PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 139, Number 4, April 2011, Pages 1291–1298
S 0002-9939(2010)10562-2
Article electronically published on September 2, 2010

# ON THE RESTRICTION OF THE HERMITIAN EISENSTEIN SERIES AND ITS APPLICATIONS

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(Communicated by Wen-Ching Winnie Li)

Abstract. We introduce a simple construction of a Siegel cusp form obtained by taking the difference between the Siegel Eisenstein series and the restricted Hermitian Eisenstein series. In addition, we present applications of the Siegel cusp form.

## 1. Introduction

In the present paper, we introduce a simple construction of a Siegel cusp form of degree 2. The Siegel cusp form is realized as the difference between the Siegel Eisenstein series and the restriction of the Hermitian Eisenstein series to the Siegel half-space.

The proposed construction has an advantage because the Fourier coefficient is explicitly computable and has a number of applications. As the first application, we introduce a new description of Cohen's function (Theorem 5.2). Second, explicit formulas for the Fourier coefficients of Igusa's cusp form of weights 10 and 12 are presented (Corollary 5.5). Finally, we refer to the $p$-adic Siegel cusp forms.

## 2. Hermitian and Siegel modular forms

We start by recalling the definition and basic characteristics of Hermitian and Siegel modular forms.

The *Hermitian half-space* of degree 2 is defined as

$$\mathbb{H}_2 := \left\{ Z \in M_2(\mathbb{C}) \;\middle|\; \tfrac{1}{2i}(Z - {}^t\overline{Z}) > 0 \right\}$$

and contains the *Siegel half-space* of degree 2,

$$\mathbb{S}_2 := \{\, Z \in \mathbb{H}_2 \mid Z = {}^tZ \,\},$$

as a submanifold. Let $\mathbb{K}$ be an imaginary quadratic number field with discriminant $d_{\mathbb{K}}$ and ring of integers $\mathcal{O} = \mathcal{O}_{\mathbb{K}}$. Then, the *Hermitian modular group* of degree 2 over $\mathbb{K}$ is defined as

$$\Gamma_2(\mathcal{O}) := \{M \in M_4(\mathcal{O}) \mid {}^t\overline{M} J_2 M = J_2\}, \quad J_2 = \begin{pmatrix} 0 & -1_2 \\ 1_2 & 0 \end{pmatrix}.$$

Received by the editors December 22, 2009 and, in revised form, April 16, 2010 and April 28, 2010.

2010 *Mathematics Subject Classification.* Primary 11F55; Secondary 11F46.

*Key words and phrases.* Siegel modular forms, Hermitian modular forms.

The group $\Gamma_2(\mathcal{O})$ acts on $\mathbb{H}_2$ by fractional linear transformation $Z \mapsto M\langle Z\rangle := (AZ+B)(CZ+D)^{-1}$, $M = \left(\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right) \in \Gamma_2(\mathcal{O})$. If $\nu$ is an abelian character of $\Gamma_2(\mathcal{O})$, then the space $M_k(\Gamma_2(\mathcal{O}), \nu)$ of Hermitian modular forms of weight $k$ and character $\nu$ with respect to $\Gamma_2(\mathcal{O})$ consists of all holomorphic functions $F : \mathbb{H}_2 \longrightarrow \mathbb{C}$ satisfying

$$F(M\langle Z\rangle) = \nu(M)\det(CZ+D)^k F(Z)$$

for all $M = \left(\begin{smallmatrix} A & B \\ C & D \end{smallmatrix}\right) \in \Gamma_2(\mathcal{O})$. We denote by $M_k(\Gamma_2(\mathcal{O}), \nu)^{sym}$ the subspace consisting of *symmetric* Hermitian modular forms characterized by

$$F({}^tZ) = F(Z).$$

A typical example of a symmetric Hermitian modular form is given by the Hermitian Eisenstein series

$$E_{k,\mathbb{K}}(Z) := \sum_{M=\left(\begin{smallmatrix} * & * \\ C & D \end{smallmatrix}\right) : \{\left(\begin{smallmatrix} * & * \\ 0 & * \end{smallmatrix}\right)\}\backslash\Gamma_2(\mathcal{O})} (\det M)^{k/2}\det(CZ+D)^{-k} \in M_k(\Gamma_2(\mathcal{O}), \det{}^{-k/2})^{sym}$$

for even $k > 4$. Moreover, $E_{4,\mathbb{K}} \in M_4(\Gamma_2(\mathcal{O}), \det{}^{-2})^{sym}$ is constructed as the Maass lift (cf. Krieg [9]).

