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A TIGHTNESS PROPERTY OF A SYMMETRIC MARKOV PROCESS AND THE UNIFORM LARGE DEVIATION PRINCIPLE

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Abstract. Previously, we considered a large deviation for occupation measures of a symmetric Markov processes under the condition that its resolvent possesses a kind of tightness property. In this paper, we prove that if the Markov process is conservative, then the tightness property implies the uniform hyper-exponential recurrence, which leads us to the uniform large deviation principle.

1. Introduction

Let E be a locally compact separable metric space and m a positive Radon measure on E with full support. Let $X = (\Omega, X_t, \mathbb{P}_x, \zeta)$ be an m-symmetric Borel right process on E. Here ζ is the lifetime of X. We assume that the process X is irreducible and strong Feller. Moreover, we assume that X possesses a *tightness property*; i.e., for any $\epsilon > 0$, there exists a compact set K such that $\sup_{x\in E} R_1 1_{K^c}(x) \leq \epsilon$. Here 1_{K^c} is the indicator function of the complement of K and R_1 is the 1-resolvent of X. In [18], [19], we consider large deviations for empirical measures of symmetric Markov processes with the tightness property.

We prove in this note that if X is conservative, $\mathbb{P}_x(\zeta = \infty) = 1$, then the tightness property implies the positive recurrence of X; in particular, the measure m turns out to be finite. Moreover, we prove that if, in addition, there exists an increasing sequence $\{K_n\}_{n=1}^{\infty}$ of compact sets such that the union of $\{K_n\}_{n=1}^{\infty}$ equals E and each part (absorbing) process X^{D_n} on D_n ($D_n := K_n^c$) is irreducible, then X possesses the following strong recurrence property: for any positive constant γ, there exists a compact set $K \subset E$ such that

$$\sup_{x\in E} \mathbb{E}_x(\exp(\gamma\sigma_K)) < \infty,$$

where σ_K is the first hitting time of K, $\sigma_K = \inf\{t > 0 : X_t \in K\}$. Wu [21] calls this property a *uniform hyper-exponential recurrence*, and we prove that the property implies the uniform large deviation principle (Theorem 2.3 and Theorem 3.12 below). As an example, a one-dimensional diffusion process satisfies the uniform hyper-exponential recurrence, and thus the uniform large deviation principle, if

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both boundaries are an entrance in Feller's classification of the boundaries (Example 3.1). On the other hand, we see that if X is not conservative, the tightness property implies a fast explosion in the sense that the lifetime ζ is exponentially integrable: for some $\gamma > 0$,

$$\sup_{x\in E} \mathbb{E}_x(\exp(\gamma\zeta)) < \infty.$$

There exist two key items in the proof of these facts: one is an inequality due to Stollman and Voigt (see (2.6)), and the other is the identification of Donsker-Varadhan's I-function (see (2.5)) with the Dirichlet form (Proposition 2.4). Combining these facts with the tightness property, we can show that the subset of probability measures on E defined by $\{u^2 \cdot m : \int_E u^2 dm = 1,\ \mathcal{E}(u,u) \le l\}$, $l > 0$, is compact with respect the weak topology, which leads us to the existence of the ground state (Lemma 2.6). Here $\mathcal{E}$ is the Dirichlet form generated by X (see (2.1)).

We finally discuss sufficient conditions for a part process on an open set to be irreducible, because this property is needed for the proof of the uniform hyper-exponential recurrence (Remark 3.7, Lemma 3.9).

2. Existence of the ground state

Let E be a locally compact separable metric space, $E_\Delta = E \cup \{\Delta\}$ the one point compactification of E, and m a positive Radon measure on E with full support. Let $X = (\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\ge 0}, X_t, \mathbb{P}_x, \zeta)$ be an m-symmetric Borel right process having left limits on $(0,\zeta)$. Here ζ is the lifetime $\zeta(\omega) = \inf\{s \ge 0 : X_s(w) = \Delta\}$ and $\{\mathcal{F}_t\}_{t\ge 0}$ is the minimal (augmented) admissible filtration.

Let $\{p_t\}_{t\ge 0}$ be the semigroup of X, $p_t f(x) = \mathbb{E}_x(f(X_t))$. By Lemma 1.4.3 in [7], $\{p_t\}_{t\ge 0}$ uniquely determines a strongly continuous Markovian semigroup $\{T_t\}_{t\ge 0}$ on $L^2(E;m)$. We define the Dirichlet form $(\mathcal{E}, \mathcal{D}(\mathcal{E}))$ on $L^2(E;m)$ generated by X:

$$\begin{cases} \mathcal{D}(\mathcal{E}) = \left\{ u \in L^2(E;m) : \lim\limits_{t\to 0} \dfrac{1}{t}(u - T_t u, u)_m < \infty \right\}, \\ \mathcal{E}(u,v) = \lim\limits_{t\to 0} \dfrac{1}{t}(u - T_t u, v)_m. \end{cases} \tag{2.1}$$

We know that the Dirichlet form $(\mathcal{E}, \mathcal{D}(\mathcal{E}))$ is *quasi-regular* ([12]).

A set $B \subset E_\Delta$ is said to be *nearly Borel* if for any probability measure μ on E_Δ there exist Borel sets $B_1,\ B_2$ of E_Δ such that $B_1 \subset B \subset B_2$ and

$$\mathbb{P}_\mu(X_t \in B_2 \setminus B_1,\ \exists t \ge 0) = 0.$$

A set $N \subset E$ is said to be *m-polar* if there exists a nearly Borel set $\widetilde{N} \subset E$ such that $N \subset \widetilde{N}$ and $\mathbb{P}_m(\sigma_{\widetilde{N}} < \infty) = 0$. A statement depending on $x \in A$ is said to hold q.e. on A if there exists an m-polar set $N \subset A$ such that the statement is true for every $x \in A \setminus N$ ("q.e." is an abbreviation of "quasi-everywhere").

