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—BOOK REVIEW SECTION—

Classical and contagious discrete distributions. Proceedings of the International Symposium held at McGill University, Montreal, Canada, August 15–20, 1963. Edited by Ganapati P. Patil. Pergamon Press, New York, and Statistical Publishing Society, Calcutta, 1965. xiv + 552 pp. $21.00.

In the historical development of statistical distribution theory, the normal distribution occupied a preeminent position until the end of the nineteenth century, when the work of Karl Pearson, particularly the well-known Pearson system of frequency curves, introduced many other continuous distributions. Although discrete combinatorial results reach far back into the orgins of probability they were almost invariably regarded as isolated instances rather than being treated as discrete distributions. Even the Poisson probabilities were not generally recognized as constituting a statistical distribution until the energetic efforts of von Bortkiewicz in the twentieth century.

Within the past twenty years, discrete distributions have been coming into their own. It is remarkable that virtually all of the new developments in probability associated with the expression "operations research" involve primarily discrete models. Inventory theory, reliability theory, information theory queueing theory among many others rely substantially upon an analysis of discrete random variables.

The present volume and the symposium to which it corresponds are tributes to the efforts of the organizer and editor, G. P. Patil. The major subdivisions (with number of papers in parentheses) are: Stochastic Systems (3), Structural Properties (3), Limit Distributions (3), Unified Models and Inference (6), Some Classical Distributions (4), Contagious Distributions (6), Inference for Mixtures of Distributions (3), Certain Distributions in the Biological Sciences (3), Finite Populations (2), and General Topics (4). The emphasis on new model formulation makes the book a valuable source of ideas for research problems and classrooms examples. With many of the research papers by distinguished authors and a valuable bibliography of over one thousand items, this is a symposium which can be recommended to buyers of books on probability.

FRANK A. HAIGHT (*Los Angeles*)

Sensitivity methods in control theory. L. Radanovic, Editor. Pergamon Press, New York, 1966. 456 pp. $13.50.

This book is a selection of papers originally presented at the International Symposium on Sensitivity Analysis held in Dubrovnik, Yugoslavia, in 1964. The above symposium was oriented towards engineers and applied mathematicians interested in control theory. An excellent introduction to the sensitivity problem is given in the editor's preface. Basically the sensitivity problem is the problem of parameter or signal variations (due to uncertainity, environmental changes, etc.) and their effects on system performance. Since such variations are always present in physical systems, sensitivity is a fundamental problem in engineering. There are, however, very few books which address themselves to this problem.

The papers are divided into five parts: 1) Basic Approaches, 2) Sensitivity Functions, 3) Compensation of Parameter Variations, 4) Synthesis of Insensitive Structures, 5) Sensitivity and Optimality.

The following resume of each of the above parts, taken directly from the editor's preface, outlines some of the main contributions to be found in the book.

> Part I of the Proceedings presents the basic approaches to the problem of sensitivity. In the paper by I. Gumowsky the concept of sensitivity is related to the problem of parametric imbedding and some specific types of imbedding are considered. Relation is further established with Lyapunov stability and the classical works of Andronov, Pontryagin and Malkin. The concept of invariant imbedding based on functional equations is introduced by Bellman, Kalaba and Sridhar. In a paper by Ya. Z. Tsypkin the sensitivity is related with the absolute stability of processes in sampled data systems. Cruz and Perkins extend the formulation of the sensitivity of feedback structures to linear

time-varying multivariable systems. In a paper by Dorato and Drenick the game theory approach to the sensitivity problem is introduced in a conceptual form. In a paper by R. Tomovic' the role of sensitivity in engineering problems is discussed.

Part II is devoted to the determination and computation of sensitivity functions. After an introductory paper by L. Bykhovski, there is a series of papers dealing with specific problems such as determination of sensitivity functions with respect to the change of system order (Kokotovic' and Rutman), sensitivity functions for discontinuous systems (De Backer, Roberts, Tsypkin and Rutman), sensitivity to parameters changing the frequency of oscillation (Roberts), sensitivity to quantization errors in hybrid computers (Vidal, Karplus and Kaludijan), sensitivity of the characteristics of nonlinear systems to the variation of amplitude of self-excited oscillations (Stojic' and Siljak). The section also includes two papers which present some straightforward methods for the determination of sensitivity functions using a structural approach (Sedler, Vuskovic' and Ciric') and a paper introducing topological approach to the evaluation of sensitivity of network functions (Milic').

