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FAILURE OF KORENBLUM'S MAXIMUM PRINCIPLE IN BERGMAN SPACES WITH SMALL EXPONENTS

VLADIMIR BOŽIN AND BOBAN KARAPETROVIĆ

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ABSTRACT. The well-known conjecture due to B. Korenblum about the maximum principle in Bergman space A^p states that for $0 < p < \infty$ there exists a constant $0 < c < 1$ with the following property. If f and g are holomorphic functions in the unit disk $\mathbb{D}$ such that $|f(z)| \le |g(z)|$ for all $c < |z| < 1$, then $\|f\|_{A^p} \le \|g\|_{A^p}$. Hayman proved Korenblum's conjecture for $p = 2$, and Hinkkanen generalized this result by proving the conjecture for all $1 \le p < \infty$. The case $0 < p < 1$ of the conjecture has so far remained open. In this paper we resolve this remaining case of the conjecture by proving that Korenblum's maximum principle in Bergman space A^p does not hold when $0 < p < 1$.

1. Introduction

1.1. **Bergman spaces and the maximum principle.** Let $\mathcal{H}(\mathbb{D})$ be the space of all functions holomorphic in the unit disk $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$. The normalized Lebesgue area measure on the unit disk $\mathbb{D}$ will be denoted by A, that is,

$$dA(z) = \frac{1}{\pi}dxdy = \frac{1}{\pi}rdrd\theta, \text{ where } z = x + iy = re^{i\theta}.$$

For $0 < p < \infty$, the unweighted Bergman space A^p is the space of holomorphic functions in $L^p(\mathbb{D}, dA)$ (see [1, 3]). Therefore $A^p = \mathcal{H}(\mathbb{D}) \cap L^p(\mathbb{D}, dA)$. If $f \in A^p$, then we write

$$\|f\|_{A^p} = \left(\int_{\mathbb{D}} |f(z)|^p dA(z)\right)^{\frac{1}{p}}.$$

For $0 < c < 1$, let $\mathbb{A}_c$ denote the annulus defined by $c < |z| < 1$. Then the maximum modulus principle states that if a function f is holomorphic in the unit disk $\mathbb{D}$ and if $|f| \le M$ in the annulus $\mathbb{A}_c$ for some positive constant M and some radius c, then $|f| \le M$ in the unit disk $\mathbb{D}$. Therefore, we have that $\|f\|_{A^p} \le M = \|M\|_{A^p}$. A natural question is whether the constant M can be replaced by an arbitrary holomorphic function in the unit disk $\mathbb{D}$, or, in other words, if f and g are holomorphic functions in the unit disk $\mathbb{D}$ with $|f(z)| \le |g(z)|$, for all $z \in \mathbb{A}_c$, does it follow that $\|f\|_{A^p} \le \|g\|_{A^p}$? For example, if the function f/g is holomorphic in the unit disk $\mathbb{D}$, then the maximum modulus principle implies that $|f(z)| \le |g(z)|$, for all $z \in \mathbb{D}$, and so $\|f\|_{A^p} \le \|g\|_{A^p}$. An important result in the theory of Bergman spaces is Korenblum's maximum principle, which we shall also refer to as the maximum principle in the Bergman space, which states:

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Maximum principle in Bergman space. Let $1 \le p < \infty$. Then there exists a constant $0 < c < 1$ with the following property. If f and g are holomorphic functions in the unit disk $\mathbb{D}$ such that $|f(z)| \le |g(z)|$ for all $c < |z| < 1$, then