Each $F \in M_k(\Gamma_2(\mathcal{O}), \det{}^l)$ possesses a Fourier expansion of the form

$$F(Z) = \sum_{0 \le H \in \Lambda_2(\mathbb{K})} A(H; F)\exp[2\pi i \mathrm{tr}(HZ)],$$

where $\Lambda_2(\mathbb{K})$ is a lattice in $Her_2(\mathbb{C})$ defined by

$$\Lambda_2(\mathbb{K}) := \left\{ \begin{pmatrix} a & t \\ \bar{t} & b \end{pmatrix} \in M_2(\mathbb{K}) \mid a, b \in \mathbb{Z},\ \sqrt{d_{\mathbb{K}}}\, t \in \mathcal{O} \right\}.$$

The *Siegel modular group* $\Gamma_2 := Sp_2(\mathbb{Z})$ also acts on $\mathbb{S}_2$ by fractional linear transformation. Let $\Gamma$ be a subgroup of $\Gamma_2$ of finite index. We denote by $M_k(\Gamma)$(resp. $S_k(\Gamma)$) the space of Siegel modular forms (resp. Siegel cusp forms) of weight $k$ with respect to $\Gamma$. For any $F \in M_k(\Gamma_2(\mathcal{O}), \det{}^{-k/2})$($k$: even), the restriction $F|_{\mathbb{S}_2}$ becomes a Siegel modular form in $M_k(\Gamma_2)$ (cf. Dern-Krieg [3], Corollary 1).

Each $F \in M_k(\Gamma_2)$ admits a Fourier expansion of the form

$$F(Z) = \sum_{0 \le T \in \Lambda_2} A(T; F)\exp[2\pi i \mathrm{tr}(TZ)],$$

where

$$\Lambda_2 = \left\{ \begin{pmatrix} a & \frac{c}{2} \\ \frac{c}{2} & b \end{pmatrix} \in M_2(\mathbb{Q}) \middle| a, b, c \in \mathbb{Z} \right\}.$$

The *Maass space* $\mathcal{M}(\Gamma_2)$ consists of Siegel modular forms $F$ characterized by

$$A(T; F) = A\left(\begin{pmatrix} a & \frac{c}{2} \\ \frac{c}{2} & b \end{pmatrix}; F\right) = \sum_{0 < d \mid \varepsilon(T)} d^{k-1} A\left(\begin{pmatrix} 1 & \frac{c}{2d} \\ \frac{c}{2d} & \frac{ab}{d^2} \end{pmatrix}; F\right),$$

where

$$\varepsilon(T) = \varepsilon\left(\begin{pmatrix} a & \frac{c}{2} \\ \frac{c}{2} & b \end{pmatrix}\right) := \max\{l \in \mathbb{N} \mid l^{-1}T \in \Lambda_2\} = \gcd(a, b, c).$$

An example of Siegel modular form in the Maass space is the Siegel Eisenstein series

$$E_k(Z) := \sum_{\left(\begin{smallmatrix} * & * \\ C & D \end{smallmatrix}\right) : \left\{\left(\begin{smallmatrix} * & * \\ 0 & * \end{smallmatrix}\right)\right\} \backslash \Gamma_2} \det(CZ+D)^{-k}.$$

If $k \geq 4$ is even, then $E_k \in \mathcal{M}_k(\Gamma_2)$.

**Proposition 2.1.** (Dern-Krieg, [3], Theorem 1, Corollary 2). *Assume that $\mathbb{K}$ is an imaginary quadratic field of class number 1. For an even integer $k$ with $k \geq 4$ and the Hermitian Eisenstein series $E_{k,\mathbb{K}}$, we have*

$$E_{k,\mathbb{K}}|_{\mathbb{S}_2} \in \mathcal{M}_k(\Gamma_2).$$

## 3. Fourier coefficient of Eisenstein series

In this section, we present explicit formulas for the Fourier coefficient of Eisenstein series for the Hermitian modular group and the Siegel modular group of degree 2.

