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HOMOGENEOUS IDEALS ASSOCIATED TO A SMOOTH SUBVARIETY

YU-HAN LIU

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ABSTRACT. In this paper we show that a smooth subvariety Z on an odd-dimensional complex projective smooth variety X is determined by the sufficiently many Hodge conjectures it solves on hypersurfaces Y on X of high degrees containing Z.

1. INTRODUCTION

1.1. In this article we consider the following situation: Let X be a complex projective smooth variety of dimension $2n+1$ with a given ample line bundle $\mathcal{O}(1)$. Let $Z \subset X$ be a smooth subvariety of dimension n. For any smooth hypersurface $Y \in |\mathcal{O}(d)|$ of X containing Z, let γ be the projection of $[Z]$ in $H^{n,n}(Y) \cap H^{2n}(Y, \mathbb{Q})_{\mathrm{van}}$, where $H^{2n}(Y, \mathbb{Q})_{\mathrm{van}}$ is the orthogonal complement of the restriction of $H^{2n}(X, \mathbb{Q})$. The cycle Z solves the Hodge conjecture for the Hodge class γ in the following sense of reversed induction on dimension: *Assume* that the Hodge conjecture is known on X. Then to prove the Hodge conjecture on Y for a given Hodge class $\tilde{\gamma} \in H^{n,n}(Y) \cap H^{2n}(Y, \mathbb{Q})$, it suffices to find a cycle Z having the same projection in $H^{2n}(Y, \mathbb{Q})_{\mathrm{van}}$ as $\tilde{\gamma}$, since then $[Z] - \tilde{\gamma}$ lifts to a Hodge class on X and is algebraic by assumption.

In general, when a vanishing Hodge class γ is given, we may consider a natural homogeneous ideal E in the homogeneous coordinate ring of X; see §3.2 below. The point is that E can be constructed using only Hodge theory, or more precisely using the Poincaré residue, and so apparently is not dependent on the algebraicity of the class γ. By considering varying hypersurfaces Y containing Z of large degrees, we then obtain a system of homogeneous ideals $\{E_Y\}$.

We will show that Z can be recovered from the system $\{E_Y\}$; see Corollary 5.2.1. The underlying philosophy is to find enough Hodge-theoretic data associated to a Hodge class γ so that an algebraic realization Z may be constructed; this work may be considered as a basic step toward understanding the Hodge conjecture from a constructive viewpoint.

1.2. In [4, Corollary (4.a.8)] it was shown that when a smooth curve $C \subset \mathbb{P}^3$ is nonspecial in the sense that $H^1(C, N_{C|\mathbb{P}^3}) = 0$, its defining equations are determined by Hodge-theoretic data. The result above may be considered as analogous.

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In [2, 6, 7, 8] the geometry of the Noether-Lefschetz locus $NL(\gamma)$ is used to construct cycles Z solving the Hodge conjecture for γ under the assumption that $NL(\gamma)$ contains a component of large dimension. The ideal E (and its generalizations) is one of the main tools used to prove these results.

2. Notation and positivity conditions

2.1. Throughout this article X will denote a fixed complex projective smooth variety of dimension $2n+1$, $\mathcal{O}(1)$ denotes a fixed ample line bundle, and Z denotes a smooth subvariety of dimension n with ideal sheaf $\mathcal{I}$. Let $N = N_{Z|X} = (\mathcal{I}/\mathcal{I}^2)^\vee$.

Denote by $S = \bigoplus S^k$ the graded algebra with $S^k = H^0(X, \mathcal{O}(k))$. Then we have $X \cong \mathrm{Proj}(S)$. Let M be the graded S-module with $M^k = H^0(X, K_X(k))$.