In part III several methods for the compensation of parameter variations are presented in the papers by J. Rissanen, S. Bingulac, H. Ur and other authors. J. Rissanen introduces functional derivative as generalization of the sensitivity function which is essential for the design of self-adjusting systems. In a comment by Rutman and Eppelman it is shown that the problem of parameter compensation in linear time-varying system can be solved 'in the large' and the necessary and sufficient conditions for parametric invariance are derived.

In part IV synthesis of insensitive systems relies upon the new concept of variable structure and the well-known principle of infinite gain. In a practical example it is shown that both the variable structure approach (Petrov, Emelyanov et al.) and the infinite gain approach (Meerov) yield insensitive structures which may be less complicated than adaptive systems.

From part V which contains material on the problems of optimal control it may be concluded that the classical sensitivity approach fails to provide methods of immediate use and that some other approaches and mathematical tools are to be sought.

In summary, the present volume contains an excellent compilation of research results in the area of sensitivity analysis. It is certainly recommended for anyone interested in control theory and control system design.

The only faults this reviewer finds with the volume is the great emphasis given to the sensitivity-function (a sensitivity function is defined as a partial derivative of a system output with respect to a system parameter) approach, in contrast to some of the other basic approaches outlined in part I, and the relatively poor quality of printing (reduced offset). However, it should be noted that the senstivity-function approach is the older and more established technique, hence it is somewhat to be expected that the greater number of contributions should be in this area. Presumably the printing process was chosen to make the proceedings available as quickly as possible.

PETER DORATO (*Farmingdale, N. Y.*)

Theory of oscillators. By A. A. Andronov, A. A. Vitt and S. E. Khaikin. Pergamon Press, New York, 1966. 815 pp. $28.00.

This book is a valuable addition to the literature on nonlinear oscillations. Its strongest points are the physical motivation for the concepts that are introduced and the numerous physical examples to illustrate the theory. Most of the discussion centers around two-dimensional systems and the general topics covered are linear systems, nonlinear conservative and nonconservative systems, theory of two-dimensional autonomous systems, coarse systems (structurally stable systems), the method of point transformations, nearly linear systems and relaxation oscillations. There are some shortcomings: the large size, the restriction to two-dimensional systems and the small amount of material devoted to asymptotic methods.

JACK K. HALE (*Providence, R. I.*)

Russian-English dictionary and reader in the cybernetical sciences. By Samuel Kotz. Academic Press, New York and London, 1966. xxi + 214 pp. $11.00.

This volume, which is constructed along the same lines as the author's earlier *Russian-English*

Dictionary of Statistical Terms and Expressions will be no doubt equally useful to English-speaking scientists. The present volume is in three parts: the dictionary, the reader, and a selected list of Soviet publications in cybernetics. In his choice of the more than 8,000 terms and expressions in the dictionary the author has been able to cover a very wide range of subjects—automatic control, symbolic logic, biology, transmission of information and the like—since he has wisely assumed that in reading a scientific paper at least two dictionaries are necessary, a general one and one pertaining to the particular field. Thus the present dictionary does not include many words (e.g., the ordinary prepositions and conjunctions or words like easy, difficulty, etc.) which, although commonly occurring in works on cybernetics, are also part of the general vocabulary of the Russian language.

For similar reasons, the author does not give any sketch of Russian grammar, although recent practice in this matter seems to show the usefulness of special grammars with examples taken from the given specific field. However, the second part—a Russian reader with an interlinear English translation—more than compensates for the absence of a grammar. The selections are well chosen and presented in such a way that they will be extremely helpful to any cyberneticist who wishes to learn Russian. Part III lists over 200 items of Soviet publications for the years 1955–1965, with a reference to comprehensive bibliographies. There is a general foreword, on the past and the future of cybernetics, by A. Porter, Head of the Department of Industrial Engineering at the University of Toronto.

S. H. Gould (*Providence, R. I.*)

Séminaire Bourbaki. W. A. Benjamin, Inc., New York, 1966. Twelve volumes (1948–1965). $12.25 per volume or $129.00 per set.

