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BOOK REVIEWS

Topics in the theory of random noise. By R. L. Stratonovich. Volume I. Gordon and Breach Science Publishers, New York and London, 1963. xiii + 292 pp. $12.50.

This is the third volume in a series of monographs on mathematics and its application. The volume is in two parts. The first part deals with the general theory of random processes and the second part with nonlinear transformations of signals and noise. The first chapter introduces the basic concepts of random variables, moments of random variables, and characteristic functions. Random functions and characteristic functionals are briefly discussed. Already at this point formal procedures employing delta function techniques are employed and this is characteristic of the remainder of the book. However, a rigorous development of the required techniques would be beyond the scope and intent of a book with this orientation. The formal techniques are well motivated by heuristic arguments. Stationary processes are introduced in the second chapter. Fourier analysis of such processes is discussed and illustrated by a number of concrete examples. The next chapter considers Gaussian and non-Gaussian random processes. Quasi-moment functions are introduced to indicate departure from normality (or Gaussian-ness). Chapter four is concerned with Markov processes. The definition of a Markov process is first given. The Chapman-Kolmogorov equation is derived. Continuous Markov processes (or diffusion processes) are studied by means of the Fokker-Planck equation. Some study is made of the circumstances under which a general process can be replaced by a Markov process. The technique used is basically that of a formal perturbation procedure. Some attention is paid to multidimensional problems. Simple classes of non-stationary processes are considered in the following chapter, for example, processes with stationary increments and periodic nonstationary processes. Systems of random points and related random functions are introduced in chapter 6. Poisson systems and shot noise are illustrative examples. The spectral density of various pulse sequences are computed. The first part concludes with a chapter on narrow-band processes. The amplitude and phase of narrow-band processes are considered. The second part of the book consists of two chapters, the first on zero-memory nonlinear transformations and the second on nonlinear transformations with memory. Particular attention is paid to the cases of piece-wise linear, polynomial and exponential transfer functions in the earlier chapter. The last chapter discusses the example of a diode detector whose action is described by a first-order nonlinear differential equation. All in all the book provides a coherent and well-motivated introduction to topics in the theory of random noise.

M. Rosenblatt

The mathematical theory of viscous incompressible flow. By O. A. Ladyzhenskaya. Translated from the Russian by R. A. Silverman. Gordon and Breach Science Publishers, New York and London, 1963. xiv + 184 pp. $9.50.

This book is a translation of Matematicheskiye voprosi dinamiki vyazkoi nezhimayemoi zhidkosti. The authoress was able to examine the galley proofs of the translation, to correct all detected misprints, to provide additional references and comments and also to incorporate an extra section on effective estimates of solutions of the nonlinear stationary problem.

The book is divided into six chapters: I. Preliminaries. II. The linearized stationary problem. III. The theory of hydrodynamical potentials. IV. The linear nonstationary problem. V. The nonlinear stationary problem. VI. The nonlinear nonstationary problem.

In considering the Navier-Stokes equations for the motion of an incompressible viscous fluid two main questions present themselves: (A) Do the equations together with suitable boundary and initial conditions have a unique solution? (B) How satisfactorily do the solutions describe the flow of a real fluid?

The only way to ascertain what the Navier-Stokes equations really have to say about the flow of real fluids is first of all to carry out a rigorous mathematical analysis of the solution of boundary value problems for these equations corresponding to actual hydrodynamical situations.

The reader accustomed to the classical methods of mathematical physics will find that in this book

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the interpretations given to what is to be understood by the solution of a problem and what it means to solve a problem will differ from those with which he is familiar. To a large extent, a precise analysis of these matters is responsible for the success of the present investigations.

The statement "it has been proved that the problem has a unique solution" can have very different meanings depending on the function space in which a solution is sought. The form in which the requirements of the problem must be satisfied is different for different spaces and different extensions of the concept of a solution of a problem, i.e. different "generalized solutions" present themselves. In fact for every problem there are infinitely many generalized solutions. In this book the kind of solution has been selected, which was introduced in 1957 in a paper by Kiselev and Ladyzhenskaya, for which it was first proved that boundary value problems have unique solutions in the large.

It is proved that in the general nonlinear case, a stationary boundary value problem has solutions for any Reynolds number R whatsoever, where both the boundaries of the object past which the flow occurs and the external forces can be quite irregular. However, these solutions are stable only for small values of R. A nonstationary boundary value problem for the Navier-Stokes equations has a unique solution for all instants of time for plane and also for axisymmetrical flows. In the general three-dimensional case it is proved that the problem has a unique solution if the external forces are conservative and if R is small at the initial instant of time.