Let us denote by $\{R_\alpha\}_{\alpha>0}$ the resolvent of X,

$$R_\alpha f(x) = \mathbb{E}_x\left(\int_0^\infty e^{-\alpha t} f(X_t)dt\right), \quad f \in \mathcal{B}_b(E),$$

where $\mathcal{B}_b(E)$ is the space of bounded Borel functions on E. We now introduce three properties of Borel right processes:

I. (**Irreducibility**) If a Borel set A is p_t-invariant, i.e., $\int_A p_t 1_{A^c} dm = 0$ for any $t > 0$, then A satisfies either $m(A) = 0$ or $m(A^c) = 0$. Here 1_{A^c} is the indicator function of the complement of A.

II. (**Strong Feller Property**) $p_t(\mathcal{B}_b(E)) \subset C_b(E)$, $t > 0$, where $C_b(E)$ is the space of bounded continuous functions.

III. (**Tightness Property**) For any $\epsilon > 0$, there exists a compact set K such that $\sup_{x\in E} R_1 1_{K^c}(x) \le \epsilon$.

Here we make remarks on the tightness property.

Remark 2.1. (i) If the measure m is finite, $m(E) < \infty$, and $\|R_1\|_{1,\infty} < \infty$, then $\|R_1 1_{K^c}\|_\infty \le \|R_1\|_{1,\infty} m(K^c)$ and property III is fulfilled. Here $\|R_1\|_{1,\infty}$ is the operator norm from $L^1(E;m)$ to $L^\infty(E;m)$.

(ii) If $R_1 1 \in C_\infty(E)$, then X is explosive and has property III. In fact, we have

$$\sup_{x\in E} R_1 1_{K^c}(x) = \sup_{x\in K^c} R_1 1_{K^c}(x) \le \sup_{x\in K^c} R_1 1(x).$$

Here $C_\infty(E)$ is the set of continuous functions vanishing at infinity. If X is a diffusion process generated by a locally elliptic operator, the property that $R_1 1 \in C_\infty(E)$ implies the compactness of R_1 as an operator on $L^\infty(E;m)$, as a result, on $L^2(E;m)$ ([5, Theorem 6.1]).

(iii) If $C_\infty(E)$ is invariant under R_1, $R_1(C_\infty(E)) \subset C_\infty(E)$, then $R_1 1 \in C_\infty(E)$ is equivalent to property III. In fact, for a compact set K, take a positive function $g \in C_\infty(E)$ such that $1_K \le g$. We then see from the invariance of $C_\infty(E)$ that $0 \le \lim_{x\to\infty} R_1 1_K(x) \le \lim_{x\to\infty} R_1 g(x) = 0$. Hence for any $\epsilon > 0$ there exists a compact set K such that

$$\limsup_{x\to\infty} R_1 1(x) \le \limsup_{x\to\infty} R_1 1_K(x) + \limsup_{x\to\infty} R_1 1_{K^c}(x) \le \sup_{x\in E} R_1 1_{K^c}(x) \le \epsilon,$$

which implies $R_1 1 \in C_\infty(E)$. Hence, if $C_\infty(E)$ is invariant under R_1 and X is conservative, $p_t 1 = 1$, then X does not have the tightness property; in particular, the Ornstein-Uhlenbeck process does not.

(iv) If the Markov process X is conservative, then property III implies that X is positive recurrent (Lemma 3.2).

It follows from property II that the transitions probability $p_t(x, dy)$ is absolutely continuous with respect to m:

$$p_t(x, dy) = p_t(x, y)m(dy) \quad \text{for each } t > 0,\ x \in E. \tag{2.2}$$

As a result, the resolvent kernel is also absolutely continuous with respect to m: $R_\beta(x, dy) = R_\beta(x, y)m(dy)$. By [7, Lemma 4.2.4] the density $R_\beta(x, y)$ is assumed to be a non-negative Borel function such that $R_\beta(x, y)$ is symmetric and β-excessive in x and in y. Under the absolute continuity condition, "quasi-everywhere" statements are strengthened to "everywhere" ones.

A positive measure μ is said to be *smooth* if there exists a positive continuous additive functional A of X such that for any positive Borel function f and γ-excessive function h ($\gamma \ge 0$), that is, $e^{-\gamma t} p_t h \le h$,

$$\lim_{t\to 0} \frac{1}{t} \mathbb{E}_{h\cdot m}\left[\int_0^t f(X_s)dA_s\right] = \int_X f(x)h(x)\mu(dx). \tag{2.3}$$

Here, $\mathbb{E}_{h\cdot m}[\,\cdot\,] = \int_X \mathbb{E}_x[\,\cdot\,]h(x)m(dx)$.

Following Z.-Q. Chen [1], we introduce classes of potentials.

Definition 2.1. A positive smooth measure μ is said to be in the class $\mathcal{K}_\infty$ if for any $\epsilon > 0$ there exist a compact subset K and a positive constant $\delta > 0$ such that for all measurable sets $B \subset K$ with $\mu(B) < \delta$,

$$\sup_{x \in E} \int_{K^c \cup B} R_1(x,y)\mu(dy) \leq \epsilon.$$

Under the condition for X being transient, the class $\mathcal{K}_\infty$ is usually defined by using the Green kernel, i.e., the 0-resolvent density, and a measure μ in the class is said to be *Green-tight*. Here we use the 1-resolvent density to deal with recurrent processes. The next lemma is proven by Z.-Q. Chen ([1, Theorem 4.2]). We give a proof for completion.