3.1. **The Hermitian modular case.** In [9], Krieg presented an explicit formula for the Fourier coefficient $A(H; E_{k,\mathbb{K}})$ for the case in which the class number of $\mathbb{K}$ is 1. To describe this, we introduce Krieg's function $G_{\mathbb{K}}(s,N) : \mathbb{N} \times \mathbb{Z} \longrightarrow \mathbb{Q}$, which is defined as

$$G_{\mathbb{K}}(s,N) = \begin{cases} \dfrac{1}{1-\chi_{\mathbb{K}}(N)}(\sigma_{s,\chi_{\mathbb{K}}}(N) - \sigma^*_{s,\chi_{\mathbb{K}}}(N)) & \text{if } N > 0,\ \chi_{\mathbb{K}}(N) \neq 1, \\ -\dfrac{B_{s+1,\chi_{\mathbb{K}}}}{2(s+1)} & \text{if } N = 0, \\ 0 & \text{otherwise,} \end{cases}$$

where $\chi_{\mathbb{K}}$ is the Kronecker character of $\mathbb{K}$, $\chi_{\mathbb{K}} = (\frac{d_{\mathbb{K}}}{*}) = \chi_{d_{\mathbb{K}}}$,

$$\sigma_{s,\chi_{\mathbb{K}}}(N) := \sum_{0<d|N} \chi_{\mathbb{K}}(d) d^s, \qquad \sigma^*_{s,\chi_{\mathbb{K}}}(N) := \sum_{0<d|N} \chi_{\mathbb{K}}(N/d) d^s,$$

and $B_{m,\chi}$ is the generalized Bernoulli number.

**Theorem 3.1.** (Krieg [9], Dern-Krieg [3]). *Assume that the class number of $\mathbb{K}$ is 1. Then, the Fourier coefficient $A(H; E_{k,\mathbb{K}})$ $(k \geq 4)$ is as follows:*
(1) *If $H > 0$, then*

$$A(H; E_{k,\mathbb{K}}) = \frac{4k(k-1)}{B_k \cdot B_{k-1,\chi_{\mathbb{K}}}} \sum_{0<d|\varepsilon(H)} d^{k-1} G_{\mathbb{K}}\Big(k-2, \frac{|d_{\mathbb{K}}|\det(H)}{d^2}\Big)$$

*where $\varepsilon(H) := \max\{l \in \mathbb{N} \,|\, l^{-1}H \in \Lambda_2(\mathbb{K})\}$.*
(2) *If $\operatorname{rank}(H) = 1$, then*

$$A(H; E_{k,\mathbb{K}}) = -\frac{2k}{B_k}\sigma_{k-1}(\varepsilon(H)).$$

(3) $A(0_2; E_{k,\mathbb{K}}) = 1$.

3.2. **The Siegel modular case.** An explicit formula for the Fourier coefficient $A(T;E_k)$ ($E_k$: Siegel Eisenstein series) was first obtained by Kaufhold [7] and Maass [10]. Second, Eichler and Zagier presented an explicit formula obtained by means of Cohen's function $H(r,N)$ (cf. [4]).

Cohen [1] introduced a function $H:\mathbb{N}\times\mathbb{Z}\longrightarrow\mathbb{Q}$ given by

$$H(r,N)=L(1-r,\chi_D)\sum_{0<d|f}\mu(d)\chi_D(d)d^{r-1}\sigma_{2r-1}(f/d)$$

in the case that $r$ and $N$ satisfy $(-1)^rN=Df^2$, where $D$ is a fundamental discriminant and $f\in\mathbb{N}$. Here, $L(s,\chi)$ is the $L$-function with character $\chi$, and $\mu$ is the Möbius function. Refer to [1] for the precise definition of $H(r,N)$.

