QUARTERLY OF APPLIED MATHEMATICS

This periodical is published quarterly by Brown University, Providence, R. I. 02912. For its support, an operational fund is being set up to which industrial organizations may contribute. To date, contributions of the following companies are gratefully acknowledged:

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The subscription price for the Quarterly is $8.00 per volume (April-January). Single issues can be purchased at $2.50, and back volumes at $8.00 per volume as far as they are available. Subscriptions and orders for single issues must be addressed to: Brown University Press, Providence, R. I. 02912.

Second-class postage paid at Providence, Rhode Island, and at Richmond, Virginia

WILLIAM BYRD PRESS, INC., RICHMOND, VIRGINIA
QUARTERLY
OF
APPLIED MATHEMATICS

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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter I and the prime ('), between alpha and α, kappa and κ, mu and μ, nu and ν, eta and η.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set above letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which follow the letter should be used.

Square roots should be written with the exponent ½ rather than with the sign √.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponents with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

\[ \exp \left( (a^2 + b^2)^{1/2} \right) \text{ is preferable to } e^{(a^2 + b^2)^{1/2}} \]

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

\[ \frac{\cos \frac{xx}{2b}}{\cos \frac{xa}{2b}} \text{ is preferable to } \frac{\cos \frac{xx}{2b}}{\cos \frac{xa}{2b}} \]
ADVANCED CALCULUS, Second Edition
A careful revision of an extremely successful text for the standard junior-senior course. Presents a modern treatment of the fundamentals of analysis in a form which emphasizes both structure and technique.

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A comparatively elementary introduction to distribution theory (theory of generalized functions) and its applications to transform analysis (operational calculus).

FINITE MATHEMATICS: With Applications in the Social and Management Sciences
by LOUIS O. KATTSOFF and ALBERT J. SIMONE, Boston College. Available in May.
A mathematics text designed for the modern student in schools of business, departments of mathematics or liberal arts colleges. Presented in the rigorous and precise language of the mathematician.
BOOK REVIEWS


The book was published in Moscow (Fizmatgiz) in 1958. The present translation has been edited by Ian N. Sneddon.


This has been translated from the original Russian edition (1960 by A. Sen and R. N. Sen. Aside from occasional misspellings of names that were transliterated from the Russian (e.g. Gordan for Jordan), the translation has been carefully and competently done.


This new edition is an unabridged and corrected republication of the work first published in 1892 by the Register Publishing Company.

Philip Davis


This table gives values of $C_n^k$ for $n$ from 1 to 100 to 10 significant figures and values of $n!$ for $n$ from 1 to 1775 to 20 significant figures.


The field of applied mathematics has made some astonishing acquisitions among which mathematical logic (through computer design, processing languages, theory of computability, etc.) is one of the more important. It was, however, a difficult task to advise mathematicians not primarily interested in foundations as to what textbook they would best consult as a concise introduction to mathematical logic. For this purpose the book under review is ideally suited. The author has succeeded in presenting the skeleton of the subject in a readable manner: Propositional Calculus, Quantification Theory (including some topics for the theory of models), Formal Number Theory (treating the notion of recursiveness, proof of the deep theorems of Gödel, Church and Tarski on the internal limitations of formal systems, incompleteness and undecidability), Axiomatic Set Theory (including a discussion of the axiom of choice), and the Theory of Computability (discussing various, equivalent, precise definitions of the intuitive concept of computability: Markov algorithms, Turing machines, recursive functions). There is also a valuable appendix with a proof (after Schütte) of the consistency of formal number theory.

E. Engeler

(Continued on p. 326)


This book consists of six chapters on various parts of analysis of contemporary interest where nonlinear functions and functionals enter in an essential fashion: Linear and nonlinear transformations; nonlinear algebraic and transcendental functions; nonlinear optimization, nonlinear programming and systems of inequalities; nonlinear ordinary differential equations; introduction to automatic control; linear and nonlinear prediction theory.

Of these chapters, only that on nonlinear optimization may be considered to be well enough organized and to contain enough material to represent a contribution to mathematical literature. The others show lack of understanding of the basic ideas and methods, lack of organization, or both. This is particularly true of the chapters on control theory and nonlinear differential equations.

The book is definitely not recommended for either students or teachers.

Richard Bellman


This volume contains translations of 33 papers in the general area of probability theory, into English, from Chinese, Hungarian, Rumanian, Russian and the other slavic languages. Twelve of the papers are short notes from Dokl. Akad. Nauk SSSR and similar transactions. The remaining, longer articles are listed below, followed by the number of the appropriate Mathematical Reviews article (M.R.) when available.


Chang Li-chien—On the ratio of an empirical distribution function to the theoretical distribution functions, Acta. Math. Sinica 5 (1955), 347–368 (not reviewed)

Dupac, Vaclav—On the Kiefer-Wolfowitz approximation method, Casopis Pest. Mat. 82 (1957), 47–75 (M.R. 19–693)

Fabian, Vaclav—Some modifications of interval estimation and choice of number of observations in a special case of a binomial random variable, Apl. Mat. 4 (1959), 35–52 (not reviewed)


(Continued on p. 334)
Mandl, Petr—On the asymptotic behavior of probabilities in groups of states of a homogeneous Markov chain, Casopis Pest. Mat. 84 (1959), 140–149 (M.R. 21–7566)


Rihter, V.—A more precise form of an inequality of S. N. Bernstein for large deviations, Vestnik Leningrad. Univ. 14 (1959), no. 1, 24–29 (not reviewed)


Wang Shou-jen—A remark on interpolation in a homogeneous random field on the lattice points in $R^n$, Advancement in Math. Volume 2 (1957), 257–262 (not reviewed)


The translations are quite variable in quality. The first of the above papers by A. Renyi, is translated so poorly that the statement of the problem is rendered unintelligible.

Frank Spitzer


About half of this slim book is devoted to basic matrix algebra. The remainder consists of applications of matrix algebra to the analysis of transistor circuits. The mathematics is elementary; the matrices are all $2 \times 2$, the most involved idea is that of an inverse matrix, which is described in terms of determinants. The book might be useful to a practicing engineer working with transistor circuits but I doubt if an applied mathematician would find it to be of more than passing interest.

B. Hazeltine