In the general case where the above conditions are not satisfied for all instants of time, there exists at least one "weak solution" for which the velocity partial derivatives which appear in the Navier-Stokes equations are summable with respect to (x, t) with exponent 5/4. It is still not known whether this solution is unique on the whole time axis $t > 0$. If the initial conditions are reasonably smooth, this solution is unique, at least during a certain time interval whose size is determined by the data of the problem.

The mathematical methods employed are based upon estimates and inequalities rigorously applied and although complete success in answering questions (A) and (B) has not yet been achieved, this book provides the first unified account of the state of the subject (as of the year 1961) to which the authoress has herself made major contributions.

L. M. Milne-Thomson

Principles of reliability. By Erich Pieruschka. Prentice-Hall, Inc., Englewood Cliffs, N. J., 1963. xvii + 365 pp. $11.25.

This book is the lastest in an ever-increasing flood which is inundating the reliability field. Like many of its predecessors, it makes little mention of related publications but primarily restricts itself to a cursory presentation of the elementals of statistics followed by the author's own interpretation of the reliability area. Unfortunately, there are no references to the author's journal publications (subjected to professional referee scrutiny) so that the reader does not have the assurance of an unbiased critic of any new material presented.

The jacket contains the usual blurb which tends to overstate the worth of this book. For example, the statement that the 'degrees of complexity are quantitatively treated' is based on the discussion in which (p. 187) the complexity of an equipment is defined as (simply) the logarithm of the reciprocal of the inherent reliability (simplicity). This book is characterized by a very uneven exposition making it difficult to determine the audience for which it was intended. For example (p. 2) the reader is shown exactly how to go from $0 \leq p \leq 1$ to $0\% \leq p \leq 100\%$ and then given an (inexact) statement of the 'theorem' of large numbers. The derivation of the mean of a binomial distribution takes nearly a page (p. 21), whereas many expressions (e.g., the approximation on p. 90) are presented without any indication of a derivation or reference. In general, the sparcity of references (given only as footnotes) would make it difficult for one to determine what is original and what might be considered 'well-known'.

Despite many shortcomings, this book contains certain sections which would possibly be useful to the reliability specialist. However, for such a reader a good portion would be completely redundant.

S. J. Amster

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Analysis of linear time-invariant systems. By William M. Brown. McGraw-Hill Book Co., Inc., New York, San Francisco, Toronto, London, 1963. xiv + 339 pp. $11.75.

Readers of this journal are undoubtedly familiar with the advances being made in the teaching of mathematics in the public school system. As a result of this "revolution" the degree of mathematical sophistication of engineering students promises to be considerably higher than that of past generations. This poses a serious challenge to the engineering schools to build curricula based on this improved foundation (and still produce engineers not mathematicians). It is within this framework that the book is to be judged.

The objectives of the book are to develop a more than superficial understanding of operational analysis, and its application to the analysis of linear time-invariant systems. This aim is attacked in two parts. Part I, consists of three chapters. A short introductory chapter (Operational Analysis, 13 pages) is employed to define concepts, terminology and to provide motivation for more extensive study. Emphasis is placed on "exponentially amplitude modulated sinusoids." Chapter 2 is a long chapter (Two-Sided Laplace Transforms with Simple Applications, 87 pages). About half of the chapter is devoted to mathematical preliminaries and fundamental theorems for Laplace Transforms. The author concentrates on functions of bounded variation and Riemann integration, though some more modern techniques are employed (i.e., the dominated convergence theorem is used to justify interchange of limit and integration). The remaining half of Chapter 2 is bisected (in terms of pages) roughly into a review of functions of a complex variable and applications to simple circuits, feedback systems and linear constant coefficient differential equations. Treatment of the delta function could be made more respectable by development of the theory of distributions or generalized functions. The author recognizes the mathematical inconsistencies in the earliest uses of the delta function, but chooses to use the delta function for notational convenience. This is not a serious shortcoming, in my view, though advocates of distribution theory will think otherwise. Part I is wrapped up in Chapter 3 (Foundations of Operational Analysis, 19 pages). It serves to document the uniqueness of the multiplicative notion of transfer function and to establish the exponential function as a basis for operational analysis of linear time-invariant systems. The concept of Area Transform is introduced as a general version of operational analysis. In essence, the inversion of the Area Transform is achieved by a two-dimensional Stieljes integral which includes integration over the entire complex plane. The Area Transform is primarily of interest as a concept rather than as a tool for engineering applications. Discussions of circuit properties in addition to linearity and time invariance (namely, real parameter systems, realizability and stability) are included in Chapter 3. The author notes that he has used Part I as a course in operational analysis for seniors and first-year graduate students.

