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OF
APPLIED MATHEMATICS

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Manuscripts submitted for publication in the Quarterly of Applied Mathematics should be sent to the Managing Editor, Professor W. Prager, Quarterly of Applied Mathematics, Brown University, Providence 12, R. I., either directly or through any one of the Editors or Collaborators. In accordance with their general policy, the Editors welcome particularly contributions which will be of interest both to mathematicians and to engineers. Authors will receive galley proofs only. Seventy-five reprints without covers will be furnished free; additional reprints and covers will be supplied at cost.

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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS
FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors’ cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for “author’s corrections.”

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; if authors wish to add material, they may do so at their own expense.

Titles: The titles should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: Only very simple symbols and formulas should be typewritten. All others should be carefully written by hand in ink. Ample space for marking should be allowed above and below all equations.

Greek letters used in formulas should be designated by name in the margin.

Square roots should be written with the exponent ½ rather than with the sign √.

Complicated exponents and subscripts should be avoided. Any complicated expression that reoccurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

\[ \exp[(a^2 + b^2)^{1/2}] \]

is preferable to \[ e^{(a^2 + b^2)^{1/2}} \].

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

\[ \frac{\cos (\pi x/2b)}{\cos (\pi a/2b)} \]

is preferable to

\[ \frac{\cos \pi x}{2b} \]

and

\[ \frac{\cos \pi a}{2b} \].

In many instances the use of negative exponents permits saving of space. Thus,

\[ \int u^{-1} \sin u \, du \] is preferable to \[ \int \sin u/u \, du \].

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

\[ (a + bx) \cos t \] is preferable to \[ \cos t (a + bx) \].

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

\[ [(a + (b + cx)^n) \cos ky] \] is preferable to \[ ((a + (b + cx)^n) \cos ky) \].

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be given as footnotes. Only in longer expository articles may references be grouped together in a bibliography at the end of the manuscript.

The following examples show the desired arrangements: for books—S. Timoshenko, Strength of materials, vol. 2, Macmillan and Co., London, 1931, p. 237; for periodicals—Lord Rayleigh, On the flow of viscous liquids, especially in three dimensions, Phil. Mag. (5) 36, 354–372 (1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors’ initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, On the flow of viscous fluids is preferable to On the Flow of Viscous Fluids, but the corresponding German title would have to be rendered as Über die Strömung zäher Flüssigkeiten.

In quoted titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: Since the printed text will not break up into pages in the same manner as the manuscript, footnotes should be numbered continuously.

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, “Eq. (25)” is acceptable, but not “the preceding Eq.” Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus “boundary conditions” should always be spelled out and not be abbreviated as “b.c.,” even if this special abbreviation is defined somewhere in the text.
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with

$$S_0 = \left[ \frac{2 \cdot 51}{14} \right]$$  \hspace{1cm} (21)

The decay of the vorticity is given by the relation

$$\frac{d\omega^3}{dt} = 70h''''(0) \cdot (\omega^3)^{3/2} - 10v \frac{\omega^3}{\lambda^3}$$  \hspace{1cm} (22)

and it will easily be found, using the relations given before, that

$$\frac{d\omega^3}{dt} = \frac{7}{3(15)^{1/2}} (\omega^3)^{3/4} \left[ S_0 - 2 \frac{G_0}{N_{\lambda,0}} (\omega^3)^{24/17} \right]$$  \hspace{1cm} (23)

In the decay of vorticity, the rate of production of vorticity by diffusive stretching of the vortex tubes and the rate of dissipation of vorticity by viscosity are each proportional to $(\omega^3)^{24/17}$. The ratio of the production of vorticity to its dissipation is constant and equal to

$$\frac{SN_{\lambda}}{2G} = 1 - \frac{51}{14} \frac{1}{G_0}$$  \hspace{1cm} (24)

The equations (23) and (21) give

$$\frac{d\omega^3}{dt} = - \frac{17}{(15)^{1/2}} (\omega^3)^{3/4} \frac{N_{\lambda,0}}{N_{\lambda,0}} (\omega^3)^{24/17}$$  \hspace{1cm} (25)

which could be found directly by differentiating equation (14).