The Bourbaki Seminars, which are held in Paris three times a year, have, by now, become a well-established tradition. Each session lasts three days and is attended by a large gathering of mathematicians, many of whom come to Paris especially for this purpose. There are about six lectures. The speakers usually belong to the Bourbaki group but frequently outsiders, including foreign visitors, are also invited. They are requested to submit the text of their lecture a few days in advance, so that it can be mimeographed and distributed to the audience. Each speaker gets a certain number of reprints to mail to interested mathematicians and finally the annual volume of the Bourbaki lectures comes out. In this way these lectures become available to a much wider audience. The entire series from 1948 to 1965 has now been reprinted in twelve bound volumes by Benjamin, Inc.

The object of a Bourbaki lecture is to present a broad but accurate account of some specific area of mathematics and to do it in such a way that it could be understood by someone who is not necessarily familiar with all the technical aspects. Although this goal has not always been attained, it is true that these lectures provide a very useful introduction to the subject for an interested reader. Moreover the precise references in the text permit him to pursue the matter further, if he so wishes, by going over to the original papers.

These lectures cover, in a broad sense, almost the whole development of mathematics during the past two decades. However they do retain the unmistakable flavour of the modern French school, both in the choice of the subject matter and its presentation. The treatment is clear, elegant and precise. In fact, sometimes it is so smooth that the reader may have trouble seeing what the central point is. Nevertheless, these lectures are extremely useful and, in my opinion, they represent the most valuable contribution to mathematics by N. Bourbaki.

Harish-Chandra (*Princeton, N. J.*)

Combinatorial group theory. By Wilhelm Magnus, Abraham Karrass and Donald Solitar. Volume 12. Interscience Publishers (A Division of John Wiley & Sons, Inc.), New York, London, Sydney, 1966. xii + 444 pp. $15.00.

This is an excellent work which will do group theory and neighbouring branches of mathematics a power of good. The authors have produced a very well-written, coherent and well-planned account of their subject, which is the hard part of group theory, the study of groups described by generators and relations, centering around the three fundamental problems posed by Max Dehn:

(i) given a presentation of a group, is there a procedure for deciding of any word in the generators whether or not it represents the identity element of the group?
(ii) given a presentation, can we decide of any two words in the generators whether they represent conjugate elements of the group?
(iii) is there a procedure for deciding of any two presentations whether the groups they describe are free?

Chapter 1, Basic Concepts, I find slightly turgid. However, it sets the scene very effectively, describing how a group is defined by a presentation, stating Dehn's problems, defining free groups, Tietze transformations and the graph of a group. Chapter 2 is concerned with deriving presentations of factor groups and subgroups from presentations of a group. The bulk of the chapter deals with subgroups, and leads the reader gently from the most general presentation in terms of arbitrarily given generators and an arbitrary "re-writing process" to more and more economical presentations obtained by selecting specially appropriate generators (Schreier) and a particularly relevant re-writing process (Reidemeister). These Reidemeister–Schreier presentations are then applied to free groups to obtain many important theorems about their subgroups.

Chapter 3, Nielsen Transformations, describes a different, rather more combinatorial treatment of free groups. The first two sections give the more or less familiar theory, and the remaining five are full of important and very satisfying applications: the connection between Nielsen transformations and the subgroup theorem for free abelian groups; knot groups and Alexander polynomials; the automorphism groups of free groups; a discussion of J. H. C. Whitehead's Theorem; other applications in topology and analysis. Chapter 4 treats free products and generalised free products with an amalgamated subgroup. The discussion is an extension of the work of Chapter 2: the authors use the last drops of freedom available in the selection of generators for a subgroup to define a "Kurosh re-writing process" which gives the Kurosh subgroup theorem for free products directly. The last section of the chapter is an attractive application, giving details of what is known about groups with one defining relation.

Chapter 5, Commutator Calculus, accounts for a quarter of the whole book. It is less directly connected with generators and relations, more concerned with the structure of the lower central series and its factors. There is a very pleasant, thorough account of the connections between free groups and free associative (power-series) algebras, free groups and free Lie algebras, and free Lie algebras and associative algebras. In particular, we get a complete account of the lower central series in a free group, and again, many applications. The last chapter, Chapter 6, is a brief summary of recent developments. It would be a pleasant survey, but it is marred by being so compressed as to be inaccurate in parts.

Generally it is an optimistic book: it deals with the soluble cases of Dehn's problems (and accordingly the emphasis throughout is on constructive proofs and algorithmic methods) but shies away from the negative side of the affair, the undecidability results of Novikov, Britton, Boone and others. It firmly makes the point that infinite group theory is justified by applicability, and accordingly applications account for a good proportion of the book. The exposition is patient, well-motivated, detailed and sensible, and is salted throughout with the best collection of exercises I have ever seen. There is a normal complement of misprints, but the general printing and layout are very good. The book fulfils its aims well, and nicely covers topics which are treated only cursorily in other texts. Generally it is well worth having both as a text (for beginning graduate students) and for reference.

