of this paper to the author, who gave the solution in 1945 while a member of the Statisti-
cal Research Group of Columbia University, which was working under contract with
the National Defense Research Committee of the Office of Scientific Research and
Development.

A COMMENT ON F. N. FRENKIEL'S NOTE "ON THIRD-ORDER CORRELATION
AND VORTICITY IN ISOTROPIC TURBULENCE"

By G. K. BATCHELOR and A. A. TOWNSEND (Cambridge University)

The short paper by Mr. F. N. Frenkiel [this Quarterly, 6, 86-90 (1948)] calls for some
comment. Mr. Frenkiel makes certain assumptions about the correlation functions and
derives relations for the decay of energy and mean-square vorticity in isotropic tur-
bulence. These relations are not in agreement with measurements which we have already
published. Mr. Frenkiel concludes that "if account is taken of probable inaccuracies in
the experimental data (in Ref. 1), it appears that the agreement (with his deductions)
may be satisfactory after all." In answer to this suggestion, we make the following points.

(1) In our opinion there is no possibility that experimental error would account for the
discrepancies between our data and Mr. Frenkiel's results. He finds \( \frac{d\lambda^2}{dt} = 7\nu \), whereas
according to our measurements, \( \frac{d\lambda^2}{dt} = 10\nu \) with a standard deviation of less than 0.4\nu.
Our measurements were corrected for the effect of finite length of the recording hot-wire
but, in any case, the correction leaves \( \frac{d\lambda^2}{dt} \) unaltered provided the wire length is small
enough to be comparable with \( \lambda \). Mr. Frenkiel also suggests lack of isotropy as a possible
source of error. The presence of isotropy to a sufficient approximation seems to us to have
been checked very well by the consistency of measurements of \( \bar{u}^2 \) and \( \lambda \) (which are related
by the energy equations for isotropic turbulence) and of \( \bar{u}^3 \), \( f_0''\lambda^4 \) and \( k_0''/\lambda \) (which are
related by the vorticity equation).

(2) There is a strong body of theory concerning the mechanism of turbulence at
high Reynolds numbers, which has been shown to link up very satisfactorily with the
measurements of Ref. 1. Any contrary results, such as that given by Mr. Frenkiel for the
variation of \( k_0''/\lambda^3 \) during decay, would therefore need strong experimental and theoretical
backing.

(3) Mr. Frenkiel gives no reasons to support his assumption that the correlation curve
is self-preserving outside the (small) region in which \( r \) is comparable with \( \lambda \). Indeed, it
leads him into inconsistencies. Later in his paper he assumes, in agreement with our
measurements, that \( f_0''\lambda^4 \) is constant during decay. This implies that the correlation
curve is also self-preserving for small values of \( r \) and it has now been assumed in effect that
\( f(r) \) is completely self-preserving. Equation (6) of his paper then leads to the con-
clusion that \( \lambda(u_0^2)^{1/2}/\nu \) is constant during decay, whereas his published result is that this
quantity varies as \( t^{-3/14} \). Again, he is willing to assume \( f_0''\lambda^4 \) independent of \( t \), and any
physical basis for this assumption would seem to lead also to \( k_0''/\lambda^3 \) being independent of \( t \)
(as our measurements show), but Mr. Frenkiel finds \( k_0''/\lambda^3 \) is proportional to \( t^{3/14} \).

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