'$. Similarly the function $\bar{\psi}'$ on C' by defining $\bar{\psi}'(y,t) = \bar{\psi}(\overline{(y,t)})$ for any $(y,t) \in C'$ is also an irreducible character of C' and fulfills the equality $\psi'(y,t) = \beta'(y,t)\bar{\psi}'(y,t)$ for any $(y,t) \in C'$.

We decompose the restriction $\mathrm{Res}^{G'}_{C'}(\alpha')$ as the sum $\sum_{1\leq i\leq n}\alpha'_i$ of irreducible characters of C' and the restriction $\mathrm{Res}^{G'}_{C'}(\bar{\chi}')$ as the sum $\sum_{1\leq j\leq m}\bar{\chi}'_j$. Then $\mathrm{Res}^{G'}_{C'}(\chi') = \sum_{1\leq i\leq n}\sum_{1\leq j\leq m}\alpha'_i\bar{\chi}'_j$. Now we claim that if ψ' is an irreducible constituent of some $\alpha'_i\bar{\chi}'_j$, then

$$\alpha'_i = \beta' \quad \text{and} \quad \bar{\chi}'_j = \bar{\psi}'.$$

Since $\pi(G',A)(\chi') = \psi'$ and $\pi(G',A)(\alpha') = \beta'$, this implies that the multiplicity of $\bar{\psi}'$ in the restriction $\mathrm{Res}^{G'}_{C'}(\bar{\chi}')$ is prime to q. In particular, the multiplicity of $\bar{\psi}$ in the restriction of $\bar{\chi}$ to $\mathcal{K}\otimes_{\mathcal{O}}\mathcal{O}_*\hat{\bar{C}}^{\circ}$ is prime to q; the uniqueness of $\bar{\psi}$ follows from [2, Th. 6.13].

Suppose $\alpha'_i \neq \beta'$. Since g is a central idempotent of $\mathcal{O}C'$ and ψ' is a character of C' provided by some irreducible $\mathcal{K}C'g$-module (refer to the isomorphisms (15.1) and (15.2)), $\alpha'_i(g)\bar{\chi}'_j(g) \neq 0$ and thus $\alpha'_i(g) \neq 0$. But since α'_i is irreducible, α'_i is a character of some irreducible $\mathcal{K}C'g$-module and the restriction $\mathrm{Res}^{C'}_{O_{p'}(C)}(\alpha'_i)$ is some multiple of β. On the other hand, since $\pi(G',A)(\alpha') = \beta'$, there are α'_{i_1}, α'_{i_2}, $\cdots$, α'_{i_l} among α'_1, α'_2, $\ldots$, α'_n such that $\alpha'_{i_1} = \alpha'_{i_2} = \cdots = \alpha'_{i_l} = \beta'$, where l is the multiplicity of β' in the restriction of α' to C'. Therefore since

$$\mathrm{Res}^{C'}_{O_{p'}(C)}(\mathrm{Res}^{G'}_{C'}(\alpha')) = \mathrm{Res}^{O_{p'}(G)}_{O_{p'}(C)}(\mathrm{Res}^{G'}_{O_{p'}(G)}(\alpha')) = \mathrm{Res}^{O_{p'}(G)}_{O_{p'}(C)}(\alpha),$$

the multiplicity of β in $\mathrm{Res}^{O_{p'}(G)}_{O_{p'}(C)}(\alpha)$ is greater than the sum of l with the multiplicity of β in $\mathrm{Res}^{C'}_{O_{p'}(C)}(\alpha'_i)$ and thus strictly exceeds l. But the multiplicity of β in $\mathrm{Res}^{O_{p'}(G)}_{O_{p'}(C)}(\alpha)$ is l. So a contradiction is produced and $\alpha'_i = \beta'$.

Note that $\bar{\chi}'_j$ is an irreducible constituent of $\mathrm{Res}^{G'}_{C'}(\bar{\chi}')$ and that $\bar{\chi}$ is a character of an irreducible $\mathcal{K}\otimes_{\mathcal{O}}\mathcal{O}_*\hat{\bar{G}}^{\circ}$-module. Therefore there is a character $\bar{\chi}_j$ of some irreducible $\mathcal{K}\otimes_{\mathcal{O}}\mathcal{O}_*\hat{\bar{C}}^{\circ}$-module such that for any $(x,s) \in C'$, $\bar{\chi}'_j(x,s) = \bar{\chi}_j(x,s)$. Then by the isomorphism (15.1), $\alpha'_i\bar{\chi}'_j$ is an irreducible character of G'. So $\psi' = \alpha'_i\bar{\chi}'_j$ and $\bar{\chi}'_j = \bar{\psi}'$. □