P. M. Neumann (*Manchester, England*)

Geometric invariant theory. By David Mumford. Academic Press Inc., New York, and Springer-Verlag, Heidelberg, Berlin, New York, 1965. v + 145 pp. $5.50.

In this monograph the author gives a detailed exposition of his results on the problem of "moduli" for algebraic curves in its most general form, namely within the frame of the Grothendieck theory of schemes. His methods rely heavily on the ideas of classical invariant theory, conveniently modernized and generalized. After a preliminary chapter 0, the book is about evenly divided: chapters 1 to 4 are concerned with invariant theory, chapters 5 to 7 with the problem of moduli.

Let us at first, for simplicity, only use the concepts of classical algebraic geometry (over the complex field). The general problem of invariant theory, as stated by the author, is then the following one: given an algebraic variety X, and an algebraic group acting (algebraically) on X, is it possible to give to the set of orbits X/G a "natural" structure of algebraic variety (what the author calls a "geometric quotient")? The word "natural" has to be made very precise, and in fact even in the classical case this is already difficult to do without the concepts of the theory of schemes. Among the conditions which constitute this very technical definition, let us only single out the fact that the natural mapping $X \to X/G$ should be a morphism, and the fact that the "regular algebraic functions" in a neighborhood of a point of X/G should exactly be those which lift to regular algebraic functions on X which are invariant by G. The classical invariant theory was almost entirely devoted to the computation of these invariant functions on X (for the projective linear group G) and of their identical relations or "syzygies"; this was of course far from solving in general the problem stated above: for instance, there is only one fundamental invariant of a quadratic form, its discriminant, but this does not at all describe the various orbits in the space of quadratic forms (of a given number of variables) under the action of the linear group (namely those on which the discriminant vanishes).

In the general theory, the author limits himself essentially to algebraic varieties over a field of characteristic O, and the group G is assumed to be reductive (i.e. isogenous to the direct product of a torus and a semi-simple algebraic group). After recalling the fact (going back to Chevalley) that an algebraic orbit space X/G always exists when X is an affine variety, he takes up the general case, which is really the interesting one (in classical invariant theory X is a projective variety). He shows that in order to obtain a "geometric quotient," one has first to disregard a set of "exceptional orbits," and restrict X to an open subset consisting of what the author calls "stable points." These exceptional orbits had already been considered by Hilbert in his papers on invariant theory, namely those where all the invariants vanish. By developing and generalizing Hilbert's ideas, the author succeeds in giving a very workable numerical criterion for a point to be stable in terms of the one-parameter subgroups of G; he shows how, by using this criterion, one may recover much of the results of classical invariant theory for binary or ternary forms; he also uses it to determine conditions under which the Chow point of an algebraic curve is stable (a result which he later uses for the construction of the variety of moduli). Finally, he obtains the stable points when G is the projective linear group acting on a product of Grassmannian varieties; when the latter reduce to projective spaces, he is even able to treat the case in which the basic field has arbitrary characteristic (and even the case in which the "basis" is a general scheme).

The problem of "moduli" consists in giving again a "natural" structure of algebraic variety to the set of isomorphism classes of all algebraic curves of a given genus g. Again, this can only be properly understood in the theory of schemes; the construction starts from the Hilbert scheme which, intuitively speaking, is an algebraic variety whose points correspond to the algebraic varieties of a projective space P_n. The author singles out in the Hilbert scheme a subscheme H_ν which represents the ν-canonical curves of genus g in P_n; the projective group G acts on H_ν, and one shows that a geometric quotient H_ν/G will be a "moduli scheme" for the curves of genus g. The construction of such a quotient is first obtained by the author when the "basis" is the field of rationals; the idea is to imbed H_ν into the Chow variety of the corresponding curves, and use his criterion of stability for the Chow points. A much more roundabout construction is needed to get the "moduli scheme" over the ring of integers. Using the Grothendieck theory of abelian schemes (which is concisely described in chapter 6) the author first constructs a "moduli scheme" over the ring of integers for abelian varieties of given dimension g, equipped with a polarization of a given degree d^2, and what the author calls a "level n" structure, essentially $2g$ sections giving in each fiber a basis of the group of points of order n in the abelian variety. He succeeds in obtaining that scheme by using the determination of stable points in a product of projective spaces (which was worked out in chapter 3). The final step uses the jacobian mapping, assigning to a curve its jacobian variety, which is naturally equipped with a canonical polarization; that mapping is injective by Torelli's theorem and the previous work done on "moduli schemes" for abelian varieties yields the final result.

