THE IDENTIFICATION OF AN INCOMPLETELY PARTITIONED NETWORK*

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In [1], the authors presented a formula for the number of trees in a class of networks defined as follows. A network with set of vertices V is incompletely partitioned if, for some positive integer n, there exist $2n$ non empty, pairwise disjoint subsets A_i and D_i ($i = 1, 2, \cdots, n$) of V such that, if x and y are two distinct vertices, then the branch connecting x and y is in the network if and only if there is no i ($i = 1, 2, \cdots, n$) such that x is in A_i and y in D_i.

If the network has m vertices, and if, for $i = 1, 2, \cdots, n$, A_i has b_i elements and D_i has c_i elements, then the number of trees in the network is

$$m^{m+n-k-2} \prod_{i=1}^{n} (m - b_i)^{c_i-1}(m - c_i)^{b_i-1}(m - b_i - c_i),$$

where $k = \sum_{i=1}^{n} (b_i + c_i)$. Special cases of this formula appear in [2] and [3].

In applying the formula, it is necessary to be able to determine whether a given network is incompletely partitioned, and, if so, to determine the sets A_i and D_i. This can be done as follows.

A bipartite network is said to be fully connected if it includes all possible branches. The following theorem can be proved from the definition in a straightforward manner. If G is an m vertex network, and G_m the complete m vertex network, then G is incompletely partitioned if and only if each component of $G_m - G$ is a fully connected bipartite network.

The number n appearing in the definition of an incompletely partitioned network is the number of components of $G_m - G$, and, for $i = 1, 2, \cdots, n$, A_i and D_i form the partition of the set of vertices of the ith component of $G_m - G$ prescribed by the definition of a bipartite network.

REFERENCES

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[3] S. Bedrosian, *Formulas for the number of trees in a network*, IRE Transactions in Circuit Theory, Vol. PGCT 8, No. 3, Sept. 1961