$$\|f\|_{A^p} \le \|g\|_{A^p}.$$

First conjectured by Korenblum (see [5]), the maximum principle was proved by Hayman in [2] for $p = 2$ with $c = \frac{1}{25} = 0.04$. Note that in [5], Korenblum proved the maximum principle for $p = 2$ and $c < \frac{1}{2}e^{-2}$ under the additional hypothesis that g/f is a holomorphic function. Then came a succession of partial results by Korenblum, O'Neil, Richards, and Zhu (see [6]), Korenblum and Richards (see [7]), Matero (see [8]), Schwick (see [10]), and others. Later, Hinkkanen in [4] improved upon Hayman's constant by showing that the result holds for $c = 0.15724$. Also, he proved that it is valid more generally in the Bergman space A^p for all $1 \le p < \infty$. In the case $p = 2$, an example due to Martin (see [5]) shows that $c < \frac{1}{\sqrt{2}}$. In this case, Wang ([11]) gave an upper bound $c < 0.69472$, and recently Wang in [14] proved that $c < 0.6778994$. On the other hand, when $p = 2$, Schuster showed that we can take $c \ge 0.21$ by using properties of the Möbius pseudodistance for the annulus (see [9]). Also, in [12], Wang obtained $c \ge 0.25018$, for all $1 \le p < \infty$, and, more recently, $c \ge 0.23917$, for all $1 \le p < \infty$ ([15]). However, the best value of the constant c, even in the case $p = 2$, is still unknown.

Actually, Korenblum conjectured that the maximum principle in Bergman space A^p holds for all $0 < p < \infty$. In [4], Hinkkanen asked whether the maximum principle in Bergman space is valid when $0 < p < 1$ (see also monograph [1]). Wang proved in [13] that the maximum principle in Bergman space holds when $0 < p < 1$ under the additional hypothesis that function g has only a simple zero at 0 in the unit disk $\mathbb{D}$. On the other hand, as we will see later, we will give a negative answer to this question.

1.2. **The main result.** In this paper we prove that Korenblum's maximum principle does not hold in the Bergman space A^p when $0 < p < 1$. Actually, we prove the following theorem.

Theorem. *Let $0 < p < 1$ and $0 < c < 1$. Then there exist functions f and g holomorphic in the unit disk $\mathbb{D}$ such that $|f(z)| < |g(z)|$ for all $c < |z| < 1$ and*

$$\|f\|_{A^p} > \|g\|_{A^p}.$$

Having in mind all previous results in this area, this theorem seems somewhat surprising. Our theorem settles the question about Korenblum's maximum principle in Bergman spaces entirely.

1.3. **Outline of the paper.** We briefly mention the main steps in the proof of the previous theorem. In Section 2, we prove that for given $0 < p < 1$ and $0 < c < 1$, there exist a positive integer n and $0 < \varepsilon < 1$ such that the following inequality holds:

$$1 + \frac{np}{2}\varepsilon^{np+2} > \left(1 + \left(\frac{\varepsilon}{c}\right)^n\right)^p.$$

For such n and $0 < \varepsilon < 1$, we define functions f and g in the following way:

$$f(z) = \frac{1}{1 + \left(\frac{\varepsilon}{c}\right)^n}(z^n + \varepsilon^n) \text{ and } g(z) = z^n.$$

Finally, in Section 3, we prove the main result by proving that the functions f and g satisfy all the required conditions.

2. Preliminaries

We will use the following elementary and useful lemma, whose proof we include for the sake of completeness.

Lemma 2.1. *Let $0 < p < 1$ and let $x \geq 0$. Then $(1+x)^p \leq 1 + px$.*

Proof. Let $\varphi(x) = 1 + px - (1+x)^p$, where $x \geq 0$. Then, we have that

$$\varphi'(x) = p - p(1+x)^{p-1} = p\left(1 - \left(\frac{1}{1+x}\right)^{1-p}\right) \geq 0,$$

because $0 < p < 1$. Hence, the function φ is nondecreasing on $[0, \infty)$. Therefore, we find that $\varphi(x) \geq \varphi(0) = 0$, or equivalently $1 + px \geq (1+x)^p$, for all $x \geq 0$. □

Remark 2.2. We note that the previous lemma also holds for $x \geq -1$.

Now, we use the previous lemma to prove our main preliminary result.

Lemma 2.3. *Let $0 < p < 1$ and $0 < c < 1$. Then, there exist $n \in \mathbb{N}$ and $0 < \varepsilon < 1$ such that*

$$1 + \frac{np}{2}\varepsilon^{np+2} > \left(1 + \left(\frac{\varepsilon}{c}\right)^n\right)^p.$$