Lemma 2.2. *If X satisfies II and III, then the measure m belongs to $\mathcal{K}_\infty$.*

Proof. By the definition of property III, there exists a compact set K such that $\sup_{x\in E}\int_{K^c} R_1(x,y)\mu(dy) \leq \epsilon/2$. Suppose that for any $\delta > 0$ there exists a Borel set $B \subset K$ with $m(B) \leq \delta$ such that $\sup_{x\in E} R_1 1_B(x) > \epsilon/2$. Then there exists a sequence $\{B_n\}_{n=1}^\infty$ of Borel subsets of K such that $m(B_n) \leq 1/2^n$ and $\sup_{x\in K} R_1 1_{B_n}(x) > \epsilon/2$. Define $A_n = \bigcup_{k=n}^\infty B_k$. Then $m(A_n)$ is less than $1/2^{n-1}$ and decreasingly converges to zero as $n \to \infty$. Hence $R_1 1_{A_n}$ decreasingly converges to zero point-wise. Since $R_1 1_{A_n}$ is continuous by the property II, $R_1 1_{A_n}$ uniformly converges to zero on K. This is contradictory to $\sup_{x\in K} R_1 1_{A_n}(x) \geq \sup_{x\in K} R_1 1_{B_n}(x) > \epsilon/2$. $\square$

We denote by $\mathcal{P}$ the set of probability measures on E. Define the function $I_\mathcal{E}$ on $\mathcal{P}$ of probability measures on E by

$$I_\mathcal{E}(\nu) = \begin{cases} \mathcal{E}(\sqrt{f}, \sqrt{f}) & \text{if } \nu = f\cdot m,\ \sqrt{f} \in \mathcal{D}(\mathcal{E}), \\ \infty & \text{otherwise.} \end{cases} \tag{2.4}$$

The space $\mathcal{P}$ is supposed to be equipped with the weak topology. Given $\omega \in \Omega$ with $0 < t < \zeta(\omega)$, let $L_t(\omega) \in \mathcal{P}$ be the normalized occupation distribution: for a Borel set A of E,

$$L_t(\omega)(A) = \frac{1}{t}\int_0^t 1_A(X_s(\omega))ds.$$

We proved the next theorem in [18].

Theorem 2.3. *Assume that X satisfies I–III.*

(i) *For each open set $G \subset \mathcal{P}$,*

$$\liminf_{t\to\infty} \frac{1}{t} \log \mathbb{P}_x\left(L_t \in G, t < \zeta\right) \geq -\inf_{\nu \in G} I_\mathcal{E}(\nu).$$

(ii) *For each closed set $K \subset \mathcal{P}$,*

$$\limsup_{t\to\infty} \frac{1}{t} \log \sup_{x\in E} \mathbb{P}_x\left(L_t \in K, t < \zeta\right) \leq -\inf_{\nu \in K} I_\mathcal{E}(\nu).$$

We define the function space $\mathcal{D}^+$ by

$$\mathcal{D}^+ = \left\{R_\alpha f:\ \alpha > 0,\ f \in L^2(E;m) \cap C_b^+(E) \text{ and } f \not\equiv 0\right\},$$

where $C_b^+(E)$ denotes the set of non-negative bounded continuous functions. We see that any function in $\mathcal{D}^+(A)$ is strictly positive by the irreducibility I. Define the operator A on $\mathcal{D}^+$ by $AR_\alpha f = \alpha R_\alpha f - f$ and the function I on $\mathcal{P}$ by

$$I(\nu) = -\inf_{\substack{u\in\mathcal{D}^+\\ \epsilon>0}} \int_E \frac{Au}{u+\epsilon} d\nu. \tag{2.5}$$

The function I is a version of the Donsker-Varadhan *I-function* introduced in [6]. Note that since the Markov process X is allowed to have a finite lifetime, the function $u = R_\alpha f \in \mathcal{D}^+$ is not alway uniformly lower-bounded by a positive constant even if f is so, and consequently the function Au/u is not always bounded. By adding a positive constant ϵ, the function $Au/(u+\epsilon)$ is bounded continuous, and consequently the I-function defined by (2.5) is lower semicontinuous on $\mathcal{P}$ with respect to the weak topology. This is a reason why we modify the Donsker-Varadhan I-function. In spite of this modification, we can identify the I-function with the Dirichlet form ([7, Theorem 6.4.2]):

Proposition 2.4.

$$I(\nu) = I_{\mathcal{E}}(\nu), \qquad \nu \in \mathcal{P}.$$

We define the subset $\mathcal{P}_M$ of $\mathcal{P}$ by

$$\mathcal{P}_M = \left\{u^2\cdot m : u \in \mathcal{D}(\mathcal{E}),\ \int_E u^2 dm = 1,\ \mathcal{E}(u,u) \le M\right\}, \quad M > 0.$$

Lemma 2.5. *The set $\mathcal{P}_M$ is compact in $\mathcal{P}$.*

Proof. Recall the inequality in [14]: for any $\beta > 0$ and any smooth measure μ,

$$\int_E u^2(x)\mu(dx) \le \|R_\beta\mu\|_\infty \cdot \left(\mathcal{E}(u,u) + \beta\int_E u^2 dm\right), \quad u \in \mathcal{D}(\mathcal{E}). \tag{2.6}$$

Combining property III with this inequality, we see that $\mathcal{P}_M$ is tight. Indeed, for any compact set $K \subset E$ and any $u^2\cdot m \in \mathcal{P}_M$,

$$\int_{K^c} u^2 dm \le \|R_1 1_{K^c}\|_\infty \cdot \left(\mathcal{E}(u,u) + \int_E u^2 dm\right) \le (M+1)\|R_1 1_{K^c}\|_\infty. \tag{2.7}$$

Since $\mathcal{P}_M = \{\nu \in \mathcal{P} : I(\nu) \le M\}$ is closed by the lower semicontinuity of I, we have the lemma. $\square$

Let λ_2 be the bottom of the spectrum:

$$\lambda_2 = \inf\left\{\mathcal{E}(f,f) : f \in \mathcal{D}(\mathcal{E}),\ \int_E f^2 dm = 1\right\}. \tag{2.8}$$

A function ϕ_0 on E is called a *ground state* of the L^2-generator for $\mathcal{E}$ if $\phi_0 \in \mathcal{D}(\mathcal{E})$, $\|\phi_0\|_2 = 1$ and $\mathcal{E}(\phi_0,\phi_0) = \lambda_2$.