Denote by $\bar{S}$ the graded algebra with $\bar{S}^k = H^0(Z, \mathcal{O}_Z(k))$, where $\mathcal{O}_Z(1) := \mathcal{O}(1)|_Z$. Then we have a natural homomorphism $S \to \bar{S}$. Let I be its kernel. Then $I^k = H^0(X, \mathcal{I}(k))$ is the vector space of sections of $\mathcal{O}(k)$ vanishing along Z. We have $Z \cong \mathrm{Proj}(\bar{S}) \cong \mathrm{Proj}(S/I)$, whose inclusion in X is induced by $S \to \bar{S}$. Let $\bar{M}$ be the graded $\bar{S}$-module with $\bar{M}^k = H^0(Z, K_X|_Z(k))$. Then we have a restriction map $M \to \bar{M}$; denote its kernel by K.

2.2. We will need various positivity assumptions:

- (B) : $H^{>0}(X, \Omega^\ell_X(kd)) = 0$ for all $k \geq 1$, $\ell \geq 0$, and $d \geq d_0$.
- (M) : $S^a \otimes M^b \longrightarrow M^{a+b}$ is surjective for all $a \geq a_0$ and $b \geq b_0$.
- (S) : $\mathcal{I}(d) \to N^\vee(d)$ is surjective on global sections for all $d \geq d_0$.
- (Z) : $\mathcal{O}_Z(c_0)$ is very ample and $N^\vee(d - c_0)$ is globally generated for some $c_0 \geq 1$ and for all $d \geq d_0$.

For sufficiently large a_0, b_0, c_0, d_0 these conditions hold since $\mathcal{O}(1)$ is ample and

- (B) : $\Omega^\ell_X = 0$ when $\ell > \dim(X)$, and the cohomological criterion of ampleness [5, Chapter III, Proposition 5.3].
- (M) : [1, Lemma 1.28].
- (S) : we have a sheaf surjection $I \to N^\vee$, and [5, Chapter III, Proposition 5.3].
- (Z) : we have $H^i(Z, \mathcal{F} \otimes \mathcal{O}(c)) \cong H^i(X, (i_*\mathcal{F})(c))$ by [5, Chapter III, Exercise 8.1] and the projection formula, for any coherent sheaf $\mathcal{F}$ on Z. Then by [5, Chapter III, Proposition 5.3] we know that $\mathcal{O}_Z(c)$ is very ample for all sufficiently large c.

3. Homogeneous ideal associated to a vanishing class

3.1. Let $Y \subset X$ be a smooth divisor in $H^0(\mathcal{O}(d))$ with d sufficiently large so that condition (B) in §2.2 is satisfied with $d_0 = d$. We can then express the Hodge decomposition of $H^{2n}(Y, \mathbb{C})_{\mathrm{van}}$ purely in terms of algebra; see for example [9, section 6.1.2] and [3, Lecture 4]. More precisely, we have residue maps

$$r^{pd} : M^{pd} = H^0(K_X(pd)) \twoheadrightarrow H^{2n-p+1,p-1}(Y)_{\mathrm{van}}.$$

In particular every vanishing Hodge class γ of degree $2n$ is of the form $r^{(n+1)d}(\omega)$ for some $\omega \in M^{(n+1)d}$.

The non-degenerate intersection pairing on $H^{2n}(Y)_{\mathrm{van}}$ can also be understood in terms of the module M: There is a non-zero linear functional

$$\lambda : W := H^0(X, K_X((n+1)d)^{\otimes 2}) \to \mathbb{C}$$

such that the pairing between the Hodge groups of complementary bi-degrees is induced by the composition

$$M^{pd} \otimes M^{(2n+2-p)d} \longrightarrow W \xrightarrow{\lambda} \mathbb{C},$$

where the first map is induced by the tensor product.