Part II (Applications and Extensions) consists of Chapters 4 through 10. Chapter 4 (Complex Parameter Systems, 13 pages) is concerned with complex representation of signals and systems. A closer tie with the conventional in-phase and quadrature components of communications would make this chapter and Chapter 9 more palatable from the engineering point of view. Several conventional measures of response time (rise time, bandwidth) are given careful coverage in Chapter 5 (System Response Time, 21 pages). "Band Pass Systems, Hilbert Transforms, and Relations Between Components of Transfer Functions" is the self-explanatory title of Chapter 6 (33 pages). "Discrete Systems and Z Transforms" are treated in Chapter 7 (26 pages) based on the convolution characterization. The Z transform is wisely treated as a notationally convenient special case of the operational analysis previously developed. Chapter 8 (Introduction to Power Spectra, Autocorrelation Functions, and Optimum Linear Systems, 30 pages) is concerned with providing some manipulative skills in statistical system analysis for use in Chapter 9. The bulk of what the author considers "exciting applications" of the previous material to radar and communications systems is contained in Chapter 9 (Introduction to Analysis and Optimization of Radar and Communication Systems, 21 pages).

The concluding chapter (Complements: Mathematics of Fourier Transforms, 50 pages) as its title implies, is intended to elaborate on some of the mathematical niceties alluded to primarily in the first part of the book. In the Preface and in a footnote to Chapter 10, the author contends that this chapter is "suitable as a basis for a serious two-credit course in the mathematics of Fourier transforms." Yet on page 281 we find "After Sec. 10-1 (sic-Riemann, Stieljes, and Lebesque integration) we shift gears and become somewhat more concise in an effort to prevent this chapter from diverging into a book in itself." These two statements do not seem compatible and in this reviewer's opinion, the treatment in Chapter 10 is too sketchy for the secondary purpose proposed.

The book builds up operational analysis from essentially a classical viewpoint as opposed to Schwartz's distribution theory, Lighthill's generalized functions, and Mikusinski's convolution quotients. I do not propose to take sides here, but merely point out that Dr. Brown has done the job well from the classical standpoint with appropriate emphasis on engineering applications. He is able to cover a wide range of material as a result of conciseness of mathematical symbolism and a good selection of problems. However, in an attempt to cover a lot of ground, uneven treatment is inevitable. Consequently, some topics in Part II are hurriedly sketched (i.e., optical synthesis of complex parameter filters).

This reviewer suggests that the first part of this book is a first step toward providing a course in operational analysis for the "new breed" in the junior year. Some distribution theoretic ideas might be incorporated. Part II contains many of the ingredients for a possible senior level course in the analysis of time-invariant systems.

M. R. AARON

Combinatorial mathematics. By Herbert John Ryser. John Wiley & Sons, Inc., New York, 1963. xiv + 154 pp. $4.00.

This is an excellent introduction to modern combinatorial mathematics for the sophisticated mathematician. The reader who is familiar with the fundamentals of algebra, geometry and number theory will find many interesting and important results in 142 pages of text. In succession, the author covers the basic aspects of inclusion and exclusion, recurrence relations, a powerful theorem due to Ramsey, matrices of zeros and ones, latin squares, finite projective planes, combinatorial designs and perfect difference sets.

The writing is lucid, and there are many illustrations given of the significant theorems. All those concerned with the combinatorial aspects of modern physics, pure and applied mathematics, circuit theory, scheduling and operations research will find the book a valuable asset.

RICHARD BELLMAN

Random processes. By Murray Rosenflatt. Oxford University Press, New York, 1962. x + 208 pp. $6.00.

Probability theory, stochastic processes, and a certain amount of information theory form the subject of this text book. It is intended for advanced undergraduate and beginning graduate students. The author's aim is not to give a systematic and rigorous treatment, but only to present selected topics and treat them intuitively. Lengthy and complicated proofs are usually omitted; mostly only the brief proofs are provided. The book is useful for students of stochastic process and it enables the reader to acquire a good working knowledge in this field.

