THE FUNDAMENTAL THEOREM OF ELECTRICAL NETWORKS*

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I wish to thank Messrs. L. C. Robbins, Jr. and W. K. Saunders for drawing my attention to what may appear to be a flaw in the reasoning of the paper with the above title. The proof there given is not convincing, and I take this opportunity to supply a fuller proof, in which the reasoning is a little delicate, but which seems to establish the result.

Consider the two propositions:

(A) \( e = 0 \) implies \( i = 0 \).

(B) \( Z' \neq 0 \).

It is stated on p. 127 of the paper cited that (A) implies (B); that is the result we have to establish here.

We have the following equations:

\[
\begin{align*}
i &= Ci', \\
e' &= C'e, \\
e' &= Z'i'.
\end{align*}
\]

These equations embody all our knowledge about the behavior of the network, and any set of vectors \( i, i', e, e' \) consistent with them are to be regarded as possible. But we must bear in mind the definition of the mesh currents \( i' \) as branch currents (components of \( i \) in branches-out-of-tree); this means that the matrix \( C \) is such that

\[
\begin{align*}
i' \neq 0 & \text{ implies } i \neq 0.
\end{align*}
\]

Choose \( e = 0 \) and \( e' = 0 \). Then (1) read

\[
\begin{align*}
i &= Ci', \\
0 &= 0, \\
0 &= Z'i'.
\end{align*}
\]

Any vectors \( i, i' \) satisfying these equations are possible.

Suppose that (B) is false, i.e. suppose \( Z' = 0 \). Then there exists a non-zero \( i' \) satisfying the last of (3), and, by (2), the \( i \) given by the first of (3) is non-zero. Thus, on the assumption that (B) is false, we have a solution of (1) with \( e = 0 \) and \( i \neq 0 \). Hence, if (B) is false, (A) is false. Therefore if (A) is true, then (B) is true. In other words, (A) implies (B), which is the required result.

*Received June 1, 1952.