24. A proof of Theorem 2. In order to avoid unnecessary repetition, we continue to keep all the notation from paragraph 5 to paragraph 23. First we consider the case that the block idempotent f of $\mathcal{O}O_{p'}(G)$ (see paragraph 2.1) is not stabilized by G. Then we have isomorphisms (see (6.2) and (8.1))

$$\mathrm{Ind}^G_K(\mathcal{O}Kd) \cong \mathcal{O}Gb \quad \text{and} \quad \mathrm{Ind}^{C_G(A)}_{C_K(A)}(\mathcal{O}C_K(A)e) \cong \mathcal{O}C_G(A)c.$$

Suppose that the $\mathcal{O}(C_K(A)\times K)$-module N induces a Morita equivalence between $\mathcal{O}Kd$ and $\mathcal{O}C_K(A)e$, which induces the Glauberman correspondence of characters from $\mathrm{Irr}(K,d)$ to $\mathrm{Irr}(C_K(A),e)$. Then it is trivial to see that the $\mathcal{O}(C_G(A)\times G)$-module $M = \mathcal{O}(C_G(A)\times G)\otimes_{\mathcal{O}(C_K(A)\times K)} N$ induces a Morita equivalence between $\mathcal{O}Gb$ and $\mathcal{O}Cc$. Take $\chi \in \mathrm{Irr}(K,d)$. Let W be a $\mathcal{K}Kd$-module providing the character χ. Since

$$\begin{aligned}(\mathcal{K}\otimes_{\mathcal{O}} M)\otimes_{\mathcal{K}G}(\mathcal{K}G\otimes_{\mathcal{K}K} W) &\cong \Big(\mathcal{K}C_G(A)\otimes_{\mathcal{K}C_K(A)}(\mathcal{K}\otimes_{\mathcal{O}} N)\otimes_{\mathcal{K}K}\mathcal{K}G\Big)\\ &\qquad\otimes_{\mathcal{K}G}(\mathcal{K}G\otimes_{\mathcal{K}K} W)\\ &\cong \mathcal{K}C_G(A)\otimes_{\mathcal{K}C_K(A)}((\mathcal{K}\otimes_{\mathcal{O}} N)\otimes_{\mathcal{K}K} W)\end{aligned}$$

and the character of $(\mathcal{K}\otimes_{\mathcal{O}}N)\otimes_{\mathcal{K}K}W$ is $\pi(K,A)(\chi)$, then by [5, Th. 2.3], the Morita equivalence induced by M induces the Glauberman correspondence of characters from $\mathrm{Irr}(G,b)$ to $\mathrm{Irr}(C,c)$. Here we note that N and M have common vertexes and source modules.

Therefore in order to prove Theorem 2, we can assume without loss of generality that f is stabilized by G. Then we have the algebra isomorphisms (see (13.2) and (15.2))

$$\mathcal{O}Gb\cong\mathcal{O}O_{p'}(G)f\otimes_{\mathcal{O}}\mathcal{O}_*\hat{\bar{G}}^\circ\bar{b}\quad\text{and}\quad\mathcal{O}Cc\cong\mathcal{O}O_{p'}(C)g\otimes_{\mathcal{O}}\mathcal{O}_*\tilde{\bar{C}}^\circ\bar{c},$$

group isomorphisms (see Lemma 22)

$$\Gamma:\hat{C}\cong\tilde{C}\quad\text{and}\quad\bar{\Gamma}:\hat{\bar{C}}\cong\tilde{\bar{C}}$$

and isomorphisms of algebras

$$\Gamma:\mathcal{O}_*\hat{C}\cong\mathcal{O}_*\tilde{C}\quad\text{and}\quad\bar{\Gamma}:\mathcal{O}_*\hat{\bar{C}}^\circ\cong\mathcal{O}_*\tilde{\bar{C}}^\circ.$$

