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This little book should be a valuable addition to the library of a mathematics or physics undergraduate or graduate who is in a hurry to increase his technical tools. By its use such a student, at least if he has already mastered Cauchy's integral theory, can be very quickly introduced to the practical applications of the Laplace transform. By dint of imposing on the functions to be transformed certain mild restrictions, not so stringent as seriously to restrict the applications, the proofs are simplified and reduced to a minimum. Thus after about twenty pages of theory the reader is ready to witness the strength of the method as applied to various branches of mathematics and physics. To absorb concepts at that pace the reader is helped by many additional pages of worked examples and exercises with answers.

The main applications are: ordinary and partial differential equations as related to physical problems, difference equations and integral equations, including the Wiener-Hopf technique. The final chapter reverts to theory, discussing the asymptotic behavior of the transform as influenced by the determining function. There is an appendix in which the main facts about the Fourier transform are proved. It is on these facts that the fundamental inversion formula of Chapter 4 was based. The appendix also includes a collection of operational formulas and a list of the common transforms.

In the reviewer's opinion the usefulness of this book would have been enhanced had more of the results been stated as explicit theorems. While the flowing informal style may be pleasant for a reader who is consuming the book in toto, the lack of stated theorems may detract from its use as a reference book. The chief strength of the book would appear to be its wealth of examples.

D. V. Widder (Cambridge, Mass.)


This monograph is a useful addition to the rapidly growing number of introductory texts on the subject of asymptotic expansions. It is concerned mainly with the approximation of definite integrals for large values of a parameter, making its scope similar to that of the recent monograph with the same title by E. T. Copson (Cambridge University Press, 1965). The subjects covered include the Euler-Maclaurin summation formula, the approximation of Laplace, the saddle-point method, and the method of stationary phase. Numerous applications are given, including the gamma function, Hankel functions, Hermite polynomials, probability theory, and ship-wave patterns. Most of the subject matter is fairly standard, exceptions being discussions of (i) methods for calculating higher coefficients in asymptotic expansions, (ii) the application of the Euler transformation.

The author's style is pleasant and easy going, with an obvious concern for instructing rather than impressing the reader. It can be recommended to graduate mathematicians and physicists, either for self-instruction or as a basis for a one-semester course. It is a pity that no exercises have been included, but in this respect the book does not differ from most other texts on asymptotics.

F. W. J. Olver (Washington, D. C.)


Feller's long awaited second volume, like the first, is not a conventional orderly textbook. The book was designed to be accessible to a reader who is unprepared for and perhaps uninterested in the rigorous mathematical background of the subject. Examples and applications are given before the concepts are rigorously defined. A single problem is sometimes treated in several different ways in different parts of the book. Each topic is illustrated by examples and applied to problems in and outside mathematics. The
difficult technique of writing such a book is carried through magnificently, just as it was in Volume 1, but of course this style has its intrinsic penalty. This is no book which a mathematician should consult to find a rigorous definition of the Markov property (which in fact is not needed in the book). The book is however an ideal source to consult to see how Markov processes are constructed and applied, using suitably chosen transition kernels.

The range of ideas touched on exhibits the catholicity of the author's interests. Any mathematician who finds nothing new to him in the book is learned indeed. The mathematical techniques used are developed as needed, some (for instance the Abelian and Tauberian theorems for Laplace transforms) with considerable simplifications over the usual development.

This volume's reference system is more useful than that of Volume 1 in that there is a greater effort to give the reader historical perspective, and it is to be hoped that if a third volume is written it will proceed further in the same direction. Inquisitive readers who look up names in an index, to count references and thereby grade their friends and others, will have to downgrade the well known probabilist W—F— who although granted a few references in the body of the book is not considered worthy of listing in the index.

