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The Quarterly prints original papers in applied mathematics which have an intimate connection with application in industry or practical science. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

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BOOK REVIEWS


The second edition of Nielsen’s book differs from the first mainly in that the material on empirical data has been reorganized and a chapter on linear programming has been added.

P. J. Davis


This is a most stimulating survey of typical problems and techniques rather than an exhaustive treatise of the subject. The first chapter treats the equation \( L(u) + \epsilon N(u) = 0 \), where \( L(u) = 0 \) is a linear algebraic or linear differential equation. To illustrate the scope of this chapter, we mention the following section headings: Lagrange Expansion; Linear (Differential) Equations with Almost Constant Coefficients; Invariant Imbedding; The Matrix Exponential; Baker-Campbell-Hausdorff Series; Nonlinear Perturbation; Poincaré-Lyapunov Theorem; Asymptotic Behavior; Iteration and Recurrence Relations; The Abel-Schröder Functional Equation; Irregular Perturbation; Equations with Small Time Lags. The second chapter is concerned with periodic solutions of nonlinear differential equations of the second order. Illustrative section headings are: Renormalization à la Lindstedt; The Van der Pol Equation; The Shohat Expansion; Carleman Linearization; Dynamic Programming and Perturbation Series; Temple's Regularization Technique. The last chapter deals with various questions regarding the linear differential equation \( u'' + k^2(x) u = 0 \). Illustrative section headings are: The Liouville Transformation; WKB Approximation; Langer Approximation; Wave Propagation and the WKB Approximation.

W. Prager


There has been a need for a textbook that would cover much of the classical material in interpolation theory and approximation theory and introduce the student to the more modern geometrical ways of thinking about these problems. The author has not only written a book which does this but has written a book that is easy to read, well-motivated, and full of exercises so the student can check his progress and reinforce the material he has just studied.

After an introductory chapter in which the functions and function spaces to be studied are introduced the subject of interpolation is considered. Many examples of interpolation are given, from interpolation at discrete points to Taylor interpolation to Fourier series, to mention three types that will be familiar to students. The next chapter deals with remainder estimates. One nice feature of this chapter, and this theme is continued through the rest of the book, is the way the author uses the added structure that one has if attention is restricted to analytic functions. The fourth chapter investigates the convergence of interpolation methods. Again analytic function theory is used in an elegant fashion.

For the last half of a one semester course the author recommends three chapters on approximation theory: (i) uniform approximation, in which an extremely nice treatment of Bernstein polynomials is given, the Stone-Weierstrass theorem is proven, and a connection between approximation and interpolation is given; (ii) best approximation, in which the idea of a normed space is given and the geometry of the spaces considered is emphasized, uniqueness is proven for strictly convex spaces and also the Tschebyscheff theory is considered; (iii) least square approximation, which again connects up interpolation and approximation through Fourier series. Again the emphasis is on the geometry of the situation.

The rest of the book deals with more specialized topics and we will only consider some of the highlights. Hilbert spaces with a reproducing kernel are considered, especially the space of square integrable

(Continued on p. 26)
analytic functions over a region $R$ in the plane. The Muntz theorem is proved using the machinery of Gram determinants that was built up in the chapter on least square approximation. The last chapter considers the problem of approximation of linear functionals. Theorems of sufficient depth are developed so that the proof of the convergence of Bernstein polynomials and Fejer's theorem on summability of Fourier series are easy corollaries.

The one omission that bothers the reviewer is the omission of Bernstein's theorem which allows one to tell how smooth a function is by the degree of approximation to it by polynomials. After Jackson's theorem is proven there is a note that more on this subject can be found in other books, but Bernstein's theorem is not even mentioned.

There are a number of books on this subject that are appearing now or will appear in the near future, but it would be hard to choose a different one for a text for a beginning course in approximation theory.