The book is divided into eight chapters. Chapter I is the Introduction. Chapters II–IV deal with probability theory, including Markov chains. Chapters V–VII deal with stochastic processes. Chapter VIII contains additional material. Information theory is treated in Chapters I and V. The book contains 86 problems for solution but no solutions are provided.

Chapter II deals with probability theory in case of finite or countable infinite sample spaces, and the notion of entropy is introduced.

Chapter III treats Markov chains by using matrix theoretical methods. Limiting theorems and functions of a Markov chain are also treated.

Chapter IV deals with probability theory in case of infinite sample spaces and the notion of stochastic processes is introduced.

Chapter V treats stationary stochastic processes, ergodic theorems, and MacMillan's theorem in information theory.

Chapter VI deals with Markov processes following Kolmogorov's and Feller's methods.

Chapter VII is concerned with stationary processes in the wide sense, spectral decomposition, linear extrapolation, and spectral estimates of normal processes.

Chapter VIII contains, among others, a decomposition theorem for stochastic processes, and a uniform mixing condition is introduced for stochastic processes.

It is a pity that the author almost completely neglects a great number of topics concerning Poisson processes.

L. Takács

Perturbation theory and the nuclear many body problem. By K. Kumar. North-Holland Publishing Co., 1962. viii + 235 pp.

The insoluble many body problem of quantum mechanics has spawned numerous approximation procedures, many of which are examined in this book. Lumped under the general heading of "perturbation theory" these procedures are divided broadly into algebraic, diagrammatic and rearrangement methods.

Chapter I is devoted to algebraic methods, and includes, among others, the traditional Brillouin-Wigner and Rayleigh-Schrödinger perturbation theory and variants thereof associated with Feenberg, as well as the multiple scattering approach of Watson. In Chapter II, diagrammatic versions of many-body perturbation theory due to Goldstone, Hugenholtz and Brueckner are developed first without, and subsequently in greater generality with the aid of second quantization. Important stress is laid on the fundamental contributions of van Hove on quasi-particle properties. Chapter III covers the reaction-matrix and propagator modification, and the approximations they give rise to.

Chapter IV comes to the heart of the matter with solution methods for the reaction matrix and their application to nuclear problems. Among other examples are cited the various solutions of Brueckner and collaborators for both infinite and finite nuclear media. The author is rather vague (and probably justifiably so) on the accuracies of the several calculations and their comparative merits.

Chapter V briefly discusses several other many body approximations including: 1. Jastrow's variational method, 2. Superposition of configurations, 3. Brenig's two-particle approximation, 4. Pseudo-potential method, and 5. Bound pairing of superconductivity.

The aim of the author has been to systemitize various seemingly divergent approximations to many body problems. This is conveniently and properly done by appropriate "encompassing" formal techniques which include the desired examples as special cases. In so doing, however, the physics of the many-body problem appears often times in the background, emphasis instead being placed on just how many diagrams or terms in a series can be included. Almost absent is even the implication that certain sets of diagrams (or terms) incorporate a certain physical significance, and are therefore important or possibly not important, in this or that regime. Apart from this aspect, the book is well presented and a welcome addition to the growing field of many-body physics.

John R. Klauder

Degrees of unsolvability. By Gerald E. Sacks. Princeton University Press, Princeton, N. J. (Annales of Mathematics Studies no. 55). x + 174 pp. $3.50.

This monograph deals with the problem of classifying number-theoretic functions according to their "computability". Two such functions f, g have the same degree ($\mathbf{f} = \mathbf{g}$) if each one is recursive in the other. The degree of f is said to be smaller or equal to that of g, ($\mathbf{f} \leq \mathbf{g}$), if f is recursive in g. In an intuitive sense, made precise by the notion of recursiveness, this means that f is computable on the basis of g. Thus all recursive functions have the same (and minimal) degree, and a richer structure of the upper semilattice of degrees under the relation $\leq$ concerns only functions that are not recursive.

The author deals exclusively with the study of the properties of the order relation $\leq$ on the set of degrees (and not with the question of determining the degrees of specific number-theoretic functions). The reader's familiarity with Kleene's "Introduction to Metamathematics" is presupposed throughout the book which is otherwise selfcontained and gives a very readable, well motivated and authoritative survey of the facts that are known to date.