Lemma 2.6 ([19])**.** *Assume that X satisfies I–III. Then there exists a ground state ϕ_0 uniquely up to a sign. ϕ_0 can be taken to be strictly positive on E.*

Proof. Let $\{u_n\}_{n=1}^\infty \subset \mathcal{D}(\mathcal{E})$ be a minimizing sequence, $\|u_n\|_2 = 1$, and $\lambda_2 = \lim_{n\to\infty}\mathcal{E}(u_n,u_n)$. We see from Lemma 2.5 that there exists a subsequence $\{u_{n_k}^2\cdot m\}_{k=1}^\infty$ such that $u_{n_k}^2\cdot m$ converges weakly to a probability measure $\nu = \phi_0^2\cdot m$, $\phi_0 \in \mathcal{D}(\mathcal{E})$, $\phi_0 \ge 0$. Since the function $I_{\mathcal{E}}$ is lower semicontinuous by Proposition 2.4, $I_{\mathcal{E}}(\phi^2 m) \le \lambda_2$. Hence the function ϕ_0 is just a ground state.

It follows from the inequality $\|\phi_0+\epsilon g\|_{\mathcal{E}}^2 \ge \lambda_2\|\phi_0+\epsilon g\|_2^2$ holding for any $g \in \mathcal{D}(\mathcal{E})$ and for any $\epsilon > 0$ that $\mathcal{E}(\phi_0,g) = \lambda_2(\phi_0,g)$. Hence $\alpha R_{\alpha-\lambda_2}\phi_0 = \phi_0$, $\alpha > \lambda_2$, which implies that ϕ_0 is strictly positive by irreducibility.

To prove the uniqueness of the ground state, we introduce a closed symmetric form $(\mathcal{E}^{\phi_0}, \mathcal{D}(\mathcal{E}^{\phi_0}))$ on $L^2(E; \phi_0^2 m)$ by

$$\begin{cases} \mathcal{E}^{\phi_0}(u,v) = \mathcal{E}(u\phi_0, v\phi_0) - \lambda_2(u\phi_0, v\phi_0), \\ \mathcal{D}(\mathcal{E}^{\phi_0}) = \{u \in L^2(E; \phi_0^2 \cdot m) : u\phi_0 \in \mathcal{D}(\mathcal{E})\}. \end{cases} \tag{2.9}$$

Since $1 \in \mathcal{D}(\mathcal{E}^{\phi_0})$, $\mathcal{E}^{\phi_0}(1,1) = 0$ and the associated resolvent $R_\alpha^{\phi_0}$ satisfies $R_\alpha^{\phi_0} f = \phi_0^{-1} R_{\alpha-\lambda_2}(f\phi_0)$, $\alpha > \lambda_2$, we see from the strict positivity of ϕ_0 that $(\mathcal{E}^{\phi_0}, \mathcal{D}(\mathcal{E}^{\phi_0}))$ is an irreducible recurrent Dirichlet form so that f is constant whenever $f \in \mathcal{D}(\mathcal{E}^{\phi_0})$, $\mathcal{E}^{\phi_0}(f,f) = 0$. Let ψ_0 be another ground state. Then $\psi_0 = f\phi_0$ with $f = \psi_0/\phi_0 \in \mathcal{D}(\mathcal{E}^{\phi_0})$, $\mathcal{E}^{\phi_0}(f,f) = \mathcal{E}(\psi_0, \psi_0) - \lambda_2 = 0$, which yields that f is constant and $\psi_0 = \pm\phi_0$. □

Let $\{u_n\}_{n=1}^\infty \subset \mathcal{D}(\mathcal{E})$ be a minimizing sequence in the proof of Lemma 2.6. We would like to emphasize that the tightness of $\{u_n^2 \cdot m\}_{n=1}^\infty \subset \mathcal{P}$ and the lower semicontinuity of the function $I_\mathcal{E}$ with respect to the weak topology are used for the proof of the existence of the ground state, while the $\mathcal{E}_1$-weak compactness of $\{u_n\}_{n=1}^\infty$ in $\mathcal{D}(\mathcal{E})$ and the $\mathcal{E}_1$-weakly lower semicontinuity of $\mathcal{E}$ are usually used (e.g. [11, Section 11.1]). If X is a Brownian motion on a Riemannian manifold or a symmetric α-stable process, then we can show by employing the Rellich theorem that for $\mu \in \mathcal{K}_\infty$ the embedding of $(\mathcal{D}(\mathcal{E}), \mathcal{E}_1)$ to $L^2(E;\mu)$ is compact ([16, Proposition 2], [17, Theorem 2.7]). Hence due to Lemma 2.2 and Proposition 2.4 we see that the resolvent R_α, $\alpha > 0$, is a compact operator on $L^2(E;m)$ and the level set $\{\nu \in \mathcal{P} : I_\mathcal{E}(\nu) \le \ell\}$ is a compact subset of $\mathcal{P}$. See [19] for another application of the existence of ground states.

3. Tightness property

In this section, we will show that the tightness property implies a strong recurrence if X is conservative and a fast explosion if X is not conservative.

Lemma 3.1. *An irreducible Borel right process X with* (2.2) *satisfies one of the next two properties:*

(a) *(Conservative)* $\mathbb{P}_x(\zeta < \infty) = 0$ *for all* $x \in E$.
(b) *(Explosive)* $\mathbb{P}_x(\zeta < \infty) > 0$ *for all* $x \in E$.