From this very sketchy summary it will be quite apparent that only mathematicians with a thorough training, both in classical and in modern algebraic geometry, may pretend to read that book with any profit.

J. DIEUDONNÉ (*Nice, France*)

Sheaf theory. By Glen E. Bredon. McGraw-Hill Book Co., New York, St. Louis, San Francisco, Toronto, London, Sydney, 1967. xi + 272 pp. \$10.50.

It has been evident for some time that the most appropriate way to define the cohomology groups of topological spaces with local coefficients is via derived functors; that is, *Grothendieck cohomology*.

This method ties in best with the Leray theory of mappings and with (cohomological) dimension theory. It is this approach that the author uses. The key to the Leray theory, and indeed to most applications of a local theory, is the continuity theorem which states that $H^n(T; F) = \overrightarrow{\lim} \{H^n(U; F) : T \subset U\}$, where U is a neighborhood of T, $T \subset X$, and F is a sheaf over X. This is not always valid when X is not a metric space, although it is when X is paracompact and T is closed in X. The author gives a number of other situations when it is valid, as, for example, when $f : X \rightarrow Y$ is a closed mapping, X is Hausdorff, and $f^{-1}(y)$ is compact for each $y \in Y$. Such theorems as the Vietoris–Begle theorem, the Hurewicz dimension theorem, and the generalized homotopy theorem for compact spaces can then be proved without various assumptions of paracompactness. Since it is not known whether or not the product of a paracompact space and a compact space is paracompact, this represents a distinct advantage over the Čech theory of sheaves. The author is able to define successfully a relative theory and to show that, except for the homotopy property, it satisfies all the properties of a desirable cohomology theory of pairs.

The last chapter of the book and the first comprehensive treatment of the theory is devoted to Borel–Moore homology. This theory is ideally suited for the study of duality between sheaf cohomology and homology. The homological counterparts of cohomological properties are also given to a large extent. These include in particular the spectral sequence of a map, the Künneth theorem, the Vietoris–Begle theorem, and the Smith theory. Although very few actual applications to the author's specialty, transformation groups, are given, many of the now standard cohomological devices used in the theory of transformation groups are introduced, as, for example, the Borel spectral sequence of an action and the Smith–Gysin sequences.

The rather extensive sets of exercises at the end of each chapter serve to establish the natural bounds of the theory and to give statements of those results needed in the body of the text but whose insertion there might tend to interrupt the smooth flow of the theory. When proofs of these exercises might afford undue difficulties, generous hints are given.

The author has managed to crowd a lot of material in a surprisingly small amount of space. He has a knack for stripping a proof of all the easily supplied detail without removing the essential points. Also, some background is required. He assumes the basic elements of homological algebra; in the chapter on comparison of cohomology theories, he assumes that the reader is familiar with the various theories being compared; he assumes known the algebraic form of the Kunneth theorem. As he states in the Introduction, some background in algebraic topology is taken for granted. The latter requirement is not absolutely necessary, however, and with care, much of the material can be given in a first course in algebraic topology.

There is the usual number of misprints, most of which are easily corrected. There are several minor mistakes, the most serious of which, on page 163 concerning the Fary sequence, can be corrected by the assumption that the support families involved are compact. Only Theorems 11.1 and 12.1 are affected. (Less stringent assumptions can be found to correct the error.)

In the reviewer's opinion, this book should become the primary source for information about sheaf cohomology theory of topological spaces, thus replacing Godement's classic in this respect. It should be pointed out, however, that there is no treatment of Grothendieck topologies and so the book is not applicable to the sheaf theoretic approach to algebraic geometry, for example.

PAUL S. MOSTERT (*New Orleans, La.*)

Differential equations: stability, oscillations, time lags. By A. Halanay. Academic Press, New York, London, 1966. xii + 528 pp. $19.50.

This translation of Professor Halanay's book on ordinary differential equations can only help to make some of the basic theory more accessible to the English-speaking student and some recent research more accessible to the-English speaking specialist. The book is hard to classify since it contains aspects of a textbook and of a research monograph. The author starts with the assumption of almost no previous knowledge of ordinary differential equations and proceeds deeply into four special topics: stability theory by Liapunov's direct method, absolute stability of control systems by Popov's method, nonlinear oscillations and systems of ordinary differential equations with time lags.