There exist some differences between the conclusions drawn here for isotropic turbulence at “large” Reynolds number of turbulence and the conclusions of Batchelor and Townsend. In particular, in the present paper it is found that the decay of vorticity is proportional to $(\omega^3)^{24/17}$ and not to $(\omega^3)^{3/2}$. The factor $G$ is considered to be constant as in the Batchelor-Townsend paper but $N_{\lambda}$ and $S$ are variable. For the case studied here, $G$ is given as function of $S$ by the equation (18), which is of the same form as that found by Batchelor and Townsend but instead of $30/7$ the constant is equal to $51/14$.

The experimental results of Batchelor and Townsend seem to agree only very roughly with these relations. However, if account is taken of probable inaccuracies in the experimental data, such as the imperfect isotropy behind grids and errors due to finite lengths of hot-wires (especially as this concerns measurements of $\lambda$), it appears that the agreement may be satisfactory after all.

**BOOK REVIEWS**


The relaxation method of approximate numerical solution of systems of equations was first introduced in 1936 by R. V. Southwell in connection with the solution of engineering problems arising in the field of structural design. In questions of the loading and deflection of complex structural frames, one is struck by the inherent precision of the conventional analytical methods of attack and the inherent lack of precision in the given data defining a problem. A number of people devised numerical schemes of calculation by which the precision of the answer could be made as good as desired and thus made comparable with the given data. Several of these methods involved successive relaxation of constraints. To Southwell
however, belongs the credit of realizing the great generality of this method of approach. In steps, he and his team of computers have solved more and more complex problems.

In the book "Relaxation Methods in Engineering Science," Southwell has presented in some detail, problems involving systems of algebraic equations and ordinary differential equations, including eigenvalue problems, which arise in engineering structures and vibrations problems.

In the present volume, the treatment of problems by the relaxation method is extended to include partial differential equations from various branches of classical physics. All of the problems are included in the general form

$$\frac{\partial}{\partial x} \left( \psi \frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left( \psi \frac{\partial \phi}{\partial y} \right) = Z,$$  \hspace{1cm} (1)

where

$$\psi = \psi(x, y, \phi, \phi_x, \phi_y), Z = Z(x, y, \phi, \phi_x, \phi_y).$$  \hspace{1cm} (2)

The first chapter is devoted to a discussion of general ideas and formulas. The relaxation method is explained and the finite difference expressions for differentials of various orders are derived. Partial differentials are then treated, and in particular the finite difference approximation to the Laplace operator is derived for triangular, square and hexagonal nets. Numerical integration and interpolation are also briefly treated in this chapter.

The remainder of the book, chapters II through VI are devoted to successively more difficult cases of (1) and (2). The added difficulties are of two classes: (a) more difficult boundary conditions, and (b) more complex forms of (1) and (2).

In chapter II, Equation (1) with $\psi = 1, Z = Z(x, y)$ is discussed in detail. (No significant numerical simplification results if $Z = 0$.) The relaxation pattern, the basic idea of relaxation, block relaxation for speedier approach to the solution, advance to a finer net, and the physical significance of these techniques in terms of the "tensioned net" are all carefully presented. The chapter concludes with several torsion problems, some of which are left for the reader to try.

In chapter II, the boundary conditions are specified values of the wanted function on specified rectangular boundaries. In chapter III, the treatment is broadened to include specified normal gradients and irregular boundaries. Several problems with known analytical solutions are solved to illustrate the accuracy attained with relatively coarse nets and then several much more difficult problems involving the torsion of irregular shafts and magnetic fields in electric machines are solved to illustrate the power of the method.

Chapter IV deals with general questions of conformal transformation. The numerical determination of the real and imaginary parts of the complex function required to transform any given domain into a circle or rectangle is considered. A treatment for suitable removal of the logarithmic singularity at the center of the circle is given as is the determination of the complex conjugate of a function. It is shown that the most accurate determination of a complex conjugate generally involves a separate relaxation for each.

