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CORRIGENDUM ON "ALMOST AUTOMORPHIC SOLUTIONS OF SEMILINEAR EVOLUTION EQUATIONS"

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ABSTRACT. In this paper we correct an error contained in our paper *Almost automorphic solutions of semilinear evolution equations*, Proceedings of the American Mathematical Society 133 (2005), 2401-2408.

We are grateful to En-Wei Zheng and Qin He for pointing out a small error in the proof of Proposition 3.3 in [2]. It is easy to fix the proof, provided we modify the statement of Proposition 3.3 by replacing $AA(X)$ by the slightly smaller space $AA_u(X)$, $(X, \|\cdot\|)$ being a Banach space. This restriction seems not to restrict the existing or potential applications in partial differential equations. Here are the definitions. A function $f \in BC(\mathbb{R}; X)$ is said to belong to $AA(X)$ [resp., $AA_u(X)$] if for every sequence of real numbers (s'_n) there exists a subsequence (s_n) such that

$$\lim_{n\to\infty} f(t+s_n) = g(t), \ \lim_{n\to\infty} g(t-s_n) = f(t)$$

pointwise on $\mathbb{R}$ [resp., uniformly on compacta of $\mathbb{R}$].

In [2] we asserted that a particular sequence in the space $BC^\delta(\mathbb{R}, Y)$ converged uniformly on $\mathbb{R}$, but our proof (based on the Arzelà-Ascoli theorem) only established uniform convergence on compacta. We treated $BC^\delta(\mathbb{R}, Y)$ as a Banach space under the norm $|\cdot|_{\delta,Y}$ for $0 \leq \delta \leq 1$. But it can also be viewed as a locally convex Fréchet space under the invariant metric

$$\rho(f,g) = \sum_{n=1}^{\infty} 2^{-n} \frac{\rho_n(f,g)}{1+\rho_n(f,g)},$$

where, for $h = f - g$,

$$\rho_n\,(f,g) = \rho_n(h,0) = \|h\|_{C[-n,n]} + \delta \sup\{\frac{|h(t)-h(s)|}{|t-s|^\delta} : t, s \in [-n,n], t \neq s\}.$$

Let $Y_{(\delta)}$ be the Fréchet space $(BC^\delta(\mathbb{R}, Y), \rho)$. This gives $Y_{(\delta)}$ the compact-open topology, in which convergence of a sequence means uniform convergence on compacta of $\mathbb{R}$ of the functions and their $\delta-$Hölder difference quotients. Then Proposition 3.3 [2] is valid if

(1) $$BC^\delta(\mathbb{R}, Y), \ BC(\mathbb{R}, X) \ \text{ are replaced by } Y_{(\delta)}, \ Y_{(0)},$$

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respectively. Since the Schauder fixed point theorem holds in locally convex spaces, Proposition 3.4 [2] holds in $AA_u(X)$, again assuming (1). The main result now follows, because $AA_u(X)$ is complete in the $Y_{(s)}$ topology. This follows from ([1], Remark p. 16).

Thus in summary we state our main results in [2] as follows.

Consider the semilinear differential equation

$$x'(t) = Ax(t) + F(t, x(t)), \quad t \in \mathbb{R}, \tag{2}$$

where the linear operator $A : D(A) \subset X \to X$ generates an exponentially stable C_0-semigroup $(T(t))_{t\geq 0}$ such that

$$F(t, x) = P(t)Q(x), \quad t \in \mathbb{R}, \quad x \in X,$$

where $P(t) \in AA_u(\mathbb{Z})$ for each $t \in \mathbb{R}$ with $\mathbb{Z} = B(X, Y)$; P is continuous from $\mathbb{R}$ to $AA_u(\mathbb{Z})$, and $Q : BC(\mathbb{R}, X) \to BC(\mathbb{R}, X)$ is continuous and satisfies the estimate

$$\|Q\varphi\|_\infty \leq \mathcal{M}(\|\varphi\|_\infty)$$

and $\mathcal{M} \in C(\mathbb{R}^+, \mathbb{R}^+)$ satisfies

$$\lim_{r\to\infty} \frac{\mathcal{M}(r)}{r} = 0.$$

Then we have

Proposition. *$BC^\delta(\mathbb{R}, Y)$ is compactly contained in $BC(\mathbb{R}, X)$; in other words, the canonical injection $id : BC^\delta(\mathbb{R}, Y) \to BC^\delta(\mathbb{R}, X)$ is compact, which implies that*

$$id : BC^\delta(\mathbb{R}, Y) \cap AA_u(X) \to AA_u(X)$$

is compact too.

Theorem. *Under the above assumptions,* (2) *has a mild solution in $AA_u(X)$.*

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