Proof. Because $0 < p < 1$, we can choose $n \in \mathbb{N}$ large enough so that $p + \frac{2}{n} < 1$. Then $n - (np+2) > 0$. Hence, we find that

$$\lim_{\varepsilon \to 0^+} \varepsilon^{n-(np+2)} = 0.$$

Therefore, we can choose $0 < \varepsilon < 1$ small enough so that

$$\varepsilon^{n-(np+2)} < \frac{nc^n}{2}. \tag{2.1}$$

We show that such n and ε satisfy the required condition. Namely, by using inequality (2.1), we find that

$$\frac{n}{2}\varepsilon^{np+2} > \left(\frac{\varepsilon}{c}\right)^n,$$

or equivalently

$$1 + \frac{np}{2}\varepsilon^{np+2} > 1 + p\left(\frac{\varepsilon}{c}\right)^n. \tag{2.2}$$

On the other hand, by using Lemma 2.1, we get

$$1 + p\left(\frac{\varepsilon}{c}\right)^n \geq \left(1 + \left(\frac{\varepsilon}{c}\right)^n\right)^p. \tag{2.3}$$

Finally, by using (2.2) and (2.3), we find that

$$1 + \frac{np}{2}\varepsilon^{np+2} > \left(1 + \left(\frac{\varepsilon}{c}\right)^n\right)^p.$$

This completes the proof of the lemma. □

Remark 2.4. We note that the previous result also follows by considering the power series of both sides, expanded in terms of ε.

3. Proof of the main theorem

In this section we prove the main result of our paper. Namely, in the following theorem we prove that Korenblum's maximum principle for the Bergman space A^p does not hold when $0 < p < 1$.

Theorem 3.1. *Let $0 < p < 1$ and $0 < c < 1$. Then there exist functions f and g holomorphic in the unit disk $\mathbb{D}$ such that $|f(z)| < |g(z)|$ for all $c < |z| < 1$ and*

$$\|f\|_{A^p} > \|g\|_{A^p}.$$

Proof. By using Lemma 2.3, for given $0 < p < 1$ and $0 < c < 1$, there exist $n \in \mathbb{N}$ and $0 < \varepsilon < 1$ such that

$$np + 2 < n \quad \text{and} \quad 1 + \frac{np}{2}\varepsilon^{np+2} > \left(1 + \left(\frac{\varepsilon}{c}\right)^n\right)^p. \tag{3.1}$$

Then, we can define functions f and g in the following way:

$$f(z) = \frac{1}{1 + \left(\frac{\varepsilon}{c}\right)^n}(z^n + \varepsilon^n) \quad \text{and} \quad g(z) = z^n,$$

for all $z \in \mathbb{D}$. Hence, the functions f and g are holomorphic in the unit disk $\mathbb{D}$. On the other hand, if $c < |z| < 1$, we have that

$$\frac{\varepsilon^n}{c^n + \varepsilon^n} > \frac{\varepsilon^n}{|z|^n + \varepsilon^n},$$

and consequently

$$\frac{c^n}{c^n + \varepsilon^n} = 1 - \frac{\varepsilon^n}{c^n + \varepsilon^n} < 1 - \frac{\varepsilon^n}{|z|^n + \varepsilon^n} = \frac{|z|^n}{|z|^n + \varepsilon^n},$$

or equivalently

$$\frac{c^n(|z|^n + \varepsilon^n)}{c^n + \varepsilon^n} < |z|^n.$$

Therefore, if $c < |z| < 1$, then, by using the previous inequality, we find that

$$|f(z)| = \left|\frac{1}{1 + \left(\frac{\varepsilon}{c}\right)^n}(z^n + \varepsilon^n)\right| \leq \frac{c^n(|z|^n + \varepsilon^n)}{c^n + \varepsilon^n} < |z|^n = |g(z)|.$$

It still remains to prove that for our functions f and g, we have

$$\|f\|_{A^p} > \|g\|_{A^p},$$

or equivalently

$$\left(\int_{\mathbb{D}} |f(z)|^p dA(z)\right)^{\frac{1}{p}} > \left(\int_{\mathbb{D}} |g(z)|^p dA(z)\right)^{\frac{1}{p}}. \tag{3.2}$$

We see that (3.2) holds if the following is true:

$$\int_{\mathbb{D}} \left|\frac{1}{1 + \left(\frac{\varepsilon}{c}\right)^n}(z^n + \varepsilon^n)\right|^p dA(z) > \int_{\mathbb{D}} |z^n|^p dA(z),$$

which is equivalent to

$$\frac{1}{\left(1 + \left(\frac{\varepsilon}{c}\right)^n\right)^p} \int_{\mathbb{D}} |z^n + \varepsilon^n|^p dA(z) > \int_{\mathbb{D}} |z|^{np} dA(z). \tag{3.3}$$