Richard Askey


This is a collection of papers presented during the conference on "Mathematical Models in Physical Sciences" held at the University of Notre Dame in 1962. The range of topics covered is very wide, and the appeal of individual papers will vary greatly from reader to reader. Among articles which can profitably be read with only a general competence in mathematics, I would number “Microscopic versus Macroscopic Models” by Grad, an excellent discussion by Kruskal of asymptotic behavior, Ulam's computer results on the limiting behavior of sets of points under a repeatedly applied non-linear transformation, Zabusky's calculations on non-linear oscillations in a chain of beads and a continuous string, and a unified variational treatment of various systems in statistics and dynamics by Drobot. Two of the contributions are for experts only: Dyson on “Models of Total Ignorance in Quantum Mechanics” and Haag on “The Mathematical Structure of the Bardeen-Cooper-Schrieffer Model.” Neyman's paper on the statistical distribution of galaxies in the universe would have benefited from some application of the theory of distribution functions as developed in modern statistical mechanics.

In addition to these contributions, there are several of which only the abstracts have been included. The discussions following each paper have also been printed, and some of these make quite interesting reading.

Stephen Prager


W. Freiberger

The book is primarily concerned with the problems of dynamics of elastic and viscoelastic systems that are of interest to civil engineers. The first two chapters review the concepts and theorems of elasticity and viscoelasticity that are required in the following. Chapter 3 treats longitudinal and transverse vibrations of strings. Vibrations of rods form the subjects of chapters 4 (longitudinal), 5 (torsional), and 6 (transverse). There follow two chapters on free and forced vibrations of beams and frames. Transverse vibrations of plates and shells are treated in chapters 9 and 10. Chapter 11 is concerned with the propagation of elastic waves, and chapter 12 presents approximate methods in dynamics of structures. Chapter 13, which deals with integral transforms, is the only one requiring a mathematical background beyond elementary calculus. Chapter 14 contains useful tables of integral transforms. Throughout the book, the presentation is clear and easy to follow.

W. Prager


This is a translation of the second edition of Gelfond's book.


As stated in the preface, the present volume takes a first step toward broader coverage of the different aspects of the computer field. It contains the following contributions: The Formulation of Data Processing Problems for Computers, by W. C. McGee; All-Magnetic Circuit Techniques, by D. R. Bennion and H. D. Crane; Computer Education, by H. E. Tompkins; Digital Fluid Logic Elements, by H. H. Glaettli; Multiple Computer Systems, by W. A. Curtin.


This volume contains translations of selected papers published in Chinese, Polish or Russian during the years 1950 to 1959. The authors and titles of the papers are listed below in the hope that they will give a cursory indication of the interest and scope of the papers included in this volume. They are Linnik, Ju. V., Linear forms and statistical criteria, I, II, 1953; Kozuljaev, P. A., On the theory of extrapolation of stationary sequences, 1950; Urbanik, K., Limit properties of Markov processes, 1957; Gnedenko, B. V., On certain problems in probability theory, 1957; Zinger, A. A. and Linnik, Ju. V., On a class of differential equations with an application to certain questions of regression theory, 1957; Wang, Shou-jen, On the estimation of regression coefficients of a random field of lattice points, 1958; Cheng Ping, Non-negative jump points of an empirical distribution function relative to a theoretical distribution function, 1958; Trybula, S., On the minimax estimation of the parameters in a multinomial distribution, 1958; Praporgescu, N., On chains of probabilities, 1958; Rezny, Zdenek, Group tests of statistical hypotheses on the basis of two control limits, 1959; Minlos, R. A., Generalized random processes and their extension to a measure, 1959; Dorogovcev, A. Ja., Statistical analysis of a difference stochastic equation, 1959; Dorogovcev,

(Continued on p. 68)

M. Rosenblatt


The two volumes of Courant and Hilbert's "Methoden der mathematischen Physik" have been regarded, since their appearance, as standard source books for applied mathematicians. And this is the second volume of the English version, contributing to "breaking through the language barrier," so to speak.