Cases of (1) with $\psi = \psi(r, z)$, of which the Laplace operator in polar coordinates $(r, z)$ is a typical example, are treated by the use of relaxation patterns which vary from point to point. Chapter V presents the necessary theory and gives a considerable number of illustrative problems from the fields of elasticity, heat conduction, and lubrication. This chapter concludes with two problems in which $\psi$ is a function of $\phi$ and its derivatives as well as $x, y$. A lubrication problem with variable viscosity and the flow of a gas through a nozzle are the illustrative problems treated in detail.

In the final chapter (VI) "Problems Involving Boundaries or Interface Not Initially Known" are treated. Usually boundary conditions consist of two parts, a given boundary and a given relation between the function and its normal derivative along this boundary. Instead, one may choose two other suitable conditions say the value of the function and the value of its normal derivative but omitting the shape of the boundary itself. Such problems arise in a natural way in many physical problems. The problems discussed involving plastic torsion, percolation of water through soils, the waterfall, and the wake behind a cylinder are all cases in which analytical methods have given few or no solutions.

No exaggerated claims are made for the ease by which the problems can be solved by the relaxation method. It is in fact pointed out that problems like those of chapters V and VI especially involve considerable labor of a type which does not appeal to many persons. It is correctly noted however that problems
of these and even more difficult types (not yet released from wartime publication restrictions) can be solved by the relaxation method, whereas they are solvable in no other way at present.

Throughout the book, it is emphasized that the only way to appreciate the significance of the relaxation method of numerical solution of partial differential equations of mathematical physics is to do problems. Thus each chapter presents the physical theory and finally accepted answer to a number of problems, leaving the numerical work as exercise for the reader.

Numerous tables of finite difference formulas for differentiation and integration are given in 10 pages of appendix.

Howard W. Emmons


This book is a very extensively revised and enlarged version of the senior author's "Introduction to the Mathematical Theory of the Conduction of Heat in Solids" which appeared in 1921. It treats very thoroughly the best available mathematical techniques for the analysis of heat conduction and includes a large number of solutions to specific problems.

The first chapter presents a discussion of the basic partial differential equation, and the mathematical formulation of initial and boundary conditions for typical systems. In the following eight chapters the Fourier methods of analysis are applied to determine transient temperature distributions under various conditions in infinite and semi-infinite solids, slabs, rods, rectangles, rectangular parallelepipeds, circular cylinders, wedges, spheres and cones. In Chapter X, the method of sources and sinks and the method of images are developed and applied to problems not easily solvable by the methods of the preceding chapters. Much, though by no means all of the material in these ten chapters was contained in the earlier book; the remainder of the book is largely new.

The next four chapters present a clearly written discussion of the Laplace transform and a thorough treatment of its application to the solution of heat conduction problems. This very powerful method, which was developed after the appearance of the earlier book, replaces the "method of contour integrals" discussed therein, and furnishes solutions to many complex problems which do not yield readily to the more elementary techniques. Many of the original results obtained by the authors are reproduced or presented for the first time in these chapters. The Laplace transform is also used to obtain the Green's functions for heat conduction in solids of various typical geometries. A final chapter includes for completeness a very brief treatment of steady state heat conduction.

Many graphs and tables not contained in the earlier book and useful in making calculations for engineering applications are included for ready reference. A very brief discussion of the available numerical and graphical methods is also given. Many references to the literature are made, but there has been no attempt to compile an exhaustive bibliography. The theory developed is a linear one, i.e. no attempt has been made to treat the problem of variable thermal properties. A minor defect is that the table of contents and the index are possibly over-concise, so that a page-by-page search is sometimes necessary to locate a desired result. The book as a whole is a clearly written, complete and excellent treatise on the subject.

S. A. Schaal

Tables of supersonic flow around yawing cones. By the staff of the Computing Section, Center of Analysis, Massachusetts Institute of Technology. Under the direction of Zdenek Kopal. Cambridge, Massachusetts, 1947. xviii + 321 pp. $5.00.

In the introduction to this volume, the theory governing the supersonic flow of a gas past a slightly yawed cone is developed. In this development, the Taylor-Maccoll solution for a symmetrically oriented cone forms the zero order terms of a perturbation expansion with regard to the yaw angle $\epsilon$. The equations relating to the terms of order $\epsilon$ are derived. The original detailed theory which has not been previously published is said to be forthcoming in a periodical.