E. Engeler

Non-linear control systems analysis. R. H. Macmillan, Editor. Pergamon Press, Oxford, London, Paris, Frankfurt, and The Macmillan Co., New York, 1963. xvi + 174 pp. \$3.75.

Non-linear control systems analysis is Vol. I of the Mechanical Engineering Division Series of The Commonwealth and International Library of Science, Technology, Engineering and Liberal Studies. This library's aim is to provide paper backs at moderate prices for 'instructors and pupils in all types of schools and educational establishments (including industry) teaching students on a full and/or part-time basis from the elementary to the most advanced levels'.

Reviewer wonders why "self instruction" is not mentioned. Such a venture of a publisher is not new; the collection "Göschen" [in German, publisher W. de Gruyter, Berlin-Leipzig] has similar aims and was flourishing as early as the 1920's, when about 800 booklets were available.

The volume under discussion is a compilation of articles published originally in the journal Process Control and Automation. The following topics are dealt with: 1) Nonlinear Control Systems by J. F. Coales; 2) The Phase Plane Method by P. Naslin; 3) The Describing Function Technique by John C. West; 4) The Application of the Describing Function by the same author; 5) Computation of Transients by P. Naslin; 6) Autonomic Control Systems by R. H. Macmillan and N. W. Rees. The authors assume that the reader is familiar with linear control systems and should get acquainted with the most striking features of non-linear systems. These features are sometimes derived, but sometimes only described. That means the presentation aims at inducing the reader to a more serious study by reading the reference literature. An engineer experienced in the field of nonlinear control may not always agree with the choice of features which are treated more elaborately than others. The idea of optimal control is treated very scantily. In contrast, relatively many pages are devoted to graphical integration of differential equations. The last chapter, "Autonomic Systems," deals with "adaptive" systems; however, by using a new name and suggesting a new subdivision of this class authors may have avoided being drawn into the heated arguments about what "adaptive" is.

I. Flügge-Lotz

Fundamental structures of algebra. By G. D. Mostow, J. H. Sampson and Jean-Pierre Meyer. McGraw-Hill Book Co., Inc., New York, Toronto, London, 1963. xvi + 585 pp. \$8.95.

The scope of this book is indicated by the chapter headings: 1. Binary operations and groups; 2. Rings, integral domains, the integers; 3. Fields, the rational numbers; 4. The real-number system; 5. The field of complex numbers; 6. Polynomials; 7. Rational functions; 8. Vector spaces and affine spaces; 9. Linear transformations and matrices; 10. Groups and permutations; 11. Determinants; 12. Rings of operators and differential equations; 13. The Jordan normal form; 14. Quadratic and hermitian forms; 15. Quotient structures; 16. Tensors. The book is exceptionally easy to read and there are many examples given which very nicely illustrate the theory.

M. E. Gurtin

Linear programming and extensions. By George B. Dantzig. Princeton University Press, 1963. xvi + 621 pp. \$11.50.

It is a real pleasure to see this long-awaited work at last in print. While many texts on linear programming have appeared in recent years, few of them rival the present volume's happy balance between the conciseness expected by the mathematical reader and the more discursive style preferred by potential users from other disciplines.

Three introductory chapters (I: The Linear Programming Concept; II: Origins and Influences; III: Formulation of a Linear Programming Model) set the stage for nine chapters devoted to mathematical methodology (IV: Linear Equation and Inequality Systems; V: The Simplex Method; VI: Proof of the Simplex Algorithm and the Duality Theorem; VII: The Geometry of Linear Programs; VIII: Pivoting, Vector Spaces, Matrices, and Inverses; IX: The Simplex Method Using Multipliers; X: Finiteness of the Simplex Method Under Perturbation; XI: Variants of the Simplex Algorithm; XII: The Price

Concept in Linear Programming). There follow eleven chapters on specific applications of linear programming (XIII: Games and Linear Programming; XIV: The Classical Transportation Problem; XV: Optimal Assignment and Other Distribution Problems; XVI: The Transshipment Problem; XVII: Networks and the Transshipment Problem; XVIII: Variables with Upper Bounds; XIX: Maximal Flows in Networks; XX: The Primal-Dual Method for Transportation Problems; XXI: Programs with Variable Coefficients; XXIII: A Decomposition Principle for Linear Programs). Where this is appropriate, methods specially designed for these problems are discussed. The next three chapters are devoted to extensions of linear programming (XXIV: Convex Programming; XXV: Uncertainty; XXVI: Discrete Variable Extremum Problems). Finally, the last two chapters are meant to give the reader some insight into the ancillary questions that arise in realistic optimization studies (XXVII: Stigler's Nutrition Mode: An Example of Formulation and Solution; XXVIII: The Allocation of Aircraft to Routes under Uncertain Demand). An extensive bibliography of nearly 400 items concludes the volume.