Hence, instead of (3.3), it is sufficient to show that

$$\int_{\mathbb{D}} |z^n + \varepsilon^n|^p dA(z) > \left(1 + \left(\frac{\varepsilon}{c}\right)^n\right)^p \int_{\mathbb{D}} |z|^{np} dA(z). \tag{3.4}$$

Since $\mathbb{D} = \mathbb{D}_\varepsilon \cup \mathbb{S}_\varepsilon \cup \mathbb{A}_\varepsilon$, where $\mathbb{D}_\varepsilon = \{z \in \mathbb{C} : |z| < \varepsilon\}$, $\mathbb{S}_\varepsilon = \{z \in \mathbb{C} : |z| = \varepsilon\}$, and $\mathbb{A}_\varepsilon = \{z \in \mathbb{C} : \varepsilon < |z| < 1\}$, we find that it is enough to show that

$$\int_{\mathbb{D}_\varepsilon} |z^n + \varepsilon^n|^p dA(z) + \int_{\mathbb{A}_\varepsilon} |z^n + \varepsilon^n|^p dA(z) > \left(1 + \left(\frac{\varepsilon}{c}\right)^n\right)^p \int_{\mathbb{D}} |z|^{np} dA(z). \tag{3.5}$$

First, we see that

$$\int_{\mathbb{D}} |z|^{np} dA(z) = \frac{1}{\pi} \int_0^{2\pi} \int_0^1 r^{np} r\, dr\, d\theta = 2 \int_0^1 r^{np+1} dr = \frac{2}{np+2}. \tag{3.6}$$

On the other hand, we have that

$$\begin{aligned} \int_{\mathbb{D}_\varepsilon} |z^n + \varepsilon^n|^p dA(z) &= \int_{\mathbb{D}_\varepsilon} \left|\varepsilon^n \left(1 + \left(\frac{z}{\varepsilon}\right)^n\right)\right|^p dA(z) \\ &= \varepsilon^{np} \int_{\mathbb{D}_\varepsilon} \left|1 + \left(\frac{z}{\varepsilon}\right)^n\right|^p dA(z) \\ &= \varepsilon^{np} \int_{\mathbb{D}_\varepsilon} \left(1 + \left(\frac{z}{\varepsilon}\right)^n\right)^{\frac{p}{2}} \left(1 + \left(\frac{\overline{z}}{\varepsilon}\right)^n\right)^{\frac{p}{2}} dA(z). \end{aligned}$$

Note that $\left|\frac{z}{\varepsilon}\right| < 1$ for all $z \in \mathbb{D}_\varepsilon$. Therefore, we get

$$\begin{aligned} \int_{\mathbb{D}_\varepsilon} |z^n + \varepsilon^n|^p dA(z) &= \varepsilon^{np} \int_{\mathbb{D}_\varepsilon} \sum_{k=0}^{\infty} \binom{p/2}{k} \left(\frac{z}{\varepsilon}\right)^{nk} \sum_{j=0}^{\infty} \binom{p/2}{j} \left(\frac{\overline{z}}{\varepsilon}\right)^{nj} dA(z) \\ &= \varepsilon^{np} \int_{\mathbb{D}_\varepsilon} \sum_{k=0}^{\infty} \binom{p/2}{k} \frac{1}{\varepsilon^{nk}} z^{nk} \sum_{j=0}^{\infty} \binom{p/2}{j} \frac{1}{\varepsilon^{nj}} \overline{z}^{nj} dA(z). \end{aligned}$$

Since, for all nonnegative integers $k \neq j$,

$$\int_{\mathbb{D}_\varepsilon} z^{nk} \overline{z}^{nj} dA(z) = 0,$$

we find that

$$\begin{aligned} \int_{\mathbb{D}_\varepsilon} |z^n + \varepsilon^n|^p dA(z) &= \varepsilon^{np} \sum_{k=0}^{\infty} \binom{p/2}{k}^2 \frac{1}{\varepsilon^{2nk}} \int_{\mathbb{D}_\varepsilon} z^{nk} \overline{z}^{nk} dA(z) \\ &= \varepsilon^{np} \sum_{k=0}^{\infty} \binom{p/2}{k}^2 \frac{1}{\varepsilon^{2nk}} \int_{\mathbb{D}_\varepsilon} |z|^{2nk} dA(z). \end{aligned} \tag{3.7}$$