The problem reduces to the determination of the coefficient of $\epsilon$ in the perturbation development and the body of the book consists of a compilation of the numerically obtained results. The range of cone angles is $5^\circ \leq \theta_1 \leq 50^\circ$. As in volume I (the corresponding tables for the unyawed cone) results are obtained for a large number of wave angles corresponding to each cone angle.

G. F. Carrier

Number seventeen in Annals of Mathematical Studies, this book is a translation from the Russian. Published in Russian in 1892 by the Mathematical Society of Kharkow, the French translation appeared in Annales de la Faculté des Sciences de Toulouse, 2nd ser., vol. 9 (1907).

The book is divided into three chapters, the first of which is labeled Preliminary Analysis; it includes definition and discussion of the general problem of stability. Chapter I also contains a discussion of systems of linear differential equations and differential equations of the disturbing motion. Chapter II is entitled Study of Steady Motion and it includes more specific treatment of the differential equations of the disturbing motion for various cases of steady motion. Chapter III is concerned with the Study of Periodic Motion and includes analysis of the characteristic equation as well as the differential equations of the disturbing motion.

This work encompasses the classical treatment of stability for continuous motion of disturbance. There is not any discussion of dynamical processes subject to discontinuous impulses.

R. Truell

---


In the first two-thirds of this treatise the developments of the solutions of the Mathieu equation are carried out and various techniques for the evaluation of the necessary parameters are given in some detail. These techniques, both numerical and analytic, are amply illustrated by appropriate numerical examples, and their ranges of validity are indicated. The solutions are developed in the form $e^{i\varphi(x)}$ where $\varphi$ is found as a trigonometric series, in the form of Maclaurin series, and in series of Bessel functions. The eigenvalue problem associated with the Mathieu equation is treated. Orthogonality conditions and several integral identities are established. The asymptotic representations and zeros of Mathieu functions are discussed. The Hill equation is briefly treated.

In the second part, various physical problems are solved in terms of Mathieu functions and the results are interpreted. Loud speaker distortion, the oscillations of an elastic column under periodic end thrust, the vibration of a membrane with an elliptical boundary, the viscous flow past an elliptic cylinder, and other problems, are used to illustrate the large class of problems in which Mathieu functions arise.

G. F. Carrier

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In this continuation of Vol. I of the same work, the function $(\pi/2x)^{1/2}J(x)$ is tabulated for the half-odd-integer values $29/2 \leq |\nu| \leq 61/2$. In the range $29/2 \leq |\nu| \leq 43/2$, the argument $x$ ranges from 0 to 10 in steps of .01 and from 10 to 25 in steps of .10. For $45/2 \leq |\nu| \leq 61/2$ the range of $x$ is 10 to 25 in steps of .10. For $1/2 \leq |\nu| \leq 61/2$ and for $-29/2 \leq \nu \leq -61/2$, the function $2\Gamma(\nu - 1)J(x)/x^\nu$ is tabulated for $0 \leq x \leq 25$ at increments of .1. The number of significant figures is at least seven. Finally, some of the zeros, maxima, and minima, of the half order Bessel functions are tabulated for $-29/2 \leq \nu \leq 45/2$.

G. F. Carrier

---


This text contains a very good introduction to the classical theory of vectors, motors, dyads, tensors, and quaternions and some of their applications to geometry and physics. Throughout the text, the main emphasis is on developing the student’s manipulative ability for dealing with these powerful tools. This is accomplished by offering very complete explanations of techniques and using numerous illustrative problems throughout the text.
The first section deals with Gibbs' Vector Analysis (Chapters I, III, V, VI). The dot, cross, triple scalar, etc. products and reciprocal basis are introduced in Chapter I. As applications, the author offers proofs of various theorems in classical plane projective geometry and vector interpretations of homogeneous, line, and plane coordinates. Following the introduction of vector differentiation and integration (Chapter III), the author treats the theory of curves (the Frenet formulas and applications). As a further illustration of vectorial methods over one dimensional fields, the kinematics of a particle is discussed. The portion of the text which treats vector fields of two and three dimensions (Chapters V, VI) discusses such topics as the gradient, the curl, the divergence, vector derivative identities, curvilinear coordinates, surface vectors, the Green and Stokes theorems and some elementary potential theory. It is the reviewer's opinion that the author uses the cross product in several instances where use of the dot product would be more advantageous. Thus, in deriving the Frenet formulas, the author works with the cross product. Here, the dot product furnishes an easier derivation and one that can be generalized to \( n \)-dimensional manifolds.

