

**NONRIGIDITY OF A CLASS
OF TWO DIMENSIONAL SURFACES
WITH POSITIVE CURVATURE AND PLANAR POINTS**

A. MEZIANI

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ABSTRACT. Existence of nontrivial infinitesimal bendings is established for an orientable surface with boundary $S \subset \mathbb{R}^3$ that has positive curvature except at finitely many planar points and such that $H_1(S) = 0$. As an application, we show that any neighborhood of such a surface S (for the C^k topology) contains isometric surfaces that are noncongruent.

INTRODUCTION

The problem considered here deals with the bendings of an orientable, embedded surface S in \mathbb{R}^3 . An infinitesimal bending of class C^k of a surface $S \subset \mathbb{R}^3$ is a deformation surface $S_\sigma = \{p + \sigma U(p), p \in S\}$, with a real parameter σ , whose first fundamental form satisfies $dS_\sigma^2 = dS^2 + O(\sigma^2)$ as $\sigma \rightarrow 0$ and such that the bending field $U : S \rightarrow \mathbb{R}^3$ is of class C^k . The bending is nontrivial if, in addition, S_σ is not obtained from S through rigid motions of the ambient space \mathbb{R}^3 .

The study of bendings of surfaces in \mathbb{R}^3 has a rich history and many physical applications. For example, it is used in the theory of elastic shells. We refer to the survey article of Sabitov ([14]) and the references therein. The results of this paper are also related to those contained in the following papers: [2], [3], [4], [5], [6], [8], [10], [12], [19], [15]. In particular, papers [3], [8], [10], [19] [20] deal with the local structure of the bending fields. Papers [6], [15], [19] discuss the bendings of rotation surfaces. Stability of shells and rigidity of convex surfaces are studied in [12] and [13]. In [2], explicit formulas for the bendings of punctured spheres are given, and in [4] the rigidity of isometric embeddings is studied.

The main result of this paper is the following theorem.

Theorem 0.1. *Let $S \subset \mathbb{R}^3$ be an embedded surface such that \bar{S} is compact, with $\partial S \neq \emptyset$, $H_1(S) = 0$, and such that \bar{S} has positive curvature except possibly at a finite number of points in S . Then, for any positive integer k , the surface S has nontrivial infinitesimal bendings of class C^k .*

As an application, we prove that any neighborhood of such a surface has noncongruent isometric surfaces. More precisely, we prove the following theorem.

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Theorem 0.2. *Let S be as in Theorem 0.1. Then, for any $\epsilon > 0$ and any integer k , there exist surfaces Σ^+ and Σ^- of class C^k contained in the ϵ -neighborhood of S , such that Σ^+ and Σ^- are isometric but not congruent.*

Our approach is through the study of the associated (complex) field of asymptotic directions on S . We prove that such a vector field generates an integrable structure on \bar{S} . We reformulate the equations for the bending field U in terms of a Bers-Vekua type equation (with singularities). Then we use recent results about the solvability of such equations to construct the bending fields.

1. INTEGRABILITY OF THE FIELD OF ASYMPTOTIC DIRECTIONS

For the surfaces considered here, we show that the field of asymptotic directions on S has a global first integral.

Let $S \subset \mathbb{R}^3$ be an orientable C^∞ surface with a C^∞ boundary. We assume that $H_1(S) = 0$. The surface S is diffeomorphic to a relatively compact domain $\Omega \subset \mathbb{R}^2$ with a C^∞ boundary. Hence,

$$(1.1) \quad \bar{S} = \{R(s, t) \in \mathbb{R}^3; (s, t) \in \bar{\Omega}\},$$

where the position vector $R : \bar{\Omega} \rightarrow \mathbb{R}^3$ is a C^∞ parametrization of \bar{S} . Let E, F, G and e, f, g be, respectively, the coefficients of the first and second fundamental forms of S . Thus,

$$\begin{aligned} E &= R_s \cdot R_s, & F &= R_s \cdot R_t, & G &= R_t \cdot R_t, \\ e &= R_{ss} \cdot N, & f &= R_{st} \cdot N, & g &= R_{tt} \cdot N, \end{aligned}$$

where $N = \frac{R_s \times R_t}{|R_s \times R_t|}$ is the unit normal of S . The Gaussian curvature of S is then $K = \frac{eg - f^2}{EG - F^2}$. We assume that \bar{S} has a positive curvature except at a finite number of planar points in S . That is, there exist $p_1 = (s_1, t_1) \in \Omega, \dots, p_l(s_l, t_l) \in \Omega$ such that

$$(1.2) \quad K(s, t) > 0, \quad \forall (s, t) \in \bar{\Omega} \setminus \{p_1, \dots, p_l\}.$$