Also, we have that

$$\int_{\mathbb{D}_\varepsilon} |z|^{2nk} dA(z) = \frac{1}{\pi} \int_0^{2\pi} \int_0^\varepsilon r^{2nk} r\, dr\, d\theta = 2 \int_0^\varepsilon r^{2nk+1} dr = \frac{\varepsilon^{2nk+2}}{nk+1} \tag{3.8}$$

holds for all nonnegative integers k. By using (3.7) and (3.8), we see that

$$\begin{aligned} \int_{\mathbb{D}_\varepsilon} |z^n+\varepsilon^n|^p dA(z) &= \varepsilon^{np}\sum_{k=0}^{\infty}\binom{p/2}{k}^2 \frac{1}{\varepsilon^{2nk}}\frac{\varepsilon^{2nk+2}}{nk+1} \\ &= \varepsilon^{np+2}\sum_{k=0}^{\infty}\binom{p/2}{k}^2 \frac{1}{nk+1}. \end{aligned} \tag{3.9}$$

Similarly, we have

$$\begin{aligned} \int_{\mathbb{A}_\varepsilon} |z^n+\varepsilon^n|^p dA(z) &= \int_{\mathbb{A}_\varepsilon} \left|z^n\left(1+\left(\frac{\varepsilon}{z}\right)^n\right)\right|^p dA(z) \\ &= \int_{\mathbb{A}_\varepsilon} |z|^{np}\left|1+\left(\frac{\varepsilon}{z}\right)^n\right|^p dA(z) \\ &= \int_{\mathbb{A}_\varepsilon} |z|^{np}\left(1+\left(\frac{\varepsilon}{z}\right)^n\right)^{\frac{p}{2}}\left(1+\left(\frac{\varepsilon}{\overline{z}}\right)^n\right)^{\frac{p}{2}} dA(z). \end{aligned}$$

Note that $\left|\frac{\varepsilon}{z}\right|<1$ for all $z\in\mathbb{A}_\varepsilon$. Hence, we get

$$\begin{aligned} \int_{\mathbb{A}_\varepsilon} |z^n+\varepsilon^n|^p dA(z) &= \int_{\mathbb{A}_\varepsilon} |z|^{np}\sum_{k=0}^{\infty}\binom{p/2}{k}\left(\frac{\varepsilon}{z}\right)^{nk}\sum_{j=0}^{\infty}\binom{p/2}{j}\left(\frac{\varepsilon}{\overline{z}}\right)^{nj} dA(z) \\ &= \int_{\mathbb{A}_\varepsilon} |z|^{np}\sum_{k=0}^{\infty}\binom{p/2}{k}\varepsilon^{nk}\frac{1}{z^{nk}}\sum_{j=0}^{\infty}\binom{p/2}{j}\varepsilon^{nj}\frac{1}{\overline{z}^{nj}} dA(z). \end{aligned}$$

Since, for all nonnegative integers $k\neq j$,

$$\int_{\mathbb{A}_\varepsilon} |z|^{np}\frac{1}{z^{nk}}\frac{1}{\overline{z}^{nj}} dA(z)=0,$$

we get

$$\begin{aligned} \int_{\mathbb{A}_\varepsilon} |z^n+\varepsilon^n|^p dA(z) &= \sum_{k=0}^{\infty}\binom{p/2}{k}^2\varepsilon^{2nk}\int_{\mathbb{A}_\varepsilon}|z|^{np}\frac{1}{z^{nk}}\frac{1}{\overline{z}^{nk}}dA(z) \\ &= \sum_{k=0}^{\infty}\binom{p/2}{k}^2\varepsilon^{2nk}\int_{\mathbb{A}_\varepsilon}|z|^{np-2nk}dA(z). \end{aligned} \tag{3.10}$$

