the Lindstedt procedure furnishes the following values
\[ c_2 = 1, \quad c_3 = \frac{15}{16}, \quad c_4 = \frac{13}{16}. \]
From the following table, reproduced from Shohat’s paper cited above, it is tempting
to conjecture that the Shohat series converges for all values of \( \lambda \) which are positive.
If so, the series should be more widely known.

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( v ) computed using (5.2)</th>
<th>( v ) (Van der Pol)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.33</td>
<td>.98</td>
<td>.99</td>
</tr>
<tr>
<td>1.0</td>
<td>.93</td>
<td>.90</td>
</tr>
<tr>
<td>2.0</td>
<td>.77</td>
<td>.78</td>
</tr>
<tr>
<td>8.0</td>
<td>.35</td>
<td>.39</td>
</tr>
<tr>
<td>10.0</td>
<td>.30</td>
<td>.31</td>
</tr>
</tbody>
</table>

The Van der Pol values were obtained using graphical techniques.

References
3. J. Lighthill, A technique for rendering approximate solutions to physical problems uniformly valid, 
   Phil. Mag. 40, 1179-1201 (1949)
   (1944)

ON MIDDLETON’S PAPER “SOME GENERAL RESULTS IN THE 
THEORY OF NOISE THROUGH NON-LINEAR DEVICES”**

By J. S. SHIPMAN (Laboratory For Electronics, Inc., Boston, Mass.)

As one of the central results of the title paper [1], Middleton obtained \( R_i(t) \), the 
correlation function for the \( i \)th zone, as a function of the input correlation function \( r_0 \) 
in the case of the \( v \)th law half-wave rectification of narrow-band normal noise (see, e.g., 
his equations (7.14) and (7.15)). Unless one resorts to series evaluations, his formulas 
are not particularly suited for numerical computation as they stand, involving as they 
do hypergeometric functions which are not well tabulated. For purposes of calculation, 
then, a reduction of the hypergeometric functions occurring in the formulas to tabulated 
functions must ordinarily be effected, usually by applying the recursion relations among 
contiguous hypergeometric functions due to Gauss.

When this reduction is accomplished, the hypergeometric functions in Middleton’s 
formulas are seen to be either polynomials in \( r_0 \) or combinations of complete elliptic 
integrals of the first and second kind, provided \( v \) is an integer (see, e.g., Middleton’s 
equations (7.16) and (7.17)). These polynomials and combinations of elliptic integrals 
turn out to be, in every case so far examined, special cases of “Bennett functions” 
recently tabulated by the author and his colleagues [2, 3]. In the present note expressions

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for $R_1(t)$ in terms of Bennett functions for the important practical cases $\nu = 1, 2$ for several values of $l$ and for $l = 0$ and $\nu = 3, 4$ are collected. The notation used is that of Middleton except for the symbol $A_{\nu}^{(r)}$ which is the Bennett function of the $\nu$th kind [3]. Thus

$$R_0(t)_{r=1} = (\beta^2 \psi / 2)[\frac{1}{2} A_{\nu}^{(1)}(r_0)],$$  
$$R_1(t)_{r=1} = (\beta^2 \psi / 2)(\cos \omega_t) A_{\nu}^{(1)}(r_0),$$  
$$R_2(t)_{r=1} = (\beta^2 \psi / 2)(\cos 2\omega_t) A_{\nu}^{(1)}(r_0),$$  
$$R_3(t)_{r=1} = 0 \quad (l = 3, 5, 7 \ldots),$$  
$$R_4(t)_{r=1} = (\beta^2 \psi / 2)(\cos 4\omega_t) A_{\nu}^{(1)}(r_0);$$

further

$$R_0(t)_{r=2} = \beta^2 \psi^2[\frac{1}{2} A_{\nu}^{(2)}(r_0)],$$  
$$R_1(t)_{r=2} = \beta^2 \psi^2(\cos \omega_t) A_{\nu}^{(2)}(r_0),$$  
$$R_2(t)_{r=2} = \beta^2 \psi^2(\cos 2\omega_t) A_{\nu}^{(2)}(r_0),$$  
$$R_3(t)_{r=2} = \beta^2 \psi^2(\cos 3\omega_t) A_{\nu}^{(2)}(r_0),$$  
$$R_4(t)_{r=2} = 0 \quad (l = 4, 6, 8, \ldots),$$  
$$R_5(t)_{r=2} = \beta^2 \psi^2(\cos 5\omega_t) A_{\nu}^{(2)}(r_0);$$

and finally

$$R_0(t)_{r=3} = 3\beta^2 \psi^3[\frac{1}{2} A_{\nu}^{(3)}(r_0)],$$  
$$R_0(t)_{r=4} = 12\beta^2 \psi^4[\frac{1}{2} A_{\nu}^{(4)}(r_0)].$$

The functions $A_{\nu}^{(r)}(r_0)$ are tabulated directly for $\nu = 1, 2$ [2, 3]; for $\nu \geq 3$ recursion formulas are given which enable Bennett functions of higher kind to be obtained from those of lower kind.

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**References**