The (complex) asymptotic directions on S are given by the quadratic equation

$$(1.3) \quad \lambda^2 + 2f\lambda + eg = 0.$$

Thus $\lambda = -f + i\sqrt{eg - f^2} \in \mathbb{R} + i\mathbb{R}^+$ except at the planar points p_1, \dots, p_l where $\lambda = 0$.

Consider the structure on $\bar{\Omega}$ generated by the \mathbb{C} -valued vector field

$$(1.4) \quad L = g(s, t) \frac{\partial}{\partial s} + \lambda(s, t) \frac{\partial}{\partial t}.$$

This structure is elliptic on $\bar{\Omega} \setminus \{p_1, \dots, p_l\}$. That is, L and \bar{L} are independent outside the planar points. The next proposition shows that L has a global first integral on \bar{S} .

Proposition 1.1. *Let S be the surface given by (1.1) whose curvature K satisfies (1.2). Then there exists an injective function*

$$Z : \bar{\Omega} \rightarrow \mathbb{C}$$

such that

1. Z is C^∞ on $\bar{\Omega} \setminus \{p_1, \dots, p_l\}$;

- 2. $LZ = 0$ on $\overline{\Omega} \setminus \{p_1, \dots, p_l\}$; and
- 3. for every $j = 1, \dots, l$, there exist $\mu_j > 0$ and polar coordinates (r, θ) centered at p_j such that in a neighborhood of p_j we have

$$(1.5) \quad Z(r, \theta) = Z(0, 0) + r^{\mu_j} e^{i\theta} + O(r^{2\mu_j}).$$

Proof. Since L is C^∞ and elliptic on $\overline{\Omega} \setminus \{p_1, \dots, p_l\}$, it follows from the uniformization of complex structures on planar domains (see [17]) that there exists a C^∞ diffeomorphism

$$Z : \overline{\Omega} \setminus \{p_1, \dots, p_l\} \longrightarrow Z(\overline{\Omega} \setminus \{p_1, \dots, p_l\}) \subset \mathbb{C}$$

such that $LZ = 0$. It remains to show that Z has the form (1.5) in a neighborhood of a planar point.

Let p_j be a planar point of S . We can assume that S is given in a neighborhood of p_j as the graph of a function $z = z(x, y)$ with $p_j = (0, 0)$, $z(0, 0) = 0$, and $z_x(0, 0) = z_y(0, 0) = 0$. The assumption on the curvature implies that

$$z(x, y) = z_m(x, y) + o(\sqrt{x^2 + y^2}^m),$$

where $z_m(x, y)$ is a homogeneous polynomial of degree $m > 2$, satisfying $z_{xx}z_{yy} - z_{xy}^2 > 0$ for $(x, y) \neq 0$. We can also assume that $z(x, y) > 0$ for $(x, y) \neq 0$. The complex structure generated by the asymptotic directions is given by the vector field

$$L = z_{yy} \frac{\partial}{\partial x} + (-z_{xy} + i\sqrt{z_{xx}z_{yy} - z_{xy}^2}) \frac{\partial}{\partial y}.$$

With respect to the polar coordinates $x = \rho \cos \phi$, $y = \rho \sin \phi$, we get

$$z = \rho^m P(\phi) + \rho^{m+1} A(\rho, \phi),$$

where $P(\phi)$ is a trigonometric polynomial of degree m satisfying $P(\phi) > 0$ and (curvature)

$$m^2 P(\phi)^2 + mP(\phi)P''(\phi) - (m - 1)P'(\phi)^2 > 0 \quad \forall \phi \in \mathbb{R}.$$