The preface, by Professor Courant, explains the genesis of the book; this English version is said to have been in preparation ever since the appearance during the last war (1943) of the Interscience Publishers reprint of volume II of the German edition, under license of the United States Government. It also explains the dedication of the book to Kurt Otto Friedrichs as "a natural acknowledgement of a lasting scientific and personal friendship." The polycephalic character of the authorship of the book is also explained (one is reminded here of the skiing picture which was distributed along with many copies of Courant and Friedrichs' book, *Supersonic Flow and Shock Waves*, showing Courant leading a crowd of readily identifiable skiers down a slope, and the resulting shock wave): "The present publication would have been impossible without the sustained unselfish cooperation given to me by friends. Throughout all my career I have had the rare fortune to work with younger people who were successively my students, scientific companions and instructors. Many of them have long since attained high prominence and yet have continued their helpful attitude. Kurt O. Friedrichs and Fritz John, whose scientific association with me began more than thirty years ago, are still actively interested in this work on mathematical physics. "..." To the cooperation of Peter D. Lax and Louis Nirenberg I owe much more than can be expressed by quoting specific details. Peter Ungar has greatly helped me with productive suggestions and criticisms. Also, Lipman Bers has rendered most valuable help and, moreover, has contributed an important appendix to Chapter IV." ...

Among younger assistants I must particularly mention Donald Ludwig whose active and spontaneous participation has led to a number of significant contributions.

It would be an impossibility to try to mention, even briefly, the topics which are discussed within the covers of this large volume. Chapter I, entitled "Introductory Remarks," describes basic concepts, problems and general lines of approach to their solution. One finds here, in particular, the Cauchy-Kowalewsky existence proof, for analytic solutions of the Cauchy problem, by the method of "majorants". There are two appendices to the chapter, the first on the equation of a minimal surface and the second on the relationship between systems of first order equations and single differential equations of higher order. Chapter II, under the title "General theory of partial differential equations of first order," centers around the "im kleinen" equivalence of a first order partial differential equation and a certain system of ordinary differential equations. The Hamilton-Jacobi theory, Hilbert's invariant integral, and contact transformations, are included. There are two appendices to the chapter, the first one on characteristic manifolds, and Haar's uniqueness proof, and the second on the theory of conservation laws, leading to not necessarily smooth, "im grossen" solutions. Chapter III carries the title "Differential equations of higher order," and opens with the normal forms for linear and quasi-linear differential operators of second order in two independent variables, followed by a classification of general equations, characteristics. An interesting section contains a lively enumeration and discussion of the chief typical problems of mathematical physics: "initial" value problems (Cauchy), "boundary value" problems (Dirichlet), "mixed" problems, Riemann's mapping problem, Plateau's problem for the equation of