W. PRAGER

Lectures on modern mathematics. Edited by T. L. Saaty. John Wiley & Sons, Inc., New York, London, 1963. ix + 175 pp. $5.75.

This book contains the write ups of six expository lectures that were given recently at George Washington University, Washington, D. C. The lectures were sponsored jointly by the university and the Office of Naval Research. There are to be eighteen in all and the remainder will be published subsequently. The lectures caused considerable stir in the Washington mathematical community and were well attended.

In setting up these lectures, it was the intention of the sponsors to invite an eminent spokesman for each of a variety of fields and have him "delineate a substantial research area, to describe it broadly and comprehensively for an audience of mathematicians who are not specialists in that area and to contribute to this description his individual evaluation of the esthetic and practical aspects of the field, its position in mathematical development as a whole, and its future, as that might be implied in the conjectural exposition of its unsolved problems."

This is such a wonderful objective and such a difficult one, that it will come as no surprise if I report that it was nowhere fulfilled. Perhaps it was an impossible assignment. Here and there one catches glimpses of an author wrestling bravely with it, as Laertes presumably wrestled with the injunctions of Polonius, and then treading the primrose path of parochialism.

I think the primary appeal of these volumes will be to experts in the individual fields and to thesis students. Each article, on its own terms, has been honestly conceived and admirably written. Each contains up to date references and long lists of interesting unsolved problems. Perhaps I am pessimistic about a larger appeal.

Here are the authors and titles of the articles: P. R. Halmos: *A Glimpse into Hilbert Space;* Laurent Schwartz: *Some Applications of the Theory of Distributions;* A. S. Householder: *Numerical Analysis;* Samuel Eilenberg: *Algebraic Topology;* Irving Kaplansky: *Lie Algebras;* Richard Brauer: *Representations of Finite Groups.*

PHILIP J. DAVIS

Lectures in statistical mechanics. By G. E. Uhlenbeck and G. W. Ford. American Mathematical Society, Providence, R. I., 1963. x + 181 pp. $7.40.

As the title implies, this volume originated from a set of lectures given by Prof. Uhlenbeck at the University of Colorado in the summer of 1960. The primary intent, according to the preface, was to interest mathematicians in the problems of statistical mechanics, but even non-mathematicians will find this an excellent introduction to the field.

After a chapter on the fundamentals of equilibrium statistical mechanics, the authors turn to the calculation of partition functions for monatomic gases by the cluster method, following this in turn with a chapter on the condensation phenomenon. The next three chapters are devoted to the treatment of irreversible processes in gases, considered from the standpoint of the Boltzmann equation; in addition to presenting the Chapman-Enskog theory of transport coefficients, this section also includes an interesting discussion of sound dispersion in a monatomic gas, particularly in the limit of high frequencies. The book concludes with a chapter on the Bogoliubov theory of transport processes in dense gases. An appendix on the quantum statistics of interacting particles has been added by Dr. E. W. Montroll.

The informal style of the lectures is retained in the text, and makes for somewhat more lively reading than is usually the case with treatises on statistical mechanics.

STEPHEN PRAGER

Multilinear analysis for students in engineering and science. By G. A. Hawkins. John Wiley & Sons, Inc., New York and London, 1963. xiv + 219 pp. $2.95.

Here is an addition to the already staggering list of books on the subject of vector and tensor calculus. The book is neither modern nor rigorous. However there is a charm in the naïveté in which it is written that makes the book exceptionally easy to read. The author states in the preface that "it is not intended that this material be used as a text for a formal course nor as a treatise, but only as an aid in the pursuit of the subject on one's own". With this thought in mind the book should serve its readers well.

M. E. GURTIN

NOTICE

Volume 22, No. 2 of the Quarterly of Applied Mathematics contains a review of Professor A. C. Eringen's book "Nonlinear Theory of Continuous Media" by Professor A. C. Pipkin, which the latter had in fact withdrawn. The erroneous publication of this review was caused by an oversight in the editorial office, for which the Managing Editor wishes to apologize to both Professor Eringen and Professor Pipkin.

W. Prager