With respect to the coordinates (ρ, ϕ) , the vector field L becomes

$$L = m(m - 1)\rho^{m-2}(P(\phi) + O(\rho))L_0$$

with

$$L_0 = \frac{\partial}{\partial \phi} + \rho(M(\phi) + iN(\phi) + O(\rho)) \frac{\partial}{\partial \rho}$$

and

$$M = \frac{P'}{mP}, \quad N = \frac{1}{m} \sqrt{\frac{m^2 P^2 + mPP'' - (m - 1)P'^2}{(m - 1)P^2}}.$$

We know (see [7]) that such a vector field L_0 is integrable in a neighborhood of the circle $\rho = 0$. Moreover, we can find coordinates (r, θ) in which L_0 is C^1 -conjugate to the model vector field

$$T = \mu_j \frac{\partial}{\partial \theta} - ir \frac{\partial}{\partial r},$$

where $\mu_j > 0$ is given by

$$\frac{1}{\mu_j} = \frac{1}{2\pi} \int_0^{2\pi} (N(\phi) - iM(\phi))d\phi = \frac{1}{2\pi} \int_0^{2\pi} N(\phi)d\phi.$$

The function $u_j(r, \theta) = r^{\mu_j} e^{i\theta}$ is a first integral of T in $r > 0$.

Now we prove that the function Z , which is defined in $\overline{\Omega} \setminus \{p_1, \dots, p_l\}$, extends to p_j with the desired form given by (1.5). Let O_j be a disc centered at p_j where L is conjugate to a multiple of T in the (r, θ) coordinates. Since u_j and Z are both first integrals of L in the punctured disc $O_j \setminus p_j$, there exists a holomorphic function h_j defined on the image $u_j(O_j \setminus p_j)$ such that $Z(r, t) = h_j(u_j(r, t))$. Since both Z and u_j are homeomorphisms onto their images, h_j is one-to-one in a neighborhood of $u_j(p_j) = 0 \in \mathbb{C}$ and since h_j is bounded,

$$h_j(\zeta) = C_0 + C_1\zeta + O(\zeta^2) \quad \text{for } \zeta \text{ close to } 0 \in \mathbb{C}$$

with $C_1 \neq 0$. This means that after a linear change of the coordinates (r, θ) (to remove the constant C_1), the function Z has the form (1.5). \square

2. EQUATIONS OF THE BENDING FIELDS IN TERMS OF L

Let S be a surface given by (1.1). An infinitesimal bending of class C^k of S is a deformation surface $S_\sigma \subset \mathbb{R}^3$, with a parameter $\sigma \in \mathbb{R}$, given by the position vector

$$(2.1) \quad R_\sigma(s, t) = R(s, t) + \sigma U(s, t),$$

whose first fundamental form satisfies

$$dR_\sigma^2 = dR^2 + O(\sigma^2) \quad \text{as } \sigma \rightarrow 0.$$

This means that the bending field $U : \overline{\Omega} \rightarrow \mathbb{R}^3$ is of class C^k and satisfies

$$(2.2) \quad dR \cdot dU = 0.$$

The trivial bendings of S are those induced by the rigid motions of \mathbb{R}^3 . They are given by $U(s, t) = A \times R(s, t) + B$, where A and B are constants in \mathbb{R}^3 and where \times denotes the vector product in \mathbb{R}^3 .

Let L be the field of asymptotic directions defined by (1.4). For each function $U : \Omega \rightarrow \mathbb{R}^3$, we associate the \mathbb{C} -valued function w defined by

$$(2.3) \quad w(s, t) = LR(s, t) \cdot U(s, t) = g(s, t)u(s, t) + \lambda(s, t)v(s, t),$$

where $u = R_s \cdot U$ and $v = R_t \cdot U$. The following theorem proved in [9] will be used in the next section.

Theorem 2.1 ([9]). *If $U : \Omega \rightarrow \mathbb{R}^3$ satisfies (2.2), then the function w given by (2.3) satisfies the equation*

$$(2.4) \quad CLw = Aw + B\overline{w},$$

where

$$(2.5) \quad \begin{aligned} A &= (LR \times \overline{LR}) \cdot (L^2R \times \overline{LR}), \\ B &= (LR \times \overline{LR}) \cdot (L^2R \times LR), \\ C &= (LR \times \overline{LR}) \cdot (LR \times \overline{LR}). \end{aligned}$$

Remark 2.1. If w solves equation (2.4), then the function $w' = aw$, where a is a nonvanishing function, solves the same equation with the vector field L replaced by the vector field $L' = aL$.