Moreover, the Gauss-Manin connection on the smooth locus of $H^0(\mathcal{O}(d))$ is given by the multiplication map. More precisely we have the following commutative diagram:

$$\begin{array}{ccc} S^d \otimes M^{pd} & \xrightarrow{\text{multiplication}} & M^{(p+1)d} \\ \downarrow & & \downarrow \\ H^0(Y, N_{Y|X}) \otimes H^{2n-p+1,p-1}(Y)_{\text{van}} & \xrightarrow{\nabla} & H^{2n-p,p}(Y)_{\text{van}} \end{array}$$

3.2. Now let γ be a vanishing Hodge class of degree $2n$ on Y; we will denote also by γ a preimage under $r^{(n+1)d}$ in $M^{(n+1)d}$. It then defines a linear functional $(-).\gamma : \omega \mapsto \lambda(\omega \otimes \gamma)$ on $M^{(n+1)d}$, non-zero if γ is non-zero, since the intersection pairing on vanishing cohomology is non-degenerate. Now we define

$$E^k := \{P \in S^k \,|\, (PM^{(n+1)d-k}).\gamma = 0\},$$

and dually

$$F^{(n+1)d-k} := \{\omega \in M^{(n+1)d-k} \,|\, (S^k\omega).\gamma = 0\}.$$

In other words, they are kernels with respect to the pairing

$$S^k \otimes M^{(n+1)d-k} \longrightarrow M^{(n+1)d} \xrightarrow{(-).\gamma} \mathbb{C}.$$

It is easy to see that $E := \bigoplus E^k$ is a homogeneous ideal in S. This construction was studied in [6, section 1.2]; in fact the ideal E here is called E_0 in [6], where a family of homogeneous ideals $\{E_r \,|\, r \geq -1\}$ is constructed.

3.3. The ideal E constructed above is considered to contain information about the Noether-Lefschetz locus $NL(\gamma) \subset H^0(\mathcal{O}(d))$ associated to γ, which consists of smooth divisors $Y \in H^0(\mathcal{O}(d))$ on which the flat transport of γ is still a Hodge class. For example, using the description of the Gauss-Manin connection above, it is easy to see that $E^d \subset S^d$ is the tangent space to $NL(\gamma)$ at Y. See [6, 1.2.3] for a geometric interpretation of some other pieces of the ideals E_r.

4. Homogeneous ideals associated to a smooth subvariety

4.1. For any given d, if $Y \in I^d$ is a smooth divisor, then we may apply §3.1 to the vanishing class γ_Y, defined as the image of $[Z]$ in the vanishing cohomology of Y, and obtain a homogeneous ideal E_Y. In the case when d is sufficiently large so that (B) is satisfied with $d_0 = d$, we denote also by $\gamma_Y \in M^{(n+1)d}$ a preimage under the residue map of γ_Y and by $(-).\gamma_Y$ the corresponding functional on $M^{(n+1)d}$.

Proposition 4.1.1. *For any fixed integer k sufficiently large so that* (B) *holds with $d_0 = k$, we have $I^k \subset E^k_Y$ for every smooth Y in I^d with d sufficiently greater than k.*

Proof. First assume $d = k$, in which case the containment is clear since I^k consists of sections of $\mathcal{O}(k)$ vanishing on Z, while E_Y^k is the tangent space to $NL(\gamma_Y)$ at the hypersurface Y; see §3.3.

Now suppose d is sufficiently large so that condition (M) is satisfied with $a_0 = d-k$ and $b_0 = nd$. Then we have $S^{d-k}M^{nd} = M^{(n+1)d-k}$. Let $P \in I^k$. Then $PS^{d-k} \subset I^d \subset E_Y^d$ by the case above for any smooth Y in I^d. Hence we have $0 = (PS^{d-k}M^{nd}).\gamma_Y = (PM^{(n+1)d-k}).\gamma_Y$, which says exactly that $P \in E_Y^k$. $\square$