The style is pleasant and the choice of subjects within the special topic is in good taste. But the lack of examples, the neglect of certain basic topics and the occasional error of omission in a proof detract from the book.

The introductory chapter gives a brief discussion of the basic existence, uniqueness and continuity theorems.

The first chapter treats stability theory and relies heavily on the direct method of Liapunov. The basic theorems and their converses are established in the usual manner and then the stability of linear constant coefficients, periodic and symplectic systems are discussed. These results are used to treat the stability by first approximation, stability under persistent perturbations, stability with respect to some argument and Perron's theorem.

The second chapter is devoted to the absolute stability of the Lurie equations of control theory. A brief introduction to the early algebraic methods used to construct Liapunov functions is given. The majority of this chapter develops the method of V. M. Popov and the frequence domain criterion for absolute stability. Unfortunately the later algebraic methods started by Yakubovich and Kalman are not discussed since these results were very new at the time of the first Rumanian edition.

The third chapter gives a fairly complete picture of nonlinear oscillations. The author discusses nonlinear periodic and almost periodic systems, the method of averaging, some special topological methods and the methods of Cesari–Hale.

The last chapter discusses ordinary differential equations with time lags (differential difference equations). In this chapter the author carries over many of the previously discussed theorems to this more general class of equations.

K. R. MEYER (*Minneapolis*)

Formulas and theorems for the special functions of mathematical physics. By Wilhelm Magnus, Fritz Oberhettinger and Raj Pal Soni. Third enlarged edition. Springer-Verlag New York Inc., New York, 1966. viii + 508 pp. $16.50.

The title is exactly right; every page is packed with definitions, integrals, differential equations, interrelations, expansions, and so on. The functions covered include the gamma, hypergeometric, Bessel, Legendre, orthogonal polynomials, Kummer, Whittaker, parabolic cylinder, incomplete gamma, and elliptic. There are also chapters on integral transforms, and on the transformation of coordinates.

How does this new work compare with other compendia? In a nutshell, it is a considerably expanded form of the two earlier editions, and an abridged form of the Bateman memorial volumes.† The last edition of "Formulas and theorems..." was in German and appeared almost twenty years ago. The new edition, in English, covers exactly the same range of functions but is over twice as long owing to an increase in the number of formulae. Every chapter in the new book has its counterpart in the Bateman volumes, with the exception of transformation of coordinates. There are, of course, many new results, but on the whole the Bateman volumes give more formulae, explanations, and functions (and, of course, cost a little more). The overlap with the recent NBS *Handbook of mathematical functions*‡ is much less. The NBS volume has greater emphasis on numerical applications, including tables, ploynomial approximations, and graphs, and gives only basic formulae.

Large compendia of this kind seem particularly prone to errors, and regrettably the present work is no exception. A casual perusal has revealed over 10 misprints. The following errors have also been noticed, many of which were communicated to the reviewer by Dr. L. Maximon:

† *Higher transcendental functions*, volumes I, II, and III, by A. Erdélyi, W. Magnus, F. Oberhettinger, and F. G. Tricomi. McGraw-Hill, New York, 1953–5.

‡ Applied mathematics series, No. 55. Government Printing Office, Washington, D.C., 1964.

p. 82, l. 9, replace $4\pi J_\nu(x)$ by $4\pi i J_\nu(x)$; p. 89, l. 3, replace $J_{2\nu+2m+1}(z)$ by $J_{2\nu+2m+1}(2z)$; p. 78, l. 15' insert factor z before $Wu(z)$; p. 179, l. 7, for $P_\nu^{-m}(\cos\vartheta)$ read $P_\nu^{-m}(\cos\vartheta')$; p. 200, l. 8 up, insert factor $z^{-2\nu-\alpha}$, and change the argument of F on next line from z^2 to z^{-2}; p. 213, in *Recurrence relations* insert factor x after $(\alpha+\beta+2n-2)_3$; p. 215, insert factor $(z-x)^{-1}$ in the integrand of the contour integral for $P_n^{(\alpha;\beta)}(x)$; p. 217, the symbol h_n in the expansion of sgn x appears to be undefined; p. 222, the last recurrence relation is obviously wrong, perhaps it should be $C_n^{\lambda+1}(x) = xC_{n-1}^{\lambda+1}(x) + (2\lambda)^{-1}(n+2\lambda)\, C_n^\lambda(x)$; p. 230, l. 9, replace $\sqrt{1-x^2}$ by $\sqrt{1-x}$: p. 239, l. 4, replace $\cos n\delta$ by $\cos m\delta$; p. 267, l. 6 up, insert factor z before $(c - a)$; p. 268, l. 6 up, change az to aw; p. 332, l. 4, for $e^{-i\pi\nu}$ read $e^{i\pi\nu}$. Incidentally, at least four of these errors were avoidable by checking the errata sheets bound with the Bateman volumes.