Proof. Suppose $O := \{x \in E : \mathbb{P}_x(\zeta < \infty) > 0\}$ is not empty. Since $g(x) := \mathbb{P}_x(\zeta < \infty)$ is an excessive function, the set O is a finely open set (e.g. [7, Theorem A.2.7]) and not m-polar. Indeed, if O is m-polar, then O is polar by the absolute continuity of the transition probability (2.2), and so $\mathbb{P}_x(\sigma_O < \infty) = 0$ for all $x \in E$, which is contradictory to the fact that $\mathbb{P}_x(\sigma_O < \infty) > 0$ for $x \in O$. (Note that [7, Theorem 4.1.2] holds for Borel right processes.) Since $O = \bigcup_{n=1}^\infty F_n$, $F_n = \{x \in E : \mathbb{P}_x(\zeta < \infty) \ge 1/n\}$, some F_n are not m-polar. Due to (2.2), we see from [7, Exercise 4.7.1] that $\mathbb{P}_x(\sigma_{F_n} < \infty) > 0$ for all $x \in E$. Note that the set F_n is finely closed and thus $X_{\sigma_{F_n}} \in F_n$ on $\{\sigma_{F_n} < \infty\}$. We then have

$$\begin{aligned} \mathbb{P}_x(\zeta < \infty) &= \mathbb{P}_x(\zeta < \infty,\ \sigma_{F_n} < \infty) + \mathbb{P}_x(\zeta < \infty,\ \sigma_{F_n} = \infty) \\ &\ge \mathbb{P}_x(\zeta(\theta_{\sigma_{F_n}}) < \infty,\ \sigma_{F_n} < \infty) \\ &= \mathbb{E}_x(\mathbb{P}_{X_{\sigma_{F_n}}}(\zeta < \infty); \sigma_{F_n} < \infty) \\ &\ge \frac{1}{n}\mathbb{P}_x(\sigma_{F_n} < \infty) > 0. \end{aligned}$$

□

3.1. **Conservative case.**

Lemma 3.2. *Assume X satisfies I–III. If X is, in addition, conservative, then it is positively recurrent.*

Proof. If X is conservative, then the tightness property (III) implies that for any $\epsilon > 0$, there exists a compact set K such that $\inf_{x\in E} R_1 1_K(x) \geq 1-\epsilon$. Since the function $R_1 1_K$ is in $L^1(E;m)$, m is finite, and thus $1 \in \mathcal{D}(\mathcal{E})$, $\mathcal{E}(1,1) = 0$. Hence X is positive recurrent ([7, Theorem 1.6.3]). $\square$

Remark 3.1. Suppose that X is conservative and its semigroup satisfies the invariance of $C_\infty(E)$, $p_t(C_\infty(E)) \subset C_\infty(E)$. Since

$$\lim_{x\to\infty} R_1 1_{K^c}(x) = 1 - \lim_{x\to\infty} R_1 1_K(x) = 1 - 0 = 1,$$

X does not have the tightness property.

Lemma 3.3. *Assume X satisfies* (2.2)*. Then*

$$\sup_{x\in X} p_t 1(x) = \operatorname*{ess\,sup}_{x\in X} p_t 1(x).$$

Proof. Let $M = \sup_{x\in X} p_t 1(x)$, $\widetilde{M} = \operatorname{ess\,sup}_{x\in X} p_t 1(x)$. Suppose $M > \widetilde{M}$ and take r so that $M > r > \widetilde{M}$. Since the function $p_t 1$ is excessive, the set $O = \{x \in X : p_t 1(x) > r\}$ is finely open and $m(O) = 0$ by the definition of $\widetilde{M}$. Hence by Lemma 4.1.4 and Theorem 4.1.2 in [7], the set O is polar and thus empty by the argument in the proof of Lemma 3.1. Therefore $p_t 1(x) \leq r$, which is contradictory to $M > r$. $\square$

Let us denote by $\|p_t\|_{p,p}$ the operator norm of p_t from $L^p(X;m)$ to $L^p(X;m)$ and put

$$-\lambda_p = \lim_{t\to\infty} \frac{1}{t} \log \|p_t\|_{p,p}, \quad 1 \leq p \leq \infty.$$

$-\lambda_p$ is the long time exponential growth bound of the semigroup $\{p_t\}_{t\geq 0}$. The next theorem gives us a probabilistic interpretation of λ_∞ (cf. [13]).

Theorem 3.4. *Assume X satisfies* (2.2)*. Then*

$$\lambda_\infty = \sup\left\{\lambda \geq 0 : \sup_{x\in E} \mathbb{E}_x(e^{\lambda\zeta}) < \infty\right\}.$$

Proof. Let γ be the right hand side of (3.4). Since for $\lambda < \gamma$,

$$\|p_t\|_{\infty,\infty} = \sup_{x\in E} \mathbb{P}_x(t < \zeta) \leq e^{-\lambda t} \sup_{x\in E} \mathbb{E}_x(e^{\lambda\zeta}),$$

$\gamma \leq \lambda_\infty$. In particular, if $\lambda_\infty = 0$, then $\gamma = 0$.

For $0 < \lambda < \lambda_\infty$, let $p_t^\lambda = e^{\lambda t} p_t$. Then since

$$\lim_{t\to\infty} \frac{1}{t} \log \|p_t^\lambda\|_{\infty,\infty} = \lambda - \lambda_\infty < 0,$$

$$\int_0^\infty \|p_t^\lambda\|_{\infty,\infty} dt = \int_0^\infty \sup_{x\in E} \mathbb{E}_x\left(e^{\lambda t}; t < \zeta\right) dt < \infty.$$

Hence

$$\sup_{x\in E} \int_0^\infty \mathbb{E}_x\left(e^{\lambda t}; t < \zeta\right) dt = \sup_{x\in E} \left(\frac{\mathbb{E}_x\left(e^{\lambda\zeta}\right) - 1}{\lambda}\right) < \infty, \tag{3.1}$$

and so $\gamma \geq \lambda_\infty$. $\square$

Let us extend the resolvent operator; for $\lambda \geq 0$,

$$R_{-\lambda} f(x) = \mathbb{E}_x \left(\int_0^\infty e^{\lambda t} f(X_t) dt \right).$$