4.2. The functional $(-).\gamma_Y$ on $M^{(n+1)d}$ corresponding to the containments $Z \subset Y \subset X$ can be described in terms of the geometry as explained in [3, Proposition on page 49]: Let $\bar{N} := N_{Z|Y}$. Then we have a short exact sequence of vector bundles

$$0 \longrightarrow \mathcal{O}_Z(-d) \longrightarrow N^\vee \longrightarrow \bar{N}^\vee \longrightarrow 0,$$

where the first map is the restriction to Z of the inclusion $\mathcal{O}(-d) \to \mathcal{I}$ given by the defining equation of Y. Then by the construction in [3, page 39] and twisting by $\mathcal{O}_Z(nd)$ we get an exact sequence

$$0 \longrightarrow \mathcal{O}_Z \longrightarrow N^\vee(d) \longrightarrow \bigwedge^2 N^\vee(2d) \longrightarrow \cdots \longrightarrow \bigwedge^n N^\vee(nd) \longrightarrow \bigwedge^n \bar{N}^\vee(nd) \longrightarrow 0$$

where all arrows except for the last two are given by contracting with the defining equation of Y (restricted to Z).

Now we have $K_Z \otimes \bigwedge^n \bar{N}^\vee \cong K_Y|_Z \cong K_X(d)|_Z \cong K_Z \otimes \bigwedge^{n+1} N^\vee(d)$ by the adjunction formula. Hence we can rewrite the sequence above as

$$0 \longrightarrow \mathcal{O}_Z \longrightarrow N^\vee(d) \longrightarrow \bigwedge^2 N^\vee(2d) \longrightarrow \cdots \longrightarrow \bigwedge^n N^\vee(nd) \longrightarrow \bigwedge^{n+1} N^\vee((n+1)d) \longrightarrow 0$$

where now every arrow is given by contracting with the defining equation of Y.

Twisting this last sequence by K_Z we get

(4.1)

$$0 \longrightarrow K_Z \longrightarrow K_Z \otimes N^\vee(d) \longrightarrow \cdots \longrightarrow K_Z \otimes \bigwedge^n N^\vee(nd) \longrightarrow K_X|_Z((n+1)d) \longrightarrow 0.$$

Every element in $\bar{M}^{(n+1)d} = H^0(Z, K_X|_Z((n+1)d))$ can be viewed as a morphism

$$\mathcal{O}_Z \longrightarrow K_X|_Z((n+1)d),$$

which gives by pulling-back (4.1) an extension of $\mathcal{O}_Z$ by K_Z, namely an element in $\mathrm{Ext}^n(\mathcal{O}_Z, K_Z) \cong H^n(Z, K_Z) \cong \mathbb{C}$.

The construction above gives the map α in the following commutative (up to a non-zero scalar multiple) diagram:

$$\begin{array}{ccccc} H^0(X, K_X((n+1)d)) & \xrightarrow{\beta} & H^0(Z, K_X|_Z((n+1)d)) & & \\ \downarrow{\scriptstyle r^{(n+1)d}} & & \downarrow{\scriptstyle \alpha} & & \\ H^n(Y, \Omega_Y^n) & \xrightarrow{\delta} & H^n(Z, K_Z^n) & \xrightarrow[\cong]{\lambda} & \mathbb{C} \end{array}$$

Here β is the restriction map from $M^{(n+1)d}$ to $\bar{M}^{(n+1)d}$ as in §2.1, δ is the restriction map, and $r^{(n+1)d}$ is the residue map as in §3.1, whose image is $H^{n,n}(Y)_{\mathrm{van}}$. The composition $\lambda \circ \delta \circ r^{(n+1)d}$ is by definition the functional $(-).\gamma_Y$, which is then equal to $\lambda \circ \alpha \circ \beta$. Note that as long as $[Z]$ is non-zero, the map δ is not zero, but the composition $\delta \circ r^{(n+1)d}$ could be zero: this happens exactly when the vanishing component γ_Y of $[Z]$ is zero.

