$$r = C \cos 2(\alpha - \beta),$$

$$\frac{\partial r}{\partial \alpha} = -2C \sin 2(\alpha - \beta) = -4C \sin(\alpha - \beta) \cos(\alpha - \beta).$$

Equations (1) now furnish

$$A = -4C \sin(\alpha - \beta), \qquad B = -4C \cos(\alpha - \beta),$$

and Eqs. (2) give

$$z = -2C(\alpha + \beta) + C \sin 2(\alpha - \beta).$$

It follows that the curves $\alpha = \text{const.}$ and $\beta = \text{const.}$ are cycloids tangent to the lines $r = C$ and $r = -C$, respectively.

From Eqs. (14) and (15), we are able to determine σ. Substituting our values for γ, A, B, r and integrating, we find that

$$\sigma = 4k(\alpha + \beta) + k \ln [1 - \sin 2(\alpha - \beta)] + \text{const.}$$

Another solution is obtained by setting $f(\alpha) + g(\beta) = \alpha - \beta$, as before, and substituting $r = e^{\alpha+\beta}\phi$ where $\phi = \phi(\alpha - \beta)$ is a function yet to be determined. After making these substitutions and carrying out the differentiations, we get

$$[\phi^2 - \phi'^2] \cos 2(\alpha - \beta) - 2\phi\phi' \sin 2(\alpha - \beta) = 0$$

which is satisfied by

$$\phi = C[\cos(\alpha - \beta) + \sin(\alpha - \beta)].$$

We thus have

$$r = Ce^{\alpha+\beta}[\cos(\alpha - \beta) + \sin(\alpha - \beta)].$$

Using relations (1) and (2), we find that

$$z = Ce^{\alpha+\beta}[\sin(\alpha - \beta) - \cos(\alpha - \beta)].$$

The curves $\alpha = \text{const.}$ and $\beta = \text{const.}$ are logarithmic spirals which intersect the straight lines through the origin at an angle of $\pi/4$. This solution corresponds to the solution obtained in **1**.

It is interesting to see that these networks of cycloids or logarithmic spirals, known in the case of plane strain, are also admissible in the case of rotational symmetry.

ON THE TREATMENT OF DISCONTINUITIES IN BEAM DEFLECTION PROBLEMS*

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In a note on the treatment of discontinuities in beam deflection problems Mr. E. Kosko[1] attributes to R. Macaulay the method whereby the number of constants of integration can be always reduced to two, independently of the number of forces. This method was, however, originated by A. Clebsch, and is discussed in his book "Theorie der Elasticität Fester Körper," 1862, page 389. In Russia it was called the Clebsch method and was widely used in textbooks on strength of materials. It was also used in German books. See, for example, A. Föppl, Festigkeitslehre, 5th ed. 1914, page 124.

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[1] Quarterly of Appl. Math. **2**, 271–272 (1944).