(Continued on p. 92)
minimal surfaces, the “jet problem” of plane hydrodynamics, . . . It is to be noticed that the formulation
of the jet, or Helmholtz problem, a problem whose solution was given by A. Weinstein, is improved over
that in the German edition, while an extensive section (in the German edition) on minimal surfaces,
a theory to which Courant himself has made outstanding contributions, has been omitted in the present
edition. There are two appendices to the chapter, the first on S. L. Sobolev’s lemma for estimating a
function by means of “$L_2$-bounds” of its derivatives, and the second on the uniqueness theorem of
Helmholtz for analytic equations with arbitrary, not necessarily analytic, Cauchy data. Chapter IV,
headed “Potential theory and elliptic differential equations,” begins with a rather systematic treatment
of potential theory and concludes with a less elementary part, where one finds, among other subjects,
Sobolev’s radiation condition for the “reduced wave equation,” E. Hopf’s maximum principle for
elliptic equations, “a priori” estimates of Schauder, and the solution of elliptic differential equations
by means of integral equations (E. E. Levi and D. Hilbert). There is an appendix on boundary value
problems for nonlinear differential equations in several variables, and a supplement (written by L. Bers)
on function theoretic aspects of the theory of elliptic partial differential equations, in particular, the
theory of pseudoanalytic functions of L. Bers and I. N. Vekua; the supplement ends with a proof of the
Schauder fixed point theorem. The concluding two chapters are concerned with hyperbolic equations
of wave propagation. Chapter V: “Hyperbolic differential equations in two independent variables,”
starts with a review of the basic concept of characteristics, which is then applied to the treatment of the
initial value problems. Among other items, one finds: characteristics and normal forms for hyperbolic
systems of first order in two variables; application to the dynamics of compressible fluids; domains of
dependence, influence and determinacy; Riemann’s method of solution; Cauchy’s problem for quasilinear
systems, and for single hyperbolic equations of higher order; and discontinuities of solutions, shocks.
There are two appendices to Chapter V, the first is devoted to the application of characteristics as
coordinates (in particular, the transition from the hyperbolic to the elliptic case through complex
domains, due to H. Lewy, P. Garabedian and H. M. Lieberstein), while the second appendix treats
transient problems and the Heaviside operational calculus. Chapter VI: “Hyperbolic differential equa-
tions in more than two independent variables,” deals primarily with Cauchy’s problem for a single
equation of arbitrary order, and with systems of such equations in several unknown functions. The
first part of the chapter handles questions of uniqueness, existence, construction and geometry of solu-
tions, while the second part concentrates on the representation of solutions in terms of the given data,
and related questions. In part I one finds, among others, geometry of characteristics for second and
higher order operators; applications to hydrodynamics, crystal optics, and magnetohydrodynamics;
propagation of discontinuities and Cauchy’s problem; oscillatory initial values and asymptotic expansion
of the solution; energy integrals and uniqueness for linear symmetric hyperbolic systems and for higher
order equations; and winds up with the existence theorem, proved by means of energy inequalities, for
symmetric hyperbolic systems. In part II, some of the topics covered are: Cauchy’s problem for equa-
tions of second order with constant coefficients, the method of spherical means, the method of plane
mean values, solution of Cauchy’s problem as a linear functional of the data (R. Courant and P. D. Lax),
ultrahyperbolic differential equations (Aseirsson’s mean value theorem), transmission of signals and
progressing waves, and Huyghens’ principle. It is to be observed that the original chapter, in the German
dition, on the classical wave equation in n dimensions, has been reviewed in the present edition, par-
alleling the recent work of A. Weinstein. The authors retain the original terminology of the German
dition in referring to a certain equation as the Darboux equation, while today a great number of mathe-
maticians refer to it as the Euler-Poisson-Darboux equation, in view of the fact that Darboux only
considered the one space variable case, whereas Poisson has already considered this equation in three-
pace, in his famous investigation of spherical mean values in connection with the wave equation. There
is a timely appendix to Chapter VI, dedicated to the theory of ideal functions (S. L. Sobolev) or distribu-
tions (L. Schwartz).

It would be easy, as in the case of any book, to mention interesting and important topics which
have not been included in the presentation. However, the reader will find plenty to occupy him in this
volume. And, as if that were not enough, he can still look forward to the third volume of the series,
which is already announced on page one of the present book, as follows: “The present volume, essentially
independent of the first, treats the theory of partial differential equations from the point of view of mathematical physics. A shorter third volume will be concerned with existence proofs and with the construction of solutions by finite difference methods and other procedures.”

J. B. Diaz


This is a translation and modernized version of a long review paper originally published in Russian more than ten years ago. About half the book is devoted to a survey of the theory of stationary random processes, mostly the theory of second order processes and their spectral representations. The second half deals with linear extrapolation and filtering. The presentation is rigorous but not completely general. The object is to present the main ideas at as elementary a level of mathematics as possible. This the author does by restricting the discussion to processes with rational spectra. The book is a valuable addition to the literature and certainly one of the most readable presentations available on this subject.

Gordon Newell


In a new book on a subject as thoroughly covered as classical electromagnetic theory one rarely finds great innovations and in this respect Poincelot’s work is no exception. The book gives a well written, detailed and elegant account of classical electromagnetic theory as of some fifteen or twenty years ago. Occasional reference is made to recent original work and a few specialized problems, for example in wave guides and coaxial lines, are dealt with on the basis of recent developments. However, many topics and methods that were either developed or that matured in the last two decades have been left out. The treatment is phenomenological and macroscopic throughout and although applications to some physical problems are discussed, no mention is made of physical mechanisms.

The book has a detailed table of contents, but unfortunately no alphabetical index.