The algebra of motors is studied in Chapter Ê. This work is used in only one application (Chapter III, pp. 117). Thus, this topic may be included or excluded in any course based on this text without affecting the continuity of the remaining material.

Tensors and their applications form the basis of Chapters IV, VII, VIII, IX. The classical Gibbs' theory of dyadic algebra is discussed in Chapter IV. Topics such as conjugate dyadics, decomposition of dyadics, symmetry, invariants of dyadics, relation between dyadics and matrices are treated. Some use of these ideas is made in Chapters V and VI. In the main, the use of dyadics in these last mentioned chapters serves to generalize the Green and Stokes vector theorems. However, fairly extensive use of dyadics is made in the treatment of Hydrodynamics (Chapter VII) and Surface Geometry (Chapter VIII). The section on Hydrodynamics discusses the Euler and Lagrange equations, the Helmholtz equations, and the Kutta-Joukowsky formula. In the Chapter on Surface Geometry, the author treats curves on a surface by means of the Darboux moving trihedral. Geodesic curvature, geodesic torsion, normal curvature, lines of curvature, asymptotic lines are among the many topics introduced. Further, the first and second fundamental forms, geodesics, the mean and total curvature, the Gauss-Bonnet formula, and the Gauss-Codazzi relations are all discussed in some detail. Chapter IX is concerned with Tensor Analysis. Transformation laws of vectors and tensors, multiplication and contraction of tensors, tensor densities, the permutation symbol, and the generalized Kronecker tensors are examined. By expanding geometric objects (dyads, triads, etc.) in terms of their tensor components and a system of base vectors and by determining the differentiation law of the base vectors, the author introduces covariant differentiation. The Riemann-Christoffel and Ricci tensors are discussed. The reviewer would like to make two criticisms of the work in this section. First, the use of dyads in the section on Surface Geometry (Chapter VIII) needlessly increases the amount of technique which students must develop and, further, detracts from the unity of the text. The same results can be obtained by the proper use of tensors (components of the dyads). Secondly, in deriving the Kutta-Joukowsky formula, the author passes from vectors to complex numbers by a vague equivalence scheme. It is known (see A. Wundheiler: Are Complex Numbers Vectors? Bull. of Am. Math. Soc., Vol. 46, No. 1, Jan. 1940, pp. 57) that "complex numbers are not isomorphic with vectors but are vectors in a metric plane with a distinguished direction." Thus, it appears desirable to show clearly the relation between complex numbers and vectors or else to treat complex numbers independently of vectors.

In the final Chapter X of the text, Quaternion Algebra is developed. Some applications are given to rigid body motions.

The author has presented in an interesting and readable manner several methods of treating geometric quantities. As a result, this book should form a valuable addition to the existing texts on Vector and Tensor methods.

N. Coburn


One of the most notable recent developments in statistical theory is sequential analysis, a sampling procedure in which the size of the sample is not fixed in advance, but depends on the results of the successive observations. The basic results are due to Professor Wald, and have been published in two papers in the Annals of Mathematical Statistics; this book is essentially an expanded and somewhat reorganized version of Professor Wald's papers, together with some additional material.
The book consists of three parts and an appendix. Part I contains background material and most of the theory, including a discussion of the sequential probability ratio test, the most important sequential procedure. Part II is a detailed description of sequential procedures for testing certain hypotheses of practical importance; the hypotheses treated concern the parameters of binomial and normal populations. Part III is a brief discussion of some possible sequential approaches to the problem of multi-valued decisions and estimation. The appendix contains those parts of the theory which require some advanced mathematics; the rest of the book is not beyond the understanding of one acquainted with elementary calculus.

Two features of the sequential method deserve especial mention. 1) The sequential procedure requires on the average fewer observations than comparable fixed-sample-size procedures, the saving in some cases of practical importance being as much as 50%. 2) Sequential theory is much simpler than that for samples of fixed size. The book is a clear presentation of an important topic, and should be invaluable to practical statisticians.