The reviewer believes that the book would have benefited considerably from a moderate increase in the effort put into editing and proof-reading. The English is decidedly clumsy in places: for example (p. 141) "$J_\nu(x)$ tends to zero with a higher magnitude than $H_\nu^{(1)}(x)$." The typography could have been made more pleasing and compact in several little ways. Cross-referencing is inadequate; typically, the ψ-function appears on p. 2 but is not defined until p. 13. Consistency is lacking: for example, "Chebyshev" is spelled in at least six ways.

These shortcomings are minor when measured against the usefulness of the authors' work, however. Although the book is no longer the only one of its kind extant, it should continue to be a valuable and compact reference tool for a large number of mathematicians, physicists, engineers, and other scientists.

F. W. J. OLVER (*Washington, D. C.*)

Topological vector spaces. By Helmut H. Schaefer. The Macmillan Co., New York, and Collier-Macmillan Ltd., London, 1966. ix + 294 pp. $10.95.

This is the most readable introduction to a subject that is coming to replace the traditional course in Banach spaces. It is an elegantly written, thorough compendium of all the average analyst needs to know on the subject. The treatment of nuclear spaces is a particularly welcome addition to the literature, unquestionably the best presentation. The chapters on ordered vector spaces and on positive operators present a choice of the highlights, without going into excessive detail.

On leafing through the pages of this book one realizes what a long way we have come since the old days of Banach spaces in the thirties, and how pliable and delicate a tool linear analysis has become.

GIAN-CARLO ROTA (*Cambridge, Mass.*)

Numerical integration. By Philip J. Davis and Philip Rabinowitz. Blaisdell Publishing Co., (A Division of Ginn and Co.), Waltham, Mass., Toronto, London, 1967. ix + 230 pp. $7.50.

The book under review provides a pleasant introduction to a variety of aspects of numerical integration. The authors restrict attention to formulas involving sums of weighted integral values. The only book in English which is comparable in this area which the reviewer knows of is the translation by A. H. Stroud of "Approximate Calculation of Integrals" by V. I. Krylov which was published by the Macmillan Company in 1962.

The differences between the two books are worth noting. The book under review contains discussions of oscillating functions, contour integration, Monte Carlo integration and multiple integration whereas Krylov's book does not. Krylov's book, on the other hand, is more detailed and contains tables to permit its use directly for solving certain problems. Davis and Rabinowitz's book is lucidly written with several discussions, while Krylov's book has a more technical appearance and is seemingly addressed to those readers already eager to apply or learn about numerical integration.

For a graduate seminar or course in numerical integration, the reviewer believes that the two books complement each other nicely. Davis and Rabinowitz provide practical advice in the form of discussions and examples and in the inclusion of an appendix representing an article "On the Practical Evaluation of Integrals" by Milton Abramowitz.

In neither book is the perhaps inherent imcompleteness of numerical integration delineated. For example, in 1957 the reviewer and A. W. Wymore published seemingly for the first time the derivation of product formulas and the use of transformation theory to derive formulas. These results are reproduced by Davis and Rabinowitz. Professor Davis himself has made significant contributions in the direction of using functional analysis to establish means for generating optimal formulas.

The book itself is attractively printed, but one section dropped out, devoid of thread or glue, when the copy was opened.

P. C. HAMMER (*University Park, Pa.*)

Probability: a survey of the mathematical theory. By John Lamperti. W. A. Benjamin, Inc., New York, Amsterdam, 1966. x + 150 pp. $8.50 (Paperback $3.95).