We then see from (3.1) that for $\lambda > 0$,

$$\|R_{-\lambda}\|_{\infty,\infty} < \infty \iff \sup_{x \in E} \mathbb{E}_x(e^{\lambda\zeta}) < \infty. \tag{3.2}$$

It holds that if $\lambda_\infty > 0$, then $\sup_{x\in E} \mathbb{E}_x(e^{\lambda_\infty \zeta}) = \infty$. Indeed, we see from (3.2) that if $\sup_{x\in E} \mathbb{E}_x(e^{\lambda_\infty\zeta}) < \infty$, then $\|R_{-\lambda_\infty}\|_{\infty,\infty} < \infty$. Noting that

$$R_{-\lambda_\infty - \epsilon} = R_{-\lambda_\infty} + \epsilon R^2_{-\lambda_\infty} + \epsilon^2 R^3_{-\lambda_\infty} + \cdots$$

([10, III, §6]), we see that if $0 < \epsilon < 1/\|R_{-\lambda_\infty}\|_{\infty,\infty}$, then $\|R_{-\lambda_\infty-\epsilon}\|_{\infty,\infty} < \infty$. Using (3.2) again, we have $\sup_{x\in E} \mathbb{E}_x(e^{(\lambda_\infty+\epsilon)\zeta}) < \infty$, which is contradictory to Theorem 3.4. Therefore, we have the next corollary.

Corollary 3.5. *Suppose $\lambda_\infty > 0$. Then*

$$\sup_{x\in E} \mathbb{E}_x\left(\exp(\lambda\zeta)\right) < \infty \iff \lambda < \lambda_\infty.$$

Z.-Q. Chen [1, Theorem 4.1] proved:

Theorem 3.6. *Suppose X is irreducible and satisfies* (2.2). *If the measure m belongs to $\mathcal{K}_\infty$, then λ_p is independent of p.*

Remark 3.7. The strong Feller property is not assumed in Theorem 3.6, while the L^p-independence is proven in [7, Theorem 6.4.3] under the assumptions (I)–(III). This extension is crucial when we show the L^p-independence for part processes of X. For a further extension of Theorem 3.6, see the recent papers [2], [3] of Z.-Q. Chen.

Combining Theorem 3.6 with Corollary 3.5, we have

Corollary 3.8. *Suppose X is irreducible and satisfies* (2.2). *If $m \in \mathcal{K}_\infty$ and $\lambda_2 > 0$, then*

$$\sup_{x\in E} \mathbb{E}_x\left(\exp(\lambda\zeta)\right) < \infty \iff \lambda < \lambda_2.$$

Let $K \subset E$ be a compact set and $D := K^c$, the complement of K. Let X^D be the part process on D:

$$X^D = \begin{cases} X_t, & t < \tau_D, \\ \Delta, & t \geq \tau_D, \quad \tau_D = \inf\{t \geq 0 : X_t \notin D\}. \end{cases}$$

Define the (quasi-regular) Dirichlet form $(\mathcal{E}^D, \mathcal{D}(\mathcal{E}^D))$ on $L^2(D; m)$ by

$$\begin{cases} \mathcal{E}^D = \mathcal{E}, \\ \mathcal{D}(\mathcal{E}^D) = \{u \in \mathcal{D}(\mathcal{E}) : u = 0 \text{ q.e. on } K\}. \end{cases} \tag{3.3}$$

Then $(\mathcal{E}^D, \mathcal{D}(\mathcal{E}^D))$ is the Dirichlet space generated by X^D ([7, Theorem 4.4.3]).

Let λ^D be the principal eigenvalue of the spectrum of $(\mathcal{E}^D, \mathcal{D}(\mathcal{E}^D))$:

$$\lambda^D = \inf\left\{\mathcal{E}(u,u) : u \in \mathcal{D}(\mathcal{E}^D),\ \int_D u^2 dm = 1\right\}. \tag{3.4}$$

Lemma 3.9. *Suppose that X satisfies I–III and is conservative. For any compact set K with non-empty interior K^o, the principal eigenvalue λ^D, $D = K^c$, is positive.*

Proof. Let $\{\phi_n\}_{n=1}^{\infty} \subset \mathcal{D}(\mathcal{E}^D) \cap C_0(D)$ be an approximating sequence in (3.4) such that $\mathcal{E}(\phi_n, \phi_n) \to \lambda^D$. Let $\{\phi_{n_k}^2 \cdot m\}_{k=1}^{\infty}$ be a subsequence of $\{\phi_n^2 \cdot m\}_{n=1}^{\infty}$ weakly converging to $\phi_0^2 \cdot m$, $\phi_0 \in \mathcal{D}(\mathcal{E})$. Then

$$1 = \limsup_{k\to\infty} \int_{E\setminus K^o} \phi_{n_k}^2 dm \le \int_{E\setminus K^o} \phi_0^2 dm,$$

and thus ϕ_0 equals 0, m-a.e. on K^o. In particular, the function ϕ_0 is not constant on E, because $m(K^o) > 0$ by the assumption on m. Hence we have $\mathcal{E}(\phi_0, \phi_0) > 0$. In fact, if $\mathcal{E}(\phi_0, \phi_0) = 0$, then ϕ_0 must be a constant by the irreducible recurrence of $(\mathcal{E}, \mathcal{D}(\mathcal{E}))$ ([9, Theorem 1.3]). We now conclude that

$$\lambda^D = \liminf_{k\to\infty} \mathcal{E}(\phi_{n_k}, \phi_{n_k}) \ge \mathcal{E}(\phi_0, \phi_0) > 0.$$

□

We write $\mathcal{K}_\infty(R_1)$ for $\mathcal{K}_\infty$ to express the dependence of the 1-resolvent. Let R_1^D be the 1-resolvent of X^D. Denote by m^D the restriction of m to D, $m^D(\bullet) = m(D \cap \bullet)$.