C. Elbaum


The book was developed for a course which follows “a basic course in statistical inference.” Its rapid start in the middle of things reflects this. It goes from sample to power in eight pages of Chapter 1. In Chapter 2 appears an interesting notation, e.g., “C[51.60 ≤ μ ≤ 68.40] = .99,” for confidence interval statements. The effect of unequal variances on the level of significance of the t test, the Behrens-Fisher problem, paired observations, and combining tests by use of -2Slog P; are also treated in Chapter 2. Analysis of variance computations, model equations, estimates by least squares, comparisons and orthogonality of them and polynomial of them, multiple comparisons a la Tukey, Newman-Keuls, Scheffe and Dunnett, and testing variance homogeneity constitute Chapter 3.

In Chapter 4 the analysis of “experiments having repeated measures” begins. This I found most tortuous. Although measurements made on the same person may well be correlated, it would seem unnecessary to draw attention to such covariances if you are supposing that the experimental errors are uncorrelated as was done. Variance heterogeneity and a very conservative F-test to allay suspicions of heterogeneity are discussed along with reliability of measurements, tests for trend, analysis of variance for ranked data and Cochran’s Q statistic.

Beginning with Chapter 5, factorial experiments or the factorial arrangement of treatments becomes the main topic and this persists through the remainder of the book. Expected values of mean squares for partially hierarchical designs, some particular cases of a Satterthwaite modified F ratio, pooling
procedures, the test for non-additivity with a single observation per cell, some observations on transformations and computations with unequal cell frequencies are included here also. In Chapter 6 these topics are illustrated with numerical computations.

In the early portion of Chapter 7 the notion of repeated measures and the associated procedure of looking at covariances are discussed. A three factor design with experimental subjects nested in one of the factors and crossed with the other two is introduced and its analysis discussed in the course of which pooling practices and tests on simple or conditional effects are treated. Tests of polynomial trends and of equality and symmetry of covariance matrices also are included in the material of Chapter 7.

Chapter 8 deals with confounding and fractional replication. Considerable attention is given to the algebra used to handle tri-level factors. Lattice designs and designs in incomplete blocks appear in Chapter 9. Chapter 10 handles Latin squares and their use in allocating balanced combinations of treatments to groups of subjects. Analysis of covariance in single factor, factorial and repeated measures cases, form the topics of Chapter 11, while rank tests, partitioning chi-square in contingency tables, Hotelling’s $T$ test and principles of least-squares estimation make up Appendix A.

The strong points of the book would seem to be in the provision of patient and detailed examples of computations and in the relatively wide coverage of designs and topics auxiliary to analysis of variance. A weakness is in the relatively minor role given to actual data or to realistic problem situations for which the test procedures were designed. Although in its scope are many topics usually found in more advanced texts, its level of discussion is mostly arithmetic and moderately algebraic but not statistically theoretical.

This is a long book since the author had a great deal to say. As a text-book it may say too much in too much detail. At any rate, it would seem that an instructor would have to cull a great deal, while the scattering of topics makes this somewhat difficult.

The only errors of note which I found were an extra $(k-1)$ in the denominator of $SS_{\text{rest}}$ on pages 50 and 108, $E(F)$ is not equal to 1.00 as on pages 119 and 122. One would have difficulties calling $\hat{e}_i - \hat{e}_i$, a “structural parameter” as on page 152, since it is the difference between two sample means of experimental errors. The computations that I looked at appeared to be carefully done and it is these which are the great merit of the book, at least for one like myself who likes to be shown and to show others how to do the arithmetic of the analysis.

C. H. Proctor


This volume contains the ten papers that were presented in April 1961 at a symposium held at the University of Maryland to commemorate the twelfth anniversary of the Institute for Fluid Dynamics and Applied Mathematics. Because of the diversity of the subjects discussed it is not likely that this volume will find its way into many private libraries, and a brief indication of its contents may therefore be helpful. The first paper is by Theodore von Kármán on the buckling of shells in which some recent experimental results are compared with the non-linear theory developed by the author and H. S. Tsien over twenty years ago. There are two papers on the theory of turbulence by Th. Theodorsen and Stanley Corrsin, and four papers on partial differential equations by Alexander Weinstein, K. O. Friedrichs, P. C. Rosenbloom, and A. P. Calderon. The remaining three papers are by J. M. Burgers on Landau damping, by E. L. Resler, Jr. on chemical kinetics, and by G. E. Uhlenbeck on kinetic theory.