Let i be a primitive idempotent of $\mathcal{O}O_{p'}(G)f$ and j be a primitive idempotent of $\mathcal{O}O_{p'}(C)g$. It is well known that the $\mathcal{O}_*\hat{\bar{G}}^\circ\bar{b}\otimes_{\mathcal{O}}\mathcal{O}Gb$-module $i\mathcal{O}O_{p'}(G)\otimes_{\mathcal{O}}\mathcal{O}_*\hat{\bar{G}}^\circ\bar{b}$ induces a Morita equivalence between $\mathcal{O}Gb$ and $\mathcal{O}_*\hat{\bar{G}}^\circ\bar{b}$, which induces the bijection (13.3) of characters, and that the $\mathcal{O}Cc\otimes_{\mathcal{O}}\mathcal{O}_*\tilde{\bar{C}}^\circ\bar{c}$-module $\mathcal{O}O_{p'}(C)j\otimes_{\mathcal{O}}\mathcal{O}_*\tilde{\bar{C}}^\circ\bar{c}$ induces a Morita equivalence between $\mathcal{O}_*\tilde{\bar{C}}^\circ\bar{c}$ and $\mathcal{O}Cc$, which induces the inverse of the bijection (15.3) of characters. Moreover since we assume $G=O_{p'}(G)C_G(A)$, $\hat{\bar{C}}=\hat{\bar{G}}$ and then it follows from Proposition 23 that $\bar{\Gamma}$ maps $\mathcal{O}_*\hat{\bar{G}}^\circ\bar{b}$ onto $\mathcal{O}_*\tilde{\bar{C}}^\circ\bar{c}$ isomorphically; in this case, it is clear that the $\mathcal{O}_*\tilde{\bar{C}}^\circ\otimes_{\mathcal{O}}\mathcal{O}_*\hat{\bar{G}}^\circ$-module ${}_{\bar{\Gamma}^{-1}}(\mathcal{O}_*\hat{\bar{G}}^\circ\bar{b})$, which is defined by the equality $\Big((1\otimes x)\otimes(1\otimes y)\Big)\cdot a=\bar{\Gamma}^{-1}(1\otimes x)a(1\otimes y^{-1})$ for any $x\in\tilde{\bar{C}}$, $y\in\hat{\bar{G}}$ and $a\in\mathcal{O}_*\hat{\bar{G}}^\circ\bar{b}$, induces a Morita equivalence between $\mathcal{O}_*\hat{\bar{G}}^\circ\bar{b}$ and $\mathcal{O}_*\tilde{\bar{C}}^\circ\bar{c}$. Then we can easily see that the $\mathcal{O}(C\times G)$-module

$$\begin{aligned}(\mathcal{O}O_{p'}(C)j\otimes_{\mathcal{O}}\mathcal{O}_*\tilde{\bar{C}}^\circ\bar{c})\otimes_{\mathcal{O}_*\tilde{\bar{C}}^\circ}\Big(i\mathcal{O}O_{p'}(G)\otimes_{\mathcal{O}}{}_{\bar{\Gamma}^{-1}}(\mathcal{O}_*\hat{\bar{G}}^\circ\bar{b})\Big)\\ \cong\mathcal{O}O_{p'}(C)j\otimes_{\mathcal{O}}{}_{\bar{\Gamma}^{-1}}(\mathcal{O}_*\hat{\bar{G}}^\circ\bar{b})\otimes_{\mathcal{O}}i\mathcal{O}O_{p'}(G)\end{aligned}$$

induces a Morita equivalence between $\mathcal{O}Gb$ and $\mathcal{O}Cc$ (note that $\mathcal{O}(C\times G)\cong\mathcal{O}C\otimes_{\mathcal{O}}\mathcal{O}G$); moreover by Proposition 23, the bijection of characters induced by this Morita equivalence coincides with the Glauberman correspondence of characters between $\mathrm{Irr}(G,b)$ and $\mathrm{Irr}(C,c)$.

Finally, employing the fifth paragraph in the proof of 1.1 in the reduced case in [5], we can prove that $\mathcal{O}O_{p'}(C)j\otimes_{\mathcal{O}}{}_{\bar{\Gamma}^{-1}}(\mathcal{O}_*\hat{\bar{G}}^\circ\bar{b})\otimes_{\mathcal{O}}i\mathcal{O}O_{p'}(G)$ has $\Delta(P)$ as a vertex.

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References

[1] M. Broué, Isométries parfaites, types de blocs, catégories dérivées. Astérisque No. **181-182** (1990), 61–92. MR1051243 (91i:20006)

[2] E. Dade, A new approach to Glauberman's correspondence. J. Algebra **270** (2003), no. 2, 583–628. MR2019631 (2004j:20010)

[3] Y. Fan and L. Puig, On blocks with nilpotent coefficient extensions. Algebr. Represent. Theory **1** (1998), no. 1, 27–73. MR1654602 (2000e:20019a)

[4] G. Glauberman, Correspondences of characters for relatively prime operator groups. Can. J. Math. **20** (1968), 1465-1488. MR0232866 (38:1189)

[5] M. E. Harris and M. Linckelmann, On the Glauberman and Watanabe correspondences for blocks of finite p-solvable groups. Trans. Amer. Math. Soc. **354** (2002), no. 9, 3435–3453. MR1911507 (2003c:20008)

[6] I. M. Isaacs, Character theory of finite groups. Academic Press, New York, 1976. MR0460423 (57:417)

[7] L. Puig, Nilpotent blocks and their source algebras. Invent. Math. **93** (1988), 77-116. MR943924 (89e:20023)

[8] L. Puig, Notes on $\mathcal{O}^*$-groups, preprint, 1998.

[9] A. Watanabe, The Glauberman character correspondence and perfect isometries for blocks of finite groups. J. Algebra **216** (1999), 548-565. MR1692989 (2000f:20015)

[10] J. Thévenaz, G-algebras and modular representation theory. Oxford Mathematical Monographs, Oxford Science Publications, The Clarendon Press, Oxford University Press, New York, 1995. MR1365077 (96j:20017)

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