**Theorem 3.2.** (Eichler and Zagier [4]). *The Fourier coefficient $A(T;E_k)$ is given as follows:*
(1) *If $T>0$, then*

$$A(T;E_k)=\frac{4k(k-1)}{B_k\cdot B_{2k-2}}\sum_{0<d|\varepsilon(T)}d^{k-1}H\Big(k-1,\frac{4\det(T)}{d^2}\Big).$$

(2) *If* $\operatorname{rank}(T)=1$, *then*

$$A(T;E_k)=-\frac{2k}{B_k}\sigma_{k-1}(\varepsilon(T)).$$

(3) $A(0_2;E_k)=1$.

## 4. Construction of the Siegel cusp form and its application

In the remainder of the present paper, we assume that

$$\mathbb{K}=\mathbb{Q}(i):\quad \text{the Gaussian field.}$$

In this case,

$$\mathcal{O}=\mathbb{Z}[i]=\{a+bi\mid a,b\in\mathbb{Z}\},\quad \chi_{\mathbb{K}}=\chi_{-4}=(\tfrac{-4}{*}).$$

Moreover, the lattice $\Lambda_2(\mathbb{K})$ appearing in the Fourier expansion of Hermitian modular forms is given as

$$\Lambda_2(\mathbb{Q}(i))=\left\{\begin{pmatrix} a & \frac{c+di}{2}\\ \frac{c-di}{2} & b\end{pmatrix}\in Her_2(\mathbb{Q}(i))\;\middle|\; a,b,c,d\in\mathbb{Z}\right\}.$$

4.1. **Restriction of the Hermitian Eisenstein series.** As mentioned in Section 2, the restriction of $F\in M_k(\Gamma_2(\mathbb{Z}[i]),\det^{k/2})$ to the Siegel half-space $\mathbb{S}_2$ becomes a Siegel modular form, namely,

$$F|_{\mathbb{S}_2}\in M_k(\Gamma_2).$$

Moreover, the relation between $A(T;F|_{\mathbb{S}_2})$ and $A(H;F)$ is

$$A(T;F|_{\mathbb{S}_2})=A\left(\begin{pmatrix} a & \frac{c}{2}\\ \frac{c}{2} & b\end{pmatrix};F|_{\mathbb{S}_2}\right)=\sum_{\substack{s\in\mathbb{Z}\\ s^2\le 4ab-c^2=4\det(T)}}A\left(\begin{pmatrix} a & \frac{c+si}{2}\\ \frac{c-si}{2} & b\end{pmatrix};F\right).$$

For even $k\ge 4$, we consider the difference between Siegel Eisenstein series $E_k$ and the restriction of Hermitian Eisenstein series $E_{k,\mathbb{Q}(i)}$:

$$(*)\qquad f_k:=E_k-E_{k,\mathbb{Q}(i)}|_{\mathbb{S}_2}$$

**Lemma 4.1.** *The Siegel modular form $f_k$ defined above is a cusp form in the Maass space $\mathcal{M}_k(\Gamma_2)$:*

$$f_k \in \mathcal{M}_k(\Gamma_2) \cap S_k(\Gamma_2).$$

*Proof.* By Proposition 2.1, we have $f_k \in \mathcal{M}_k(\Gamma_2)$. (Note that the class number of $\mathbb{Q}(i)$ is 1.) It is known that the Maass space $\mathcal{M}_k(\Gamma_2)$ contains only Eisenstein series and cusp forms (cf. [4]). Therefore, it is enough to consider the zero Fourier coefficient $A(0_2; f_k)$. Since $A(0_2; E_k) = A(0_2; E_{k,\mathbb{Q}(i)}|_{\mathbb{S}_2}) = 1$, we have $A(0_2, f_k) = 0$. This shows that $f_k \in S_k(\Gamma_2)$. □

**Theorem 4.2.** *Let $k$ be an even integer such that $k \geq 4$. Then there exists a Siegel cusp form $F_k$ contained in the Maass space $\mathcal{M}_k(\Gamma_2)$ such that the Fourier coefficient $A(T; F_k)$ is given as*

$$A(T; F_k) = \sum_{0<d|\varepsilon(T)} d^{k-1}\alpha_k(4\det(T)/d^2),$$

$$\alpha_k(N) = H(k-1, N) - \frac{B_{2k-2}}{B_{k-1,\chi_{-4}}} \sum_{\substack{s\in\mathbb{Z} \\ s^2\leq N}} G_{\mathbb{Q}(i)}(k-2, N-s^2).$$