3. MAIN RESULTS

Theorem 3.1. *Let S be a surface given by (1.1) such that its curvature K satisfies (1.2). Then, for every $k \in \mathbb{Z}^+$, the surface S has a nontrivial infinitesimal bending $U : \bar{\Omega} \rightarrow \mathbb{R}^3$ of class C^k .*

Remark 3.1. It should be mentioned that without the assumption that $K > 0$ up to the boundary ∂S , the surface could be rigid under infinitesimal bendings. Indeed, let T^2 be a standard torus in \mathbb{R}^3 . It is known (see [1] or [13]) that if S consists of the portion of T^2 with positive curvature, then S is rigid under infinitesimal bendings. Here the curvature vanishes on ∂S .

Before we proceed with the proof, we give a consequence of Theorem 3.1.

Theorem 3.2. *Let S be as in Theorem 3.1. Then for every $\epsilon > 0$ and for every $k \in \mathbb{Z}^+$, there exist surfaces Σ^+ and Σ^- of class C^k in the ϵ -neighborhood of S (for the C^k -topology) such that Σ^+ and Σ^- are isometric but not congruent.*

Proof. Let $U : \bar{\Omega} \rightarrow \mathbb{R}^3$ be a nontrivial infinitesimal bending of S of class C^k . Consider the surfaces Σ_σ and $\Sigma_{-\sigma}$ defined by the position vectors

$$R_{\pm\sigma}(s, t) = R(s, t) \pm \sigma U(s, t).$$

Since $dR \cdot dU = 0$, then $dR_{\pm\sigma}^2 = dR^2 + \sigma^2 dU^2$. Hence Σ_σ and $\Sigma_{-\sigma}$ are isometric. Furthermore, since U is nontrivial, then Σ_σ and $\Sigma_{-\sigma}$ are not congruent (see [16]). For a given $\epsilon > 0$, the surfaces $\Sigma_{\pm\sigma}$ are contained in the ϵ -neighborhood of S if σ is small enough. □

Proof of Theorem 3.1. First we construct nontrivial solutions w of equation (2.4) and then deduce the infinitesimal bending fields U . For this, we use the first integral Z of L to transform equation (2.4) into a Bers-Vekua type equation with singularities. Let $Z_1 = Z(p_1), \dots, Z_l = Z(p_l)$ be the images of the planar points by the function Z . The pushforward of equation (2.4) via Z gives rise to an equation of the form

$$(3.1) \quad \frac{\partial W}{\partial \bar{Z}} = \frac{A(Z)}{\prod_{j=1}^l (Z - Z_j)} W + \frac{B(Z)}{\prod_{j=1}^l (Z - Z_j)} \bar{W},$$

where $w(s, t) = W(Z(s, t))$ and $A, B \in C^\infty(Z(\bar{\Omega} \setminus \{p_1, \dots, p_l\})) \cap L^\infty(Z(\bar{\Omega}))$. The local study of the solutions of such equations near a singularity is considered in [11], [18], [20]. To construct a global solution of (3.1) with the desired properties, we proceed as follows. We seek a solution W in the form $W(Z) = H(Z)W_1(Z)$, where

$$H(Z) = \prod_{j=1}^l (Z - Z_j)^M,$$

where M is a (large) positive integer to be chosen. In order for $W = HW_1$ to solve (3.1), the function W_1 needs to solve the modified equation

$$(3.2) \quad \frac{\partial W_1}{\partial \bar{Z}} = \frac{A(Z)}{\prod_{j=1}^l (Z - Z_j)} W_1 + \frac{B(Z)}{\prod_{j=1}^l (Z - Z_j)} \frac{\overline{H(Z)}}{H(Z)} \bar{W}_1.$$

Since A and $B\bar{H}/H$ are bounded functions on $Z(\bar{\Omega})$, a result of [18] gives a continuous solution W_1 of (3.2) on $Z(\bar{\Omega})$. Furthermore, such a solution is C^∞ on $Z(\bar{\Omega} \setminus \{Z_1, \dots, Z_l\})$ since the equation is elliptic and the coefficients are C^∞ outside the Z_j 's. Note also that since $Z(\Omega) \neq \mathbb{C}$, equation (3.2) has bounded nontrivial solutions.