W. H. Reid


This volume contains the Proceedings of the Thirteenth Symposium in Applied Mathematics of the American Mathematical Society held in New York City in April 1960. The twenty papers which it contains cover a wide variety of subjects including hydrodynamic stability, turbulence theory, surface waves, and control theory. Considered individually, these papers provide a valuable, often critical, assessment of recent advancements in the subjects discussed. When they are considered collectively, however, two major trends appear. One is concerned with the effect of nonlinear processes and the other
with random processes. Although these two processes are often treated separately, the present volume makes it clear that ultimately both must be considered to obtain an adequate description of many important phenomena.

W. H. Reid


This volume was prepared following the recommendations of a Conference on Mathematical Tables held at Cambridge, Mass., in September 1954 under the joint auspices of NSF and MIT; it may be briefly described as a modernized version of the classical tables of Jahnke and Emde and contains sections on the following subjects: Mathematical constants (D. S. Liepman); Physical constants and conversion factors (A. G. McNish); Elementary analytical methods (M. Abramowitz); Elementary transcendental functions (R. Zucker); Eponential integral and related functions (W. Gautschi and W. F. Cahill); Gamma function and related functions (P. J. Davis); Legendre functions (I. A. Stegun); Bessel functions of integer order (F. W. J. Olver); Bessel functions of fractional order (H. A. Antosiewicz); Integrals of Bessel functions (Y. L. Luke); Struve functions and related functions (M. Abramowitz); Confluent Hypergeometric functions (L. J. Slater); Coulomb wave functions (M. Abramowitz); Hypergeometric functions (F. Oberhettinger); Jacobian elliptic functions and theta functions (L. M. Milne-Thomson); Elliptic integrals (L. M. Milne-Thomson); Weierstrass elliptic and related functions (T. H. Southard); Parabolic cylinder functions (J. C. P. Miller); Mathieu functions (G. Blanch); Spheroidal wave functions (A. N. Lowan); Confluent Hypergeometric functions (L. J. Slater); Coulomb wave functions (M. Abramowitz); Hypergeometric functions (F. Oberhettinger); Jacobian elliptic functions and theta functions (L. M. Milne-Thomson); Elliptic integrals (L. M. Milne-Thomson); Weierstrass elliptic and related functions (T. H. Southard); Parabolic cylinder functions (J. C. P. Miller); Mathieu functions (G. Blanch); Spheroidal wave functions (A. N. Lowan); Orthogonal polynomials (U. W. Hochstrasser); Bernoulli and Euler polynomials, Riemann zeta function (E. V. Haynsworth and K. Goldberg); Combinatorial analysis (K. Goldberg, M. Newman and E. Haynesworth); Numerical interpolation, differentiation, and integration (P. J. Davis and I. Polonsky); Probability functions (M. Zelen and N. C. Severo); Miscellaneous functions (I. A. Stegun); Scales of notation (S. Peavy and A. Schopf); Laplace transforms.

No generally valid statement can be made regarding argument range and step or number of digits given, except that the last is usually considerably greater than in Jahnke-Emde. For example, Jahnke-Emde tabulates the Bessel function $J_0(x)$ to 4 decimal places for $x = 0.0(0.01)15.5$, whereas the present volume gives 15 decimal places for $x = 0.0(0.1)15.0$.

W. Prager


This is a most timely work on the general theory of iterative algorithms for the solution of a single equation or a system of equations. While the material is presented in the proper historical perspective, the general manner of presentation and a considerable number of individual results strike this reviewer as new.