The chapter headings give some idea of the books contents as well as of the unconventional approach. I) The exponential and the uniform densities. II) Special densities, Randomization. III) Densities in higher dimensions. Normal densities and processes. IV) Probability measures and spaces. [Until this chapter, starting on p. 101, in which the needed measure theory is sketched, the author relies on a feeling for computations with distributions and expectations obtained from Volume 1. The topics in this chapter are introduced somewhat apologetically: “most of the book should be readable without the present chapter” “in effect, measure theory justifies formal manipulations.” Since the stress in most of the book is on qualitative properties of distributions and of sequences of distributions, for example the asymptotic distributions related to sums of independent random variables, little measure theory is needed explicitly, aside from that involved in the basic definitions themselves. Inexperienced readers of this book will tend to underestimate the role of measure theory in modern probabilistic analysis.] V) Probability distributions in $\mathbb{R}^r$. VI) A survey of some important distributions and processes. [Includes material on infinitely divisible distributions, renewal processes, random walks, Markov chains, martingales.] VII) Laws of large numbers. Applications in analysis. [Most of this chapter is devoted to applications of a trivial variance calculation. The author proves with this tool that the Bernstein polynomials determined by a continuous function converge uniformly to it; he derives the general form of an absolutely monotone function; he finds an expression for a monotone function $e^{-\alpha}$ in terms of its moments, inverts Laplace transforms, and so on.] VIII) The basic limit theorems. [Various versions of the central limit theorem. Instead of the usual characteristic function technique the work is based on an exceedingly elegant study of the operator $F u$: given a distribution function $F$, $F u$ takes the continuous bounded function $u$ on $(-\infty, \infty)$ into the convolution of $F$ and $u$. Properties of regularly varying functions are also treated in this chapter.] IX) Infinitely divisible distributions and semigroups. X) Markov processes and semigroups. XI) Renewal theory. [Simplified proofs of the basic theorems.] XII) Random walks in $\mathbb{R}^d$. [“treats random-walk problems with emphasis on combinatorial methods and the systematic use of ladder variables.”] XIII) Laplace transforms. Tauberian theorems. Resolvents. XIV) Applications of Laplace transforms. [To renewal theory, branching processes, diffusion, first passage times . . .]. XV) Characteristic functions. XVI) Expansions related to the central limit theorem. XVII) Infinitely divisible distributions. XVIII) Applications of Fourier methods to random walks. XIX) Harmonic analysis. [Topics related to stationary processes.]

Feller's books are unique combinations of theory and application. They will look fresh long after purely theoretical books written in the present rapidly developing state of probability theory are obsolete.

J. L. Doob (Urbana, Ill.)


One of the striking accomplishments of measure theory and functional analysis is the present form of probability and potential theory, which have been almost unrecognizably transformed under their influence. Large parts of a combination of Markov process and martingale theory are equivalent to large
parts of potential theory. Many problems can be considered either from a probabilistic or a potential theoretic point of view, and the technique of either field can be used to solve them. The interplay between the two fields has been a great stimulus to research in both.

Meyer's book is not a book on the combined theories. It is a book which covers certain mathematical topics which are basic to the intersection of the two theories. When a probabilist wishes to learn modern potential theory he can consult a reasonable number of papers and monographs. When a potential theorist wishes to learn probability he finds that there are several probability texts which cover the topics he needs but that it is difficult to extricate these topics from the surrounding material. In accordance with this situation Meyer does not give a systematic introduction to potential theory. There is no axiomatic theory of harmonic and superharmonic functions, for example. But probability theory is developed from its elementary concepts to martingale theory, with a minimum of attention to anything not needed for martingale theory. Markov processes are not defined. The Hunt potential theory of these processes is considered to be an application for which this book prepares a reader, and which a projected later volume will cover. A newcomer to potential theory might have some difficulty in deciding what it is after reading this book. Certainly he would never guess what it has been! The notions of capacity and energy do not appear at all in the sections on potential theory. Capacity appears in the book as a certain kind of set function, in the early sections on measure theory; the application to potential theory is not given. Energy appears only as a concept connected with certain stochastic processes (as developed in Meyer's own research.) In general Meyer likes to see beauty bare and avoids applications and examples in this severe and unadorned book.

The book is divided into three parts. Part A ‘Introduction to probability theory’ sketches the necessary parts of measure theory and the basic probability definitions. It can be read with no prior knowledge of probability. A valuable section contains the author's nontopological approach to analytic sets and Choquet capacities. As he remarks, perhaps one should study capacities before measures. The emphasis on additivity of set functions may be a pedagogical mistake! This part concludes with a chapter on continuous parameter stochastic processes, including separability, measurability, sample function properties and stopping times. The author's own unjustly neglected refinements of separability are included. Part B 'Martingale theory' contains the most elegant treatment of this subject which has appeared, although a reader would do well to consult Meyer's references to see elementary examples and more applications. It is a sign of the development of probability theory that it is now possible to write such a good introduction to the subject with exactly one page devoted to independence (plus a later proof of the Hewitt-Savage 0-1 law) and no mention of Bernoulli. Part B contains a detailed discussion of Meyer's theorem on the decomposition of a supermartingale and related topics associated with the analysis of a monotone family of \( \sigma \)-algebras by means of stopping times and martingales. Part C 'Analytic tools of potential theory' is not the analogue of Part A. It is not an introduction to potential theory (although it can be read without prior knowledge of the subject) but a treatment of various aspects of potential theory and various mathematical techniques used in it. The chapters are: Kernels and resolvents; Construction of resolvents and semigroups; Convex cones and extremal elements. The Hunt result on the construction of resolvents and semigroups from suitable kernels is obtained. The last chapter is a useful treatment of Choquet's generalization of the Krein-Milman theorem and associated ideas.