Then, we find that

$$\begin{aligned} \int_{\mathbb{A}_\varepsilon} |z|^{np-2nk} dA(z) &= \frac{1}{\pi}\int_0^{2\pi}\int_\varepsilon^1 r^{np-2nk} r\,dr\,d\theta \\ &= 2\int_\varepsilon^1 r^{np+1-2nk}dr \\ &= 2\frac{1-\varepsilon^{np+2-2nk}}{np+2-2nk} \\ &= \frac{1-\varepsilon^{np+2-2nk}}{\frac{np+2}{2}-nk}, \end{aligned} \tag{3.11}$$

for all nonnegative integers k. We note that $np+2 \neq 2nk$, for all nonnegative integers k, because $n > np+2 > 2$. Also, note that

$$\frac{1-\varepsilon^{np+2-2nk}}{\frac{np+2}{2}-nk} = \int_{\mathbb{A}_\varepsilon} |z|^{np-2nk} dA(z) \geq 0,$$

for all nonnegative integers k. By using (3.10) and (3.11), we have that

$$\begin{aligned} \int_{\mathbb{A}_\varepsilon} |z^n+\varepsilon^n|^p dA(z) &= \sum_{k=0}^{\infty} \binom{p/2}{k}^2 \varepsilon^{2nk} \frac{1-\varepsilon^{np+2-2nk}}{\frac{np+2}{2}-nk} \\ &= \sum_{k=0}^{\infty} \binom{p/2}{k}^2 \frac{\varepsilon^{2nk}-\varepsilon^{np+2}}{\frac{np+2}{2}-nk} \\ &= \varepsilon^{np+2} \sum_{k=0}^{\infty} \binom{p/2}{k}^2 \frac{\varepsilon^{2nk-(np+2)}-1}{\frac{np+2}{2}-nk}, \end{aligned} \tag{3.12}$$

where

$$\frac{\varepsilon^{2nk-(np+2)}-1}{\frac{np+2}{2}-nk} \geq 0, \text{ for all nonnegative integers } k.$$

Finally by using (3.6), (3.9), and (3.12), the proof of (3.5) reduces to the proof of

$$\varepsilon^{np+2} \sum_{k=0}^{\infty} \binom{p/2}{k}^2 \left(\frac{1}{nk+1} + \frac{\varepsilon^{2nk-(np+2)}-1}{\frac{np+2}{2}-nk} \right) > \left(1+\left(\frac{\varepsilon}{c}\right)^n\right)^p \frac{2}{np+2},$$

or equivalently

$$\varepsilon^{np+2} \sum_{k=0}^{\infty} a_k > \left(1+\left(\frac{\varepsilon}{c}\right)^n\right)^p \frac{2}{np+2}, \tag{3.13}$$

where we denote

$$a_k = \binom{p/2}{k}^2 \left(\frac{1}{nk+1} + \frac{\varepsilon^{2nk-(np+2)}-1}{\frac{np+2}{2}-nk} \right) \geq 0,$$

for all nonnegative integers k. Note that

$$\sum_{k=0}^{\infty} a_k \geq a_0 = 1 + \frac{\varepsilon^{-(np+2)}-1}{\frac{np+2}{2}} = 1 + \frac{2}{np+2}\left(\varepsilon^{-(np+2)}-1\right). \tag{3.14}$$

Hence, by using (3.14) instead of (3.13), it is sufficient to show that

$$\varepsilon^{np+2}\left(1+\frac{2}{np+2}\left(\varepsilon^{-(np+2)}-1\right)\right) > \left(1+\left(\frac{\varepsilon}{c}\right)^n\right)^p \frac{2}{np+2},$$

which is equivalent to

$$\frac{2}{np+2}\varepsilon^{np+2}\left(\frac{np}{2}+\varepsilon^{-(np+2)}\right) > \left(1+\left(\frac{\varepsilon}{c}\right)^n\right)^p \frac{2}{np+2},$$

and hence it is sufficient to show that

$$1+\frac{np}{2}\varepsilon^{np+2} > \left(1+\left(\frac{\varepsilon}{c}\right)^n\right)^p.$$

But this is true because of (3.1). This finishes the proof of the theorem. □

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Faculty of Mathematics, University of Belgrade, Studentski trg 16, Serbia
Email address: `bozinv@mi.sanu.ac.rs`

Faculty of Mathematics, University of Belgrade, Studentski trg 16, Serbia
Email address: `bkarapetrovic@matf.bg.ac.rs`