Chapter 1 introduces basic concepts and notations including a classification of iteration functions (one-point and multipoint, with and without memory), order, informational usage, and informational efficiency of an iteration function. Chapter 2 discusses iteration functions without regard to their structure. The existence and uniqueness of the iterative solution to a fixed-point problem is established when the iteration function fulfills a Lipschitz condition. Linear and superlinear convergence are investigated and an iteration calculus is developed for iteration functions with a certain number of continuous derivatives. Chapter 3 prepares the ground for the subsequent analysis of convergence and order of one-point iteration functions with and without memory. Chapter 4 is concerned with iteration functions generated by direct or inverse hyperosculatory interpolation. Chapters 5 and 6 treat one-point iteration functions without and with memory, respectively. The former chapter contains a fundamental theorem whereby there exist optimal iteration functions of all orders for the determination of a simple zero of $f(x) = 0$, an iteration function of the order $p$ depending explicitly on at least $f$ and its $p - 1$ first derivatives. Whereas the one-point iteration formulas studied in Chapter 5 are of integral order, those in Chapter 6 are of nonintegral order. Chapter 6 contains a remarkable conjecture, according to which
it would, for instance, be impossible to construct a one-point iteration function with memory that is
of second order and does not require the evaluation of derivatives. Chapter 7 deals with multiple roots.
In particular, a necessary and sufficient condition is established for an iteration function to be of second
order for roots of arbitrary multiplicity. Chapters 8 and 9 bring the theory of multipoint iteration
functions. Whereas various techniques of deriving such iteration functions are discussed in the former
chapter, two special families of functions are considered in the latter. These depend on several parameters
some of which are chosen to achieve a preassigned order while the rest are chosen to obtain desirable
properties. For the first family, for instance, \( f \) is evaluated at several points but \( f' \) only at a single point.
Chapter 10 is concerned with iteration functions that do not require the evaluation of derivatives and
Chapter 11 with systems of equations. Chapter 12 is essentially a compilation of the iteration functions
discussed in earlier chapters. Appendices A through D review interpolation, the evaluation of the deriv-
atives of the inverse function, computational efficiency (as distinguished from informational efficiency),
and techniques of convergence acceleration. Appendix E contains numerical examples, and Appendix F
discusses areas for future research.

Throughout the book, the presentation not only is exceptionally clear and easy to follow, it also
avoids the all too common mistake of overwhelming the reader with a great display of powerful mathe-
matical tools that are then used to obtain comparatively trivial results. To sum up: this stimulating
work is strongly recommended.

W. Prager

Nonlinear integral equations. Edited by P. M. Anselone. The University of Wisconsin

The volume contains the following papers presented at the Second Advanced Seminar conducted
by the Mathematics Research Center, U. S. Army (Madison, April 22-24, 1963): Direct Iteration,
Existence, and Uniqueness (A. Wouk), Applications of the Fixed Point Theorem by Russian Mathemati-
cians (H. P. Thielman), Equations in Partially Ordered Spaces (H. F. Bueckner), Newton's Method
and Variations (R. H. Moore), On Nonlinear Integral Equations of the Hammerstein Type (C. L. Dolph
and G. J. Minty), Variational Methods for Nonlinear Integral Equations (L. B. Rall) Problem in
Qualitative Behavior of Solutions of Nonlinear Volterra Equations (J. A. Nohel). The Numerical
Solutions of Nonlinear Integral Equations (B. Noble), Some Nonlinear Integral Equations of Hydro-
dynamics (D. H. Hyers), Nonlinear Integral Equations of Radiative Transfer (T. W. Mullikin).

1964. XII + 280 pp. $8.95.

The first edition appeared in 1958. This second edition includes new sections on sensitivity analysis
integer programming, and the decomposition algorithm. The section on available computer codes and
the bibliography have been brought up to date.

Problems in the sense of Riemann and Klein. By J. Plemelj. Edited and translated by

The first part of this stimulating book is concerned with the theory of ordinary linear differential
equations with analytic coefficients. This is applied to the hypergeometric equation and the Fuchsian
equation of the second order. Mapping problems for second order Fuchsian differential equations are
discussed and Klein's theorems concerning mapping with the ratio of two independent solutions of a
Fuchsian equation are presented. The final chapter of the first part treats the oscillation aspects for
linear second order differential equations.

The second part begins with a review of the theory of Fredholm integral equations. This is applied
to the Dirichlet problem for the potential equation. Plemelj's formulas for the solution of certain singular
integral equations are presented as well as a problem of Riemann, which points the way to the explicit
solution of certain integral equations with Cauchy kernels.

W. Prager