The writing is compact but clear. The book is a masterful sophisticated treatment of a difficult subject, actually of many difficult subjects whose interrelations are not brought out in this volume. Most of the material presented has appeared only in the periodical literature. It is to be hoped that Meyer will actually write a second volume in which the techniques he has developed in the first will be applied and the (to the uninitiated) seemingly unconnected topics integrated.

J. L. Doob (Urbana, Ill.)


In the last twenty years or so, there has been a great increase of research activity on the fundamental aspects of non-linear continuum mechanics. This has been directed mainly to the formulation of rational theories of the mechanics of non-linear materials, whether elastic or viscoelastic, fluid or solid. It is with
these theories that *The Non-Linear Field Theories of Mechanics* is chiefly concerned. The range of topics covered is wide. In addition to the more fully developed theories, such as finite elasticity, the mechanics of materials with memory and theories based on constitutive equations of the differential type, very recent theories whose status is less clear are discussed. Indeed some of the topics, such as the work of Adkins and Green on the mechanics of mixtures and of Ericksen, Toupin and others on continuum mechanical theories in which the deformation is described by tensor fields additional to the usual displacement field, were developed while the book was being written.

The book is impressive in its scholarly approach. In style it leans to mathematical formalism rather than to physical insight and this will, in large measure, determine its audience. The theory is elegantly developed and the proofs of theorems often represent a considerable improvement on those given in the original papers.

It is inevitable, however, since work of such recent vintage is discussed, that there will be some measure of disagreement with the points-of-view adopted.

For example, the inequality of Coleman and Noll (*C-N* condition), is uncritically stated. This and various related conditions—*GCN, GCN₀, GCN₀⁺* conditions, and so on—are discussed at length and various of their implications are derived. It is, however, stated in §51 that for pure homogeneous deformation of a cube of isotropic hyperelastic material in equilibrium subject to pairs of equal and oppositely directed normal forces, it is reasonable to expect that the greater stretch will occur in the direction of the greater force and that this follows from the *C-N* condition. Now, one may question the general validity of the conclusion that the greater stretch will necessarily occur in the direction of the greater force—and hence of the *C-N* condition from which it is derived—on both mathematical and physical grounds.

Consider the deformation of a unit cube of isotropic elastic material by three pairs of equal and oppositely directed forces, each of which has magnitude $F$ and is uniformly distributed over the face on which it acts. If all of the forces are increased proportionately from zero, we may expect that the cube will remain a cube when the forces reach their final magnitudes $F$. However, suppose that we first increase the forces on two pairs of opposite faces to their final values $F$, while applying no forces to the third pair of faces, the cube meanwhile becoming a rectangular block with dimensions in two directions much greater than that in the third direction. Now apply a third set of forces $F$ to the major surfaces of this sheet-like block. It is quite reasonable to suppose that these forces, distributed as they are over a large surface, may not necessarily be adequate to return the block to cubic form. We may then have a situation in which the principal stretches are unequal—and possibly grossly unequal—with equal forces. This conclusion is substantiated by the detailed mathematical analysis of Rivlin for a neo-Hookean incompressible material, in a paper (Phil. Trans. A, 240, 491, (1948)) which is not referred to in the book. In §53, the authors appear to anticipate this conflict with the conclusions of the paper of Rivlin by explicitly limiting the application of the *C-N* condition to compressible materials. It is, however, hardly reasonable to suppose that the qualitative conclusions reached in the incompressible case will cease to have validity when the material is, say, slightly compressible, particularly in view of the physical reasonableness of these conclusions.

At various other points in the book, work equally open to question is presented in a manner which would hardly lead the reader to realize its tentative, or even questionable character. Thus, the author’s aim stated in the introduction “Instead of completeness, we have attempted to achieve permanence” is not entirely realized. Indeed, in the present state of the subject, this aim is perhaps one which could not be achieved without sacrificing much of the value of the book.

Nevertheless, the bulk of the book remains admirable and as a whole it will stand as a major contribution to the subject with which it deals.

R. S. RIVLIN (Providence, R. I.)