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BOOK REVIEWS

Stress concentration around holes. By G. N. Savin. English edition prepared from the Russian by Eugene Gros. Pergamon Press, New York, 1961. xi + 430 pp. $15.00.

This is an excellent translation of a revised and amplified version of G. N. Savin's well-known book on two dimensional states of stress around holes. Most of the problems treated are drawn from the Russian literature and many of the results are due to the author and his students. Included are: circular, elliptic, triangular and rectangular holes (the latter two with rounded corners); holes with edge beads; holes in isotropic and anisotropic elastic plates under tension, shear, bending (in and out of the plane of the plate) and gravity loading; plastic zone around a circular hole; circular ring; one or more holes in an infinite plate, semi-infinite plate, infinite strip and cylindrical shell. The problems are solved, for the most part, by Muskhelishvili's method and the solutions are exploited in great detail, ending with formulas, tables, contour diagrams and graphs.

R. D. Mindlin

Queues. By D. R. Cox and Walter L. Smith. Methuen & Co., Ltd., London, and John Wiley & Sons, Inc., New York, 1961. xii + 180 pp. $3.75.

This monograph is designed as an introduction to the theory of queues for readers with a modest background in mathematics and probability theory. It includes a representative sample of the queuing models that can be treated exactly and a discussion of some crude methods for investigating complicated models. After a classification of types of queues and some qualitative argument regarding the existence of stationary distributions for queue lengths, the exact solutions are obtained concisely. The subsequent treatment of machine interference and other more complex queuing problems is highly empirical. Although the book is a useful introduction to both the science and art of queuing theory, it suffers from a corresponding non-uniformity of style.

G. F. Newell

Direct methods in crystallography. By M. M. Woolfson. Oxford University Press, New York, 1961. viii + 144 pp. $4.80.

This work is one of a series, Monographs on the Physics and Chemistry of Materials. It treats one aspect of the problem of working out the structures of crystals from X-ray diffraction data, namely, determining the signs of the structure factors for centrosymmetric crystals. The structure factors are the coefficients in the three-dimensional Fourier series which represents the electron density throughout a crystal; the magnitudes only are determined experimentally. For crystals having centers of symmetry, the structure factors are real, so the problem is to determine the sign for each. "Direct methods" is taken to mean the determination of signs from the magnitudes on the basis of physical restrictions on the electron density function: non-negativity, etc.

This monograph serves a very useful function for crystallographers. It surveys a large amount of material available only in original journal articles and monographs, extracts the essential contributions, and fits them together into a useful pattern. The treatment exhibits the author's extensive theoretical and practical experience and seems to be very well balanced. The book is also well-timed: it appears that the chief principles have by now been established.

Detailed directions are provided for actual use of the various sign-determining procedures, both those which may be managed with desk calculators and those requiring large computers. Four examples are left to the reader to work out; answers are supplied.

Everyone engaged in crystal structure determination will find this work valuable; many others may be interested in the problems which it describes.

G. B. Carpenter

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BOOK REVIEWS

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Methods of least squares and principles of the theory of observations. By Yu. V. Linnik. Edited by N. L. Johnson. Pergamon Press, New York, Oxford, London, Paris, 1961. xii + 360 pp. $12.50.

This work provides an elegant and thoroughly modern presentation of the method of least squares. The first three chapters contain the necessary background material on matrix algebra (which is used extensively), probability and statistics, respectively. After that, the subject is developed rigorously and to considerable depth, but always illuminated by practical examples and interspersed with accounts of computational methods.

The translation is clumsy and the quality of production of the book as poor as one has come to associate with Pergamon translations. The publishers' apology about the production and price reads strangely when it is remembered that several of their translations are available in much superior editions at lower prices. Anyone able to read the language would be well-advised to purchase the excellent German translation published in 1961 by the Deutcher Verlag der Wissenschaften, Berlin; it is entirely re-set, beautifully printed and costs approximately $8.50.

W. Freiberger

Analysis of nonlinear control systems. By Dunstan Graham and Duane McRuer. John Wiley & Sons, Inc., New York, London, 1961. xiii + 482 pp. $9.75.

This book is significantly different from others in this field. It does not deal with nonlinear mechanics or nonlinear oscillations including nonlinear control systems as a special part, but it concentrates on the latter exclusively.

In the control field the authors do not present a number of interesting examples (classical and recent) but, after a brief description of the difficulties introduced by nonlinearities, they discuss the essential techniques for analyzing nonlinear control systems.