*Proof.* If we normalize $f_k$ as

$$F_k := \frac{B_k \cdot B_{2k-2}}{4k(k-1)} f_k,$$

then $F_k \in \mathcal{M}_k(\Gamma_2) \cap S_k(\Gamma_2)$ satisfies the required properties. □

*Remark* 4.3. As described later, the Siegel cusp form $F_k$ vanishes identically for small $k$ (see the proof of Theorem 5.2). We shall refer to the non-vanishing property of $F_k$ here. We fix $T = \begin{pmatrix} 1 & 1/2 \\ 1/2 & 1 \end{pmatrix} \in \Lambda_2$. Then, by Theorem 4.2, the corresponding Fourier coefficient is given by

$$A(T; F_k) = -\frac{B_{k-1,\chi_{-3}}}{k-1} + \frac{B_{2k-2}}{B_{k-1,\chi_{-4}}} \left\{(3^{k-2}-1) + 2(2^{k-2}-1)\right\}.$$

We consider the asymptotic approximation of the right side as $k \to \infty$. In general, we have

$$|B_{2m}| \sim 4\sqrt{\pi m}\left(\frac{m}{\pi e}\right)^{2m} \quad (m \to \infty)$$

and

$$|B_{2m-1,\chi}| \sim \frac{2}{\sqrt{f}}\left(\frac{f}{2\pi}\right)^{2m-1}(2m-1)! \quad (m \to \infty),$$

where $f = f_\chi$ is the conductor of $\chi$ (cf. [12], Exercises 4.3(b)). From these approximations, we see that the Fourier coefficient $A(T; F_k)$ does not vanish for sufficiently large $k$.

## 5. Applications

5.1. **Cohen's function and Krieg's function.** As an application of Theorem 4.2, we present a new description of Cohen's function.

**Proposition 5.1.** (Cohen [1], Proposition 4.2). *Set*

$$s_2(n) := \sum_{d|n}((n/d)^2 - 2d^2)(\tfrac{-4}{d}) \text{ for } n > 0,$$

$$s_2(0) = 1/2,$$

$$s_4(n) = \sum_{d|n}((n/d)^4 - 2d^4)(\tfrac{-4}{d}) + (3/2)\sum_{n=x^2+y^2}(x^4 - 6x^2y^2 + y^4) \text{ for } n > 0,$$

$$s_4(0) = -5/2.$$

*Then*

$$H(3,N) = -\frac{1}{126}\sum_{\substack{s\in\mathbb{Z}\\ s^2\le N}} s_2(N-s^2),$$

$$H(5,N) = \frac{1}{330}\sum_{\substack{s\in\mathbb{Z}\\ s^2\le N}} s_4(N-s^2).$$

Using Theorem 4.2, we present a new description of $H(r,N)$.

**Theorem 5.2.** *Let $G_{\mathbb{Q}(i)}(s,N)$ be Krieg's function for $\mathbb{K} = \mathbb{Q}(i)$. Then,*

$$H(3,N) = \frac{1}{63}\sum_{\substack{s\in\mathbb{Z}\\ s^2\le N}} G_{\mathbb{Q}(i)}(2, N-s^2),$$

$$H(5,N) = -\frac{1}{165}\sum_{\substack{s\in\mathbb{Z}\\ s^2\le N}} G_{\mathbb{Q}(i)}(4, N-s^2),$$

$$H(7,N) = \frac{1}{183}\sum_{\substack{s\in\mathbb{Z}\\ s^2\le N}} G_{\mathbb{Q}(i)}(6, N-s^2).$$

*Proof.* The fact that $S_k(\Gamma_2) = \{0\}$ for $k = 4,6,8$ implies that $F_k \equiv 0$ if $k = 4,6,8$, where $F_k$ is the Siegel cusp form given in Theorem 4.2. Therefore, we obtain

$$H(k-1,N) = \frac{B_{2k-2}}{B_{k-1,\chi_{-4}}}\sum_{\substack{s\in\mathbb{Z}\\ s^2\le N}} G_{\mathbb{Q}(i)}(k-2, N-s^2)$$

for $k = 4,6,8$. Thus, we obtain the required formulas. □

5.2. **Igusa's cusp forms.** In [5], Igusa studied the structure of the graded ring of Siegel modular forms of degree 2.