Lemma 3.10. *Let K be a compact set. Then $m^D \in \mathcal{K}_\infty(R_1^D)$, $D = K^c$.*

Proof. Let $\widetilde{K}$ and δ be a compact set and a positive constant in Definition 2.1. We can suppose that the interior of $\widetilde{K}$ contains K. Let G be a relatively compact open set such that $K \subset G \subset \bar{G} \subset \widetilde{K}$ and $m(G \setminus K) < \delta$. Then $\widetilde{K} \cap G^c$ is a compact subset of D and

$$R_1^D 1_{(\widetilde{K}\cap G^c)^c} = R_1^D 1_{\widetilde{K}^c \cup (G\setminus K)} \le R_1 1_{\widetilde{K}^c} + R_1 1_{G\setminus K} \le \epsilon.$$

Moreover, $R_1^D 1_B \le R_1 1_B$ for any Borel set $B \subset \widetilde{K} \cap G^c$. □

It follows from (2.7) that

$$\int_D u^2 dm = \int_E u^2 1_D dm \le \|R_1 1_D\|_\infty \cdot \left(\mathcal{E}(u,u) + \int_E u^2 dm\right), \quad u \in \mathcal{D}(\mathcal{E}^D),$$

and thus

$$1 \le \|R_1 1_D\|_\infty \cdot (\lambda^D + 1). \tag{3.5}$$

The tightness property implies that there exists a sequence $\{K_n\}_{n=1}^{\infty}$ of compact sets such that $\bigcup_{n=1}^{\infty} K_n = E$ and $\|R_1 1_{K_n^c}\|_\infty \to 0$ as $n \to \infty$. Hence we see from (3.5) that for $D_n = K_n^c$,

$$\lambda^{D_n} \uparrow \infty \quad \text{as} \quad n \to \infty. \tag{3.6}$$

Note that if X is conservative, then the lifetime of X^D equals the hitting time of K. Combining Lemma 3.10 with Corollary 3.8, we know that if X^{D_n} is irreducible, then

$$\sup_{x\in D_n} \mathbb{E}_x(\exp(\gamma \sigma_{K_n})) < \infty \Longleftrightarrow \gamma < \lambda^{D_n}. \tag{3.7}$$

Note that

$$\sup_{x\in D} \mathbb{E}_x(\exp(\sigma_K)) = \sup_{x\in E} \mathbb{E}_x(\exp(\sigma_K)). \tag{3.8}$$

Indeed, let $x_0 \in K \setminus K^r$, where K^r is the regular set of K, $\mathbb{P}_x(\sigma_K = 0) = 1$. Then since

$$\begin{aligned}\mathbb{E}_{x_0}(\exp(\sigma_K)) =& \mathbb{E}_{x_0}(\exp(\sigma_K); X_t \in K) + \mathbb{E}_{x_0}(\exp(\sigma_K); X_t \in D)\\ \leq& e^t\, \mathbb{P}_{x_0}(X_t \in K) + e^t\, \mathbb{E}_{x_0}(\exp(t + \sigma_K(\theta_t)); X_t \in D)\\ \leq& e^t\, \mathbb{P}_{x_0}(X_t \in K) + e^t\, \mathbb{E}_{x_0}(\mathbb{E}_{X_t}(\exp(\sigma_K)); X_t \in D)\\ \leq& e^t\, \mathbb{P}_{x_0}(X_t \in K) + e^t \sup_{x\in D} \mathbb{E}_x(\exp(\sigma_K))\end{aligned}$$

and

$$\mathbb{P}_{x_0}(X_t \in K) \leq \mathbb{P}_{x_0}(\sigma_K \leq t) \longrightarrow 0 \quad \text{as } t \downarrow 0,$$

we have (3.8) and thus

$$\sup_{x\in E} \mathbb{E}_x(\exp(\gamma\sigma_{K_n})) < \infty \Longleftrightarrow \gamma < \lambda^{D_n}. \tag{3.9}$$

Hence we have from (3.6) and (3.9) the following:

Lemma 3.11. *Suppose that X satisfies I–III and is conservative. If there exists an increasing sequence $\{K_n\}_{n=1}^{\infty}$ of compact sets such that $\bigcup_{n=1}^{\infty} K_n = E$ and X^{D_n}, $D_n = K_n^c$, are irreducible, then X has the following property:*

(H) *For any $\gamma > 0$ there exists a compact set K such that* $\sup_{x\in E} \mathbb{E}_x(\exp(\gamma\sigma_K)) < \infty.$

Property (H) is said to be a *uniform hyper-exponential recurrence* ([21]). We will give sufficient conditions for the part process X^D being irreducible (Lemma 4.2, Lemma 4.3).

Noting that

$$p_t(x, U) = 0 \text{ for } \forall t > 0 \quad \Longleftrightarrow \quad \mathbb{P}_x(\sigma_U < \infty) = 0,$$

we see that if X is irreducible, the semigroup $\{p_t\}_{t\geq 0}$ is *topological transitive*; that is, for all non-empty open sets U and $x \in E$, there exists $t > 0$ such that $p_t(x, U) > 0$. Therefore, Theorem 1.2 in Wu [21] leads us to:

Theorem 3.12. *Suppose X satisfies I–III and is conservative. If there exists an increasing sequence $\{K_n\}_{n=1}^{\infty}$ of compact sets such that $\bigcup_{n=1}^{\infty} K_n = E$ and X^{D_n}, $D_n = K_n^c$, are irreducible, then the uniform large deviation principle holds: for each open set G of $\mathcal{P}$,*

$$\liminf_{t\to\infty} \frac{1}{t} \log \inf_{x\in E} \mathbb{P}_x(L_t \in G) \geq - \inf_{\mu\in G} I_{\mathcal{E}}(\mu).$$