David Blackwell


This “Guide” is planned for the “large group of people who cannot maintain an independent awareness of the mathematical and physical reference literature but who can make effective use of a classified guide. This group includes scientists, engineers, librarians, and students . . . Often the techniques that overcome obstacles in one field can be effectively used in another. For example, linear electrical networks are the most intensively studied vibration systems. By means of electromechanical analogues, all these results from the electrical field are applicable to the mechanical field.” The bibliography of approximately 1800 entries, classified under some 150 headings, is preceded by over 70 pages of detailed discussion of the principles of reading and study, self-directed education, literature search, and too little on the use of periodicals. Suggestions are offered on problems of library technique and use.

As the intended audience is so inhomogeneous, both in interests and in level of accomplishment, the author has necessarily assumed almost complete lack of knowledge on the part of the reader. An experienced worker, however, would do well to read all of the, what might appear to him as a somewhat pedantic and overdetailed, discussion for the valuable hints it contains. Unfortunately a difficulty arises in the use of the bibliography whose primary functions should be to tell the reader about the essential treaties in each field, books which bring the information up to date, and also periodicals in which reviews and research reports are published. In some cases, the author has done this carefully, but in many classifications, the research worker or engineer who is branching out, possibly temporarily, into an unfamiliar field is confronted with too many books about which too little is said for him to make an intelligent selection.

In its present form, the “Guide” is most useful to those who have an elementary understanding of the topics considered and who require detailed or advanced knowledge. It is to be hoped that frequent revisions or supplements will be issued to keep the “Guide” valuable. As an example, illustrating this necessity, Applied Mechanics Reviews will soon be published as a periodical covering the current world literature in many of the classifications. Obviously, no mention of this valuable addition to the review journals can appear in the present “Guide.”

D. C. Drucker


The preface states that since the publication of the first edition of these tables [reviewed in this Quarterly, 2, 276 (1944)] no errors have been reported in the tabular material. Accordingly, this material could be reproduced from the negatives which were used for the first edition. Some corrections and revisions were made in the Introduction, however, which now contains the relations between the tabulated functions on the 45° ray and the $ber$ and $bei$ functions.

W. Prager

This table of nomograms includes an introduction containing a note on the history of real and complex hyperbolic functions and notation, earlier tables and graphical representations of complex hyperbolic functions, the use of nomograms for: \( \cosh(b + ja) = p + jq \), \( \sinh(b + ja) = p + jq \), \( \tanh(b + ja) = r \frac{\theta}{\text{e}^{i\theta}} \), \( x + jy = r \frac{\theta}{\text{e}^{i\theta}} \) and \( R \frac{\alpha}{\phi} = 1 + r \frac{\phi}{\alpha} \) as well as the use of nomograms for reflection losses and reflection phase shift and for resonance circuits and filters. Approximately ten pages of formulas for circular and hyperbolic functions include real functions, gudermannian angle, integrals, differentials, series developments, functions of imaginary argument, functions of complex argument and inverse complex hyperbolic functions.

A table of multiples of \( \pi/2 \) from 1 to 100 is present as well as some formulas for four terminal networks and transmission lines.

All of the material mentioned above is contained in thirty five pages (8½ x 12). In addition there are fifty pages of nomograms as follows:

- \( \cosh(b + ja) = p + jq \) 13 nomograms covering the range \( b = 0 \) to \( b = 4.0 \) nepers
- \( \sinh(b + ja) = p + jq \) 13 nomograms covering the range \( b = 0 \) to \( b = 4.0 \) nepers
- \( \tanh(b + ja) = r \frac{\theta}{\text{e}^{i\theta}} \) 16 nomograms covering the range \( b = 0 \) to 2.0 nepers

Eight additional nomograms of elementary functions related chiefly with transmission line and network calculations.

This book of nomograms has been prepared with unusual care and accuracy, and its contents should be of considerable value to electrical and acoustical engineers and possibly to physicists. Any extensive numerical calculations involving impedances or reflection and transmission coefficients can be facilitated by means of these nomograms.

Rohn Truell