The function $W(Z) = H(Z)W_1(Z)$ is therefore a solution of (3.1). It is C^∞ on $Z(\bar{\Omega} \setminus \{Z_1, \dots, Z_l\})$ and vanishes to order M at each point Z_j . Consequently, the function $w(s, t) = W(Z(s, t))$ is C^∞ on $\bar{\Omega} \setminus \{p_1, \dots, p_l\}$ and vanishes to order $M\mu_j$ at each planar point (μ_j is the positive number appearing in Proposition 1.1).

Now, we construct the bending field U from the solution w of (2.4) and the relation $w = LR \cdot U$. Set $w = gu + \lambda v$, where λ is the asymptotic direction given in (1.3). The functions u and v are uniquely determined by

$$v = \frac{w - \bar{w}}{2i\sqrt{eg - f^2}} \quad \text{and} \quad u = \frac{w + \bar{w} + 2fv}{2g},$$

provided that the function w vanishes to a high order at the planar points (for this, it is enough that W vanishes to a large order at the points Z_j). These functions are C^∞ outside the planar points. Since at each planar point p_j the order of vanishing of K is m_j , the functions v and u vanish to order $M\mu_j - m_j$. It follows from $LR \cdot U = w$ that $R_s \cdot U = u$ and $R_t \cdot U = v$. The condition $dR \cdot dU = 0$ implies that

$$(3.3) \quad R_{ss} \cdot U = u_s, \quad R_{tt} \cdot U = v_t, \quad \text{and} \quad 2R_{st} \cdot U = u_t + v_s.$$

In terms of the components (x, y, z) of R and (ξ, η, ζ) of U , we have

$$(3.4) \quad \begin{cases} x_s \xi + y_s \eta + z_s \zeta & = u \\ x_t \xi + y_t \eta + z_t \zeta & = v \\ x_{ss} \xi + y_{ss} \eta + z_{ss} \zeta & = u_s \\ x_{tt} \xi + y_{tt} \eta + z_{tt} \zeta & = v_t \\ 2x_{st} \xi + 2y_{st} \eta + 2z_{st} \zeta & = u_t + v_s \end{cases}$$

(the consistency of this overdetermined system is guaranteed by equation (2.4)). Note that at each point $p \in \bar{\Omega}$ where $K > 0$, the functions ξ , η , and ζ are uniquely determined by u , v , and u_s (or v_t). Indeed, at such a point the determinant of the first three equations of (3.4) is

$$R_{ss} \cdot (R_s \times R_t) = |R_s \times R_t|e \neq 0.$$

With our choice that w (and so u and v) vanishes to an order larger than that of the curvature at each planar point, the functions ξ , η , and ζ are also uniquely determined to be 0 at each planar point. To see why, assume that at p_j we have $x_s y_t - x_t y_s \neq 0$. Then, after solving the first two equations for ξ and η in terms of ζ , u , and v , the third equation becomes

$$(3.5) \quad R_{ss} \cdot (R_s \times R_t) \zeta = \begin{vmatrix} x_s & y_s & u \\ x_t & y_t & v \\ x_{ss} & y_{ss} & u_s \end{vmatrix}.$$

Since the zero, p_j , of $R_{ss} \cdot (R_s \times R_t)$ is isolated and since u and v vanish to a high order at p_j , the function ζ is well defined by (3.5). Consequently, for any given $k \in \mathbb{Z}^+$, a nonzero solution w of (2.4) which vanishes at high orders (M large) gives rise to a unique field of infinitesimal bending U of S , so that it is C^∞ on $\bar{\Omega} \setminus \{p_1, \dots, p_l\}$ and vanishes to an order k at each p_j . Such a field is therefore of

class C^k at each planar point. It remains to verify that U is not trivial. If such a field were trivial ($U = A \times R + B$), then the vanishing of $dU = A \times dR$ at p_j together with $dR \cdot dU = 0$ would give $A = 0$ and so $U = B = 0$, since $U = 0$ at p_j . This would result in $w \equiv 0$, which is a contradiction. \square

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