The chapter headings give a good idea of the contents of the book: II. General Techniques for Solving Nonlinear Control Problems. III. Introduction to Quasi-Linearization and the Describing Function Technique. IV. Sinusoidal Describing Functions for Isolated Nonlinear Elements. V. Quasi-Linear Closed Loop Systems with Periodic Outputs. VI. Random Input Describing Functions. VII. The Phase Plane Method. VIII. Trajectories and Stability. IX. Relay Servos, Switching and Programmed Controllers. X. Epilog and Consequence.

The authors give different interpretations of the describing function and stress the fact that the full substitute for a nonlinearity is the equivalent linear operator (describing function) plus the "remnant," that means the limitations of the describing function technique are clearly recognized. Tables and figures show nonlinear elements and their describing functions.

As the heading of Chapter VII indicates, the authors concentrate on phase *plane* methods and pay very little attention to higher-order control systems which demand a phase *space* representation. The second method of Liapounoff for investigating stability of systems is briefly discussed; essentially, the reader is provided with a literature survey and the necessary references.

The problem of "optimum control" is not dealt with in its generality; only "optimum switching" for time optimum control is discussed. There is no reference to Pontryagin's maximum principle; perhaps because the authors feel that these time optimum controls are controls "programmed" for performing a certain task and not "feedback" controls in the general sense.

Reviewer feels that this book will be appreciated by many control engineers who have to deal with nonlinear feedback control systems, since it reflects the wide experience of its authors in using analytical means for discussing practical problems.

I. Flügge-Lotz

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BOOK REVIEWS

(*Continued from p. 266*)

Elements of queueing theory with applications. By Thomas L. Saaty. McGraw-Hill Co., Inc., New York, Toronto, London, 1961. xv + 423 pp. $11.50.

This book contains 423 pages plus 910 references. The author and publisher must be congratulated for being first with the most. The emphasis is on tedious derivations of explicit exact formulas for various types of queues (a disease of the subject). Although the theory of birth and death processes, which has been carefully developed in the literature, is used as the basis for the stochastic theory of queues, the former theory has been so greatly condensed in this book as to be unintelligible (and in parts incorrect). The point of view is that a problem is solved if one can obtain a formula, even if the formula is too awkard to be understood. The book should serve as a useful survey of the literature on queues and related subjects. It is not recommended as an introduction for the curious but uninitiated.

G. F. Newell

An introduction to celestial mechanics. By Theodore E. Sterne. Interscience Publishers Inc., New York, London, 1960. xi + 206 pp. $4.50.

This polished little book is an excellent introduction for the physicist and engineer who is concerned with artificial satellites. Approximately one-third of the material is concerned with this subject On the other hand, lunar theory receives a perfunctory paragraph and there is not even a statement of the restricted three body problem.

The first chapter deals with the standard questions of elliptic motion. Outstanding is the collection of pretty exercises concerning artificial satellites and space travel.

The second chapter discusses the shape and gravitational potential of the earth. [It appears from recent work that the figure 1/297 for the flattening of the earth, quoted on page 37, should be revised downwards.]

The third chapter concerns standard matters of spherical astronomy and the determination of orbital elements from observations.

Chapter four introduces Lagrange's equations and the Hamilton-Jacobi theory. Although the physicist is accustomed to this kind of mathematics and is more efficient at it than the mathematician himself, the engineer will find this the most difficult part of the book. The introduction to perturbation theory is made through the general Hamilton-Jacobi equation rather than through simple examples more likely to be familiar to the engineer. There is an excellent discussion, pages 112–113, of the danger of confusing the presence of secular terms with instability.

Chapter five, which is the heart of the book, concerns the author's own solution of the earth satellite perturbation problem, including the effect of atmospheric drag and the oblateness of the earth. On the other hand no account is taken of radiation pressure, recently found to have a serious effect on certain satellites.

The final chapter describes numerical techniques, including that of Encke.

For its small size this book delivers a great deal of information.

Harry Pollard

Tables of spectrum lines. By A. N. Zaidel′, V. K. Prokof′ev, S. M. Raiskii. Pergamon Press, London, New York, Paris, Los Angeles, and Veb Verlag Tecknik, Berlin, 1961. XLIII + 550 pp. $14.00.

This table, for those actively engaged in spectral analysis, lists over 40,000 of the most important spectral lines (compared with about 109,000 listed by Harrison) of 60 elements in order of wavelength as well as some 23,000 lines of 93 elements arranged by elements. A third part of this book contains tables of ionization potentials, molecular weights, boiling points of elements and oxides, carbon arc data, and sensitive lines of the elements.

Rohn Truell