**Theorem 5.3** (Igusa [5])**.** *There exist two Siegel cusp forms $X_{10}$ and $X_{12}$ of weights 10 and 12, respectively, and the graded ring $\bigoplus_{k\in 2\mathbb{Z}} M_k(\Gamma_2)$ is generated by four Siegel modular forms $E_4$, $E_6$, $X_{10}$, and $X_{12}$, where $E_k$ is the Siegel Eisenstein series:*

$$\bigoplus_{k\in 2\mathbb{Z}} M_k(\Gamma_2) = \mathbb{C}[E_4, E_6, X_{10}, X_{12}] \ \ (\textit{polynomial ring}).$$

If $k = 10, 12$, then the Siegel cusp form of weight $k$ is uniquely determined up to constant. Then, $X_{10}$ and $X_{12}$ are normalized as

$$A\left(\begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix}; X_{10}\right) = A\left(\begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix}; X_{12}\right) = 1.$$

Then, all of the Fourier coefficients of $X_k$ $(k = 10, 12)$ are rational integers (cf. Igusa [6]).

**Theorem 5.4.** *Let $f_k$ be the Siegel cusp form defined in $(*)$. Then,*

$$f_k = E_k - E_{k,\mathbb{Q}(i)}|_{\mathbb{S}_2} = c_k \cdot X_k \qquad (k = 10, 12),$$

*where*

$$c_{10} = -\frac{2^{14} \cdot 3^4 \cdot 5^2 \cdot 7 \cdot 11}{277 \cdot 43867}, \quad c_{12} = -\frac{2^{14} \cdot 3^6 \cdot 5^6 \cdot 7^2 \cdot 13}{19 \cdot 131 \cdot 593 \cdot 691 \cdot 2659}.$$

*Proof.* Since $S_k(\Gamma_2) = \mathbb{C} \cdot X_k$ for $k = 10, 12$ (Theorem 5.3), $f_k = c_k \cdot X_k$ for some constant $c_k$. The explicit values of $c_k$ are obtained by direct calculation of the Fourier coefficients of both sides. □

**Corollary 5.5.** *For a primitive $T \in \Lambda_2$ (i.e., $\varepsilon(T) = 1$), we have*

$$A(T; X_{10}) = -\frac{5263}{12800} H(9, 4\det(T)) - \frac{43867}{12096000} \sum_{\substack{s \in \mathbb{Z} \\ s^2 \le 4\det(T)}} G_{\mathbb{Q}(i)}(8, 4\det(T) - s^2)$$

$$A(T; X_{12}) = \frac{1161983}{151200000} H(11, 4\det(T)) - \frac{77683}{453600000} \sum_{\substack{s \in \mathbb{Z} \\ s^2 \le 4\det(T)}} G_{\mathbb{Q}(i)}(10, 4\det(T) - s^2).$$

*Proof.* By Theorems 4.2, 5.4, we have

$$A(T; X_k) = c_k^{-1} \cdot \frac{4k(k-1)}{B_k \cdot B_{2k-2}} \alpha_k(4\det(T))$$
$$= \frac{4k(k-1)}{c_k \cdot B_k \cdot B_{2k-2}} \Big\{ H(k-1, 4\det(T)) - \frac{B_{2k-2}}{B_{k-1,\chi_{-4}}} \sum_{\substack{s \in \mathbb{Z} \\ s^2 \le 4\det(T)}} G_{\mathbb{Q}(i)}(k-2, 4\det(T) - s^2) \Big\}.$$

□

*Remark* 5.6. ***p*-adic Siegel cusp forms:** Combining the results from [8] and [11] we immediately see that for a prime $p \equiv 3 \pmod 4$ and $k_m := 2 + (p-1)p^m$ the sequence $f_{k_m}$ converges $p$-adically to a true modular form $\widetilde{f}$ for $\Gamma_0(p)$. Based on some numerical evidence we conjecture that $\widetilde{f}$ is a cusp form.

## Acknowledgement

The authors thank the referee for many valuable comments.

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