Example 3.1 (One-dimensional diffusion processes)**.** Let us consider a one-dimensional diffusion process $X = (X_t, \mathbb{P}_x, \zeta)$ on an open interval $I = (r_1, r_2)$ such that $\mathbb{P}_x(X_{\zeta-} = r_1 \text{ or } r_2,\ \zeta < \infty) = \mathbb{P}_x(\zeta < \infty),\ x \in I$, and $\mathbb{P}_a(\sigma_b < \infty) > 0$ for any $a, b \in I$. The diffusion X is symmetric with respect to its canonical measure m and satisfies I and II. The boundary point r_i of I is classified into four classes: *regular boundary, exit boundary, entrance boundary, and natural boundary* ([8, Chapter 5]):

(a) If r_2 is a regular or exit boundary, then $\lim_{x\to r_2} R_1 1(x) = 0$.
(b) If r_2 is an entrance boundary, then $\lim_{r\to r_2} \sup_{x\in(r_1,r_2)} R_1 1_{(r,r_2)}(x) = 0$.
(c) If r_2 is a natural boundary, then $\lim_{x\to r_2} R_1 1_{(r,r_2)}(x) = 1$ and thus $\sup_{x\in(r_1,r_2)} R_1 1_{(r,r_2)}(x) = 1$.

Therefore, tightness property III is fulfilled if and only if no natural boundaries are present. As a corollary of equation (3.6), if r_2 is an entrance boundary, for any $\lambda > 0$ there exists $r_1 < r < r_2$ such that

$$\sup_{x>r} \mathbb{E}_x(\exp(\lambda\sigma_r)) < \infty,$$

where σ_r is the first hitting time of $\{r\}$. Therefore, if both the boundaries are entrance, then the uniform large deviation holds. Let $p_t(x, y)$ be the transition probability density of X. We see from [15] that if X is *uniformly ergodic*, that is, there exists a positive constant M such that

$$p_1(x, z) \leq M \cdot p_1(y, z) \quad \text{for any } x, y, z \in I,$$

then it satisfies the uniform large deviation principle. Nevertheless, we do not know that a one-dimensional diffusion process with entrance boundaries always satisfies the uniform ergodicity.

We see from [8] that the Ornstein-Uhlenbeck process on the one-dimensional space $\mathbb{R}$ has natural boundaries and its semigroup keeps $C_\infty(\mathbb{R})$ invariant. Hence due to Remark 3.1, we see that the Ornstein-Uhlenbeck process does not possess the tightness property. Moreover, it is known in [21] that the Ornstein-Uhlenbeck process does not satisfy the uniform large deviation, while it satisfies the locally uniform large deviation.

4. Irreducibility of part processes

In this section, we consider conditions for part processes being irreducible. If X is a diffusion process generated by a locally uniform elliptic operator, then its part process on a domain is irreducible ([7, Corollary 4.6.4, Example 4.6.1]). More generally, we have:

Lemma 4.1. *Assume $R_1 1 \in C_\infty(E)$. If $D \subset E$ is a connected open set, then X^D satisfies I–III.*

Proof. By the assumption, X is a doubly Feller process; that is, it satisfies the strong Feller property and the invariance of $C_\infty(E)$. We then know from Chung [4] that X^D has the strong Feller property. Hence Exercise 4.6.3 in [7] leads us to this lemma. □

We next treat jump processes. Let $(N(x, dy), H_t)$ be a Lévy system of X. We make the next assumption:

$$\text{(J)} \quad \begin{cases} (i) & \text{If } m(B) > 0, \text{ then } N(x, B) > 0 \text{ for any } x \in E. \\ (ii) & \{x \in E : \mathbb{P}_x(\inf\{t > 0 : H_t > 0\} = 0) = 1\} = E. \end{cases}$$

Lemma 4.2. *Assume* (J). *Then for any compact set $F \subset D$ with $m(F) > 0$, $\mathbb{P}_x(\sigma_F < \tau_D) > 0$.*

Proof. Let $x \notin D \setminus F$ and take $r > 0$ such that $B(x, r) \cap F = \emptyset$. Then

$$\mathbb{E}_x\left(\sum_{0<s<\tau_D} 1_{B(x,r)}(X_{s-}) 1_F(X_s)\right) = \mathbb{E}_x\left(\int_0^{\tau_D} 1_{B(x,r)}(X_s) N(X_s, F) dH_s\right).$$

The right hand side is positive by the assumption, which leads us to the lemma. □

Lemma 4.3. *Assume* (J). *Let $K \subset D$ be a set with $m(K) > 0$. Then $R_1^D(x, K) > 0$ for any $x \in D$.*

Proof. Since

$$\int_D R_1^D(x,K)dm = \int_D R_1^D 1(x) 1_K(x) dm > 0,$$

the set $\{x \in D : R_1^D(x,K) > 0\}$ is of positive m-measure. Take a compact set F such that $F \subset \{x \in D : R_1^D(x,K) > 0\}$ and $m(F) > 0$. Then

$$\begin{aligned} R_1^D(x,K) &= \mathbb{E}_x\left(\int_0^{\tau_D} e^{-t} 1_K(X_t)dt\right) \geq \mathbb{E}_x\left(\int_{\sigma_F}^{\tau_D} e^{-t} 1_K(X_t)dt; \sigma_F < \tau_D\right) \\ &= \mathbb{E}_x\left(e^{-\sigma_F} R_1^D(X_{\sigma_F}, K); \sigma_F < \tau_D\right). \end{aligned}$$

The right hand side is positive by Lemma 4.2. □

4.1. **Explosive case.** If X is explosive, then the principal eigenvalue λ is positive. Indeed, if the ground state ϕ_0 satisfies $\mathcal{E}(\phi_0, \phi_0) = 0$, then $\phi_0 = 0$ by transience. It follows from Corollary 3.8 that

$$\sup_{x \in E} \mathbb{E}_x(\exp(\gamma\zeta)) < \infty \iff \gamma < \lambda_2.$$

In particular, property (b) in Lemma 3.1 can be strengthened to $\mathbb{P}_x(\zeta < \infty) = 1$.

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