ON HYPERSONIC SIMILITUDE*

By WALLACE D. HAYES (*North American Aviation, Inc.*)

A recent paper by H. S. Tsien[1] presents a law of similitude for two-dimensional potential hypersonic flows over a slender body. Hypersonic flow is fluid flow for which the Mach number is much greater than one. If the transformation

$$x = b\xi, \quad (1a) \qquad y = \delta\eta, \quad (1b) \qquad \phi = a_0\delta f(\xi, \eta) \quad (1c)$$

is made, where b and δ are the length and thickness of the body and a_0 and M are the velocity of sound and the Mach number in the undisturbed stream, the two-dimensional potential equation is transformed into

$$[1 - (\gamma - 1)Kf_\xi - \tfrac{1}{2}(\gamma + 1)f_\eta^2]f_{\eta\eta} = K^2 f_{\xi\xi} + 2Kf_\eta f_{\xi\eta}, \tag{2}$$

where

$$K = M\delta/b \tag{3}$$

is a fundamental similarity parameter. The boundary conditions may be expressed

$$f_\xi = f_\eta = 0 \quad \text{at} \quad \xi = -\infty, \tag{4a}$$

$$f_\eta = KH'(\xi) \quad \text{at} \quad \eta = H(\xi), \tag{4b}$$

where $y = H \cdot \delta$ defines the body shape. Since the equation is not linearized it is not permissible to satisfy the shape boundary condition at $\eta = 0$. Two potential flows with similar bodies and the same value of K are thus given by the same mathematical solutions and are similar. The drag and lift coefficients for bodies of similar shape may be expressed

$$C_D = \frac{1}{M^3}\Delta(K) \tag{5a}$$

$$C_L = \frac{1}{M^2}\Lambda(K) \tag{5b}$$

Professor Tsien also demonstrated the analogous law for axially symmetric flow.

It is the purpose of this note to point out that this similarity law is both much simpler in concept and much more general in scope than has been previously indicated; it is, in fact, applicable to three-dimensional flow with shock waves and rotation.

The two-dimensional hypersonic potential equation expressed by Eq. (2) is identical with the one-dimensional non-stationary potential equation in y and t under the transformation

$$t = T\xi \tag{6}$$

with (1b) and (1c), provided the replacement

$$T = b/Ma_0 \tag{7}$$

* Received Jan. 16, 1947.

[1] H. S. Tsien, *Similarity laws of hypersonic flows*, Jour. Math. Phys. **25**, 247–251 (1946).

is made in the definition of K. Also, the boundary conditions (4a), (4b) are the same if now $y = H \cdot \delta$ defines the position of the single boundary as a function of time. This shows that with the slender body hypersonic assumptions, $M \gg 1$, there is no intrinsic dependence upon the axial variable from the point of view of an observer stationary with respect to the undisturbed fluid and the problem becomes a non-stationary one in one fewer space variables. A change of δ with K kept constant is merely a scale transformation in the non-stationary system.

The difference between this point of view and the actual case is that the disturbances at two points on the same streamline on the body are assumed to be in phase. This difference is negligible if the ratio of the signal time between the two points to the time phase difference between the two points is large. Using the fact that in the hypersonic flow over a slender body there are appreciable changes in the velocity of sound but not in the flow velocity, the ratio of these times is equal to the local Mach number, and this parameter being large ensures the validity of the point of view. It is clear that the concept applies to three-dimensional as well as to two-dimensional flow. The presence of shock waves in a flow of extremely high Mach number can change the order of magnitude of the local sound velocity. However, a simple investigation shows that this sound velocity cannot be greater in order of magnitude than $Ma_0\beta$, where β is the inclination of the surface causing the shock and is assumed small but of order $1/M$ or larger. Thus with shocks present, the local Mach number will remain of order β^{-1} or larger. Hence the consideration of the hypersonic flow about a slender body as a non-stationary problem in one less dimension remains valid when shock waves and the resultant entropy changes are present.

The general similitude may be expressed thus: If a slender body of the shape

$$g(\xi, \eta, \zeta) = 0 \tag{8}$$

where

$$x = b\xi, \quad (9a) \qquad y = \delta\eta, \quad (9b) \qquad z = \delta\zeta \quad (9c)$$

is placed in a uniform stream of large Mach number the problem is identical with a non-stationary problem in y, z, and t where

$$t = x/Ma_0 \tag{10}$$

and is characterized by the parameter K as given by Eq. (3). The boundary condition satisfied on the surface is

$$Kg_\xi + \left(\frac{v}{a_0}\right)g_\eta + \left(\frac{w}{a_0}\right)g_\eta = 0 \tag{11}$$

where v and w are the velocity components in the y and z directions, respectively. The drag and lift coefficients based upon an area of magnitude $b\delta$ are given by Eqs. (5a) and (5b).