Given the sequence (4.1), an element $\mathcal{O}_Z \to K_X|_Z((n+1)d)$ in $\bar{M}^{(n+1)d}$ maps to zero in $\mathrm{Ext}^n(\mathcal{O}_Z, K_Z)$ if and only if it lifts to a morphism $\mathcal{O}_Z \to K_Z \otimes \bigwedge^n N^\vee(nd)$; hence we can describe the kernel of $(-).\gamma_Y$ as follows:

Lemma 4.2.1. *Given an element $P_1 \in I^d$ defining a smooth divisor Y, let Q_1 be its image in $H^0(N^\vee(d))$. Then the kernel of the functional $(-).\gamma_Y$ on $M^{(n+1)d}$ consists of elements whose images in $\bar{M}^{(n+1)d}$ lie in $Q_1 \wedge H^0(K_Z \otimes \bigwedge^n N^\vee(nd))$ under the wedge product map*

$$H^0(N^\vee(d)) \otimes H^0(K_Z \otimes \bigwedge^n N^\vee(nd)) \xrightarrow{\wedge} \bar{M}^{(n+1)d}.$$

5. Recovering a smooth subvariety from its associated ideals

5.1. We will need the following lemmas:

Lemma 5.1.1. *Let ℓ be an integer sufficiently large so that $K_X(\ell)$ is base-point-free. If P in S^k satisfies $PM^\ell \subset K = \ker(M \to \bar{M})$, then it also satisfies $PS^\ell \subset I$.*

Proof. The condition $PM^\ell \subset K$ implies that $P\omega$ vanishes along Z for every $\omega \in M^\ell = H^0(K_X(\ell))$. Since the line bundle $K_X(\ell)$ is base-point-free, for every point $z \in Z$ we can find a global section ω not vanishing at z. Hence P must vanish at z, for every $z \in Z$. □

Lemma 5.1.2. *Suppose $\mathcal{O}_Z(c_0)$ is very ample and $N^\vee(d-c_0)$ is globally generated; namely condition (Z) holds. For every non-zero element $R \in S^{c_0}$ there is an element $Q' \in H^0(N^\vee(d-c_0))$ such that $RQ' \neq 0$ in $H^0(N^\vee(d))$.*

Proof. For any $z \in Z$ at which R is non-zero, find Q' also not vanishing at z. □

Lemma 5.1.3. *Suppose $\mathcal{O}_Z(c_0)$ is very ample. If $\eta \in \bar{M}^{(n+1)d}$ lies in $R\bar{M}^{(n+1)d-c_0}$ for every non-zero $R \in \bar{S}^{c_0} = H^0(Z, \mathcal{O}_Z(c_0))$, then $\eta = 0$.*

Proof. The condition implies that η as a global section of $K_X|_Z((n+1)d)$ vanishes along the divisor $Z(R)$ on Z. But $\mathcal{O}_Z(c_0)$ is very ample; hence every point $z \in Z$ is contained in $Z(R)$ for some non-zero R. □

5.2. For any fixed k we denote by $d \gg k$ that condition (M) is satisfied for $a_0 = d-k$ and $b_0 = nd$; notice that if $d_1 \geq d$ and $d \gg k$, then $d_1 \gg k$. With the notation as in §4.1, define

$$J^k_d := \bigcap_{\text{smooth } Y \in I^d} E^k_Y$$

and

$$J^k := \bigcap_{d \gg k} J^k_d.$$

Then $J := \bigoplus J^k$ is a homogeneous ideal, and we have $I \subset J$ in all sufficiently large degrees by Proposition 4.1.1.

Theorem 5.2.1. *For every element $P \in J$ of sufficiently large degree, there is an integer ℓ such that $PS^\ell \subset I$.*

Proof. Fix an integer k sufficiently large so that condition (B) holds with $d_0 = k$, and fix any $d \gg k$ so that moreover conditions (S) and (Z) are satisfied with $d_0 = d$ and some c_0. Let $P \in S^k$ be an element which does not satisfy the conclusion of the statement, and we will show that P does not lie in J.

