PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 141, Number 12, December 2013, Pages 4425–4426
S 0002-9939(2013)11733-8
Article electronically published on August 9, 2013

CORRIGENDUM TO "INTEGRAL EQUATIONS, IMPLICIT FUNCTIONS, AND FIXED POINTS"

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(Communicated by James E. Colliander)

Abstract. In the paper of this title in Proc. Amer. Math. Soc. **124** (1996), no. 8, 2383–2390, we defined a large contraction, proved a fixed point theorem based on it, and then used it to extend Krasnoselskii's fixed point theorem. In that last-mentioned theorem we neglected to add the assumption that the mapping set be closed. In this correction we state the relevant theorems, add the assumption, and explain why it is necessary.

1. Statement of the correction

In [1] we introduced the definition of a large contraction as follows.

Definition 1.1. Let (M,ρ) be a metric space and $B : M \to M$. B is said to be a *large contraction* if $\varphi,\, \psi \in M$, with $\varphi \neq \psi$, then $\rho(B\varphi, B\psi) < \rho(\varphi,\psi)$ and if $\forall \varepsilon > 0$ $\exists \delta < 1$ such that $[\varphi,\psi \in M,\, \rho(\varphi,\psi) \geq \varepsilon] \Rightarrow \rho(B\varphi, B\psi) \leq \delta\rho(\varphi,\psi)$.

We then proved the following result.

Theorem 1.2. *Let (M,ρ) be a complete metric space and B be a large contraction. Suppose there is an $x \in M$ and an $L > 0$ such that $\rho(x, B^n x) \leq L$ for all $n \geq 1$. Then B has a unique fixed point in M.*

As an application we extended a classical result of Krasnoselskii, which we now state. However, we neglected to ask that the set M be closed. In the proof we applied the mapping B to M and claimed that there is a fixed point using Theorem 1.2. It is clear that unless we ask that M be closed, we cannot say that M is complete, and, hence, Theorem 1.2 would not apply. Thus, we now state Theorem 1.3 in the correct way.

Theorem 1.3. *Let $(S, \|\cdot\|)$ be a Banach space, and let M be a closed, bounded, convex nonempty subset of S. Suppose that A, $B : M \to M$ and that*

(i) $$x, y \in M \Rightarrow Ax + By \in M,$$

(ii) *A is continuous and AM is contained in a compact subset of M, and*

(iii) *B is a large contraction.*

Then $\exists\, y \in M$ with $Ay + By = y$.

Received by the editors February 3, 2012.
2010 *Mathematics Subject Classification.* Primary 45D05, 26B10, 47H10.

Acknowledgments

The correction was communicated to the author by Professor Bo Zhang of Fayetteville State University. The author finds that in [2], Professor Sehie Park quoted Theorem 1.3, making the correction, but did not mention that the original was wrong. The author is very grateful to both Professors Zhang and Park. The author deeply regrets any errors that other authors may have made as a result of the original omission.

References

[1] T. A. Burton, *Integral equations, implicit functions, and fixed points*, Proc. Amer. Math. Soc. **124** (1996), no. 8, 2383–2390, DOI 10.1090/S0002-9939-96-03533-2. MR1346965 (96j:45001)

[2] Sehie Park, *Generalizations of the Krasnoselskii fixed point theorem*, Nonlinear Anal. **67** (2007), no. 12, 3401–3410, DOI 10.1016/j.na.2006.10.024. MR2350896 (2009a:47111)

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