By Lemma 5.1.1 the assumption on P implies that for all sufficiently large integers e' we have $PM^{e'} \not\subseteq K = \ker(M \to \bar{M})$. Replacing d with a larger integer if necessary, we can find $e := (n+1)d-k$ sufficiently large satisfying $PM^e \subset M^{(n+1)d}$ and $PM^e \not\subseteq K$. Let $\eta \in \bar{M}^{(n+1)d}$ be any non-zero element in the image $\overline{PM^e}$ of PM^e in $\bar{M}^{(n+1)d}$.

Now consider the following commutative diagram:

$$\begin{array}{ccc} \bar{S}^{c_0} \otimes H^0(N^\vee(d-c_0)) \otimes H^0(K_Z \otimes \bigwedge^n N^\vee(nd)) & \xrightarrow{\wedge} & \bar{S}^{c_0} \otimes \bar{M}^{(n+1)d-c_0} \\ \downarrow & & \downarrow \\ H^0(N^\vee(d)) \otimes H^0(K_Z \otimes \bigwedge^n N^\vee(nd)) & \xrightarrow{\wedge} & \bar{M}^{(n+1)d} \end{array}$$

where the horizontal maps are the wedge product and the vertical ones are multiplication.

We claim that there exists a non-zero element $Q_1 \in H^0(N^\vee(d))$ such that η does not lie in $Q_1 \wedge H^0(K_Z \otimes \bigwedge^n N^\vee(nd))$. Suppose to the contrary that for every non-zero Q_1 we have

$$\eta \in Q_1 \wedge H^0(K_Z \otimes \bigwedge^n N^\vee(nd)).$$

Then by Theorem 5.1.2, for every non-zero element $R \in \bar{S}^{c_0}$ we can find $Q' \in H^0(N^\vee(d-c_0))$ such that $RQ' \neq 0$ and therefore $\eta \in RQ' \wedge H^0(K_Z \otimes \bigwedge^n N^\vee(nd))$. But this implies

$$\eta \in R\bar{M}^{(n+1)d-c_0},$$

which implies that $\eta = 0$ by Lemma 5.1.3, a contradiction.

Note that the set of non-zero Q_1 satisfying the condition $\eta \notin Q_1 \wedge H^0(K_Z \otimes \bigwedge^n N^\vee(nd))$ is a Zariski open subset of $H^0(N^\vee(d))$, and we just showed it is non-empty.

By condition (S) we can lift any Q_1 as above to an element $P_1 \in I^d$, and we can find such a P_1 defining a smooth divisor Y on X containing Z. So by Lemma 4.2.1 we know that PM^e is not contained in the kernel of $(-).\gamma_Y$. In other words, $P \notin E_Y^k$; hence $P \notin J^k$ as required. □

Corollary 5.2.1. *The subvariety Z can be recovered from J. More precisely, $Z \cong$ $\mathrm{Proj}(S/J)$.*

Proof. Indeed, in all sufficiently large degrees, J lies between the ideal I and its saturation in S; see [5, Ex. 2.14 and Ex. 5.10]. □

Remark 5.2.2. In its stated form Corollary 5.2.1 requires the knowledge of all smooth divisors Y containing Z, and such knowledge clearly already determines Z. But the proof gives more: Suppose the saturation $\bar{I}$ of I is generated by its degree e homogeneous piece $\bar{I}^e$ and we have $Z \subset Y$ with $\deg(Y) = d$ sufficiently large. This defines E_Y, which may be viewed as a first approximation of I. The proof of Corollary 5.2.1 says that for every element P in E_Y^e but not in $\bar{I}^e$ we can find another Y' containing Z and possibly of higher degree, so that P does not lie in $E_{Y'}$. Therefore we can take $E_Y \cap E_{Y'}$ as a refinement. Since $\bar{I}^e$ is finite dimensional, a finite intersection $(E_Y \cap E_{Y'} \cap E_{Y''} \cap \ldots)^e$ will be contained in $\bar{I}^e$, and then Z is determined by these finitely many divisors $Y, Y', Y'', \ldots$.

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Department of Mathematics, Princeton University, Princeton, New Jersey 08544
E-mail address: yuliu@math.princeton.edu