This little book should be a pleasure to use as a text in a one-semester upper level course for the mathematically mature. Professor Lamperti has exercised judgment in the selection of topics from the field, and skill in their exposition in the text. The selection of theorems, and of lemmas used in their proof, is exceptionally happy; problems are included directly in the text at the points of their greatest relevance to the development. There are only four chapters: Foundations, Laws of Large Numbers and Series, Limiting Distributions . . . , Stochastic Processes. The last chapter is devoted largely to the Brownian motion process, although some topics in Markov processes are touched, and a heuristic discussion of applications to boundary value and eigenvalue problems is included. A real gem.

V. E. Beneš (*Murray Hill, N. J.*)

Variational principles. By B. L. Moiseiwitsch. Interscience Publishers (A Division of John Wiley & Sons), London, New York, Sydney, 1966. x + 310 pp. $14.00.

This book presents a general survey of the application of variational methods in mathematical physics. The book is essentially divided into two parts. The first part, consisting of the first three chapters, is devoted to the variational formulation of physical laws in various areas of classical and quantum mechanics. Chapter 1 reviews the canonical formulation of classical mechanics based on Hamilton's principle. Chapter 2 begins by treating the connection between classical mechanics and geometrical optics and the corresponding relation between wave mechanics and wave optics based on the use of Hamilton's characteristic function. After a very brief summary of the formalism of quantum mechanics the author then discusses Schwinger's quantum action principle, which reduces to Hamilton's principle for variations of the dynamical variables which vanish at the terminal points. Chapter 3 presents the formalism of classical field theory based on the Lagrangian and Hamiltonian formalism. Discussion of such a wide range of topics within a book of this scope must necessarily be brief and, at times, summary, and students would be ill-advised to use this first part of the book in their initial contact with either classical or quantum mechanics. The author's intention in these chapters is to present a review of the variational formulation of physical laws, and within this context the discussion is clearly written and eminently readable.

The second part of the book is more technical in nature and is devoted to the application of variational methods to eigenvalue problems and to scattering theory. This part should prove particularly useful to students of quantum mechanics.

David Lurié (*Dublin, Ireland*)

Fourier transforms and the theory of distributions. By J. Arsac. Translated by Allen Nussbaum and Gretchen C. Heim. Prentice-Hall, Inc., Englewood Cliffs, N. J., 1966. xv + 318 pp. $10.50.

In writing this book, the author, who is Director of Numerical Analysis Services at the Meudon (Paris) Observatory, notes in his preface

> ". . .that there are already a number of excellent related treatments. However, these are either very mathematical or else extremely simple, with emphasis on applications. In the present work, I have tried to merge the viewpoints of the mathematicians and the physicists and run the middle course between extreme rigor and none at all."

As it turned out, the book does not present a homogeneous mixture of the two viewpoints. The author, who can clearly qualify as both mathematician and physicist, tends to switch hats abruptly.

In the first half of the book, he lays out the mathematical foundations of the theory, being a little more meticulous, perhaps, than the practitioner would demand. Only the stickler would complain, for example, that the proof of the Riemann–Lebesgue lemma is incomplete, or accuse the author of a lot of handwaving in glossing over the fine points of the Schwartz theory.

The second half of the book is devoted to applications such as diffraction, complex impedances, wave propagation, filters of limited bandwidth, resolving power theory, effects of noise, and numerical methods.

It is here, when the author gets engrossed in some of the applications, that the mathematics undergoes a metamorphosis which only those with a lot of practical experience will be able to interpret properly. These people know, for example, what a "function met in practice" really is and what is meant by the "width of its maximum"; they know how to choose a function which is "smooth as possible" without overdoing it; they know that a function [met in practice] which vanishes outside $(-1, 1)$ necessarily has a derivative of some order which is discontinuous at the end points [not really discontinuous, but for all practical purposes, "discontinuous"]; they know that some problems cannot [and probably should not] be precisely formulated; they know that "minimum" or "optimum" means "as good as you can do without a lot of extra work and data". Anyone else will fret while the author makes three attempts to define the object-image problem, and they will not see the merit in his claim that a certain numerical procedure for estimating a Fourier transform "minimizes" the noise.

Those who are interested in reading the book should be warned that they will encounter numerous errors. Most of these appear to be the result of careless proofreading and are frequently of a type that require more than a momentary pause to correct.

B. F. Logan (*Murray Hill, N. J.*)

Statistical methods in economics. By Harold W. Guthrie. Richard D. Irwin, Inc., Homewood, Illinois, 1966. xiv + 373 pp. $8.25.

Four criteria might be used when judging the adequacy of a text for use in a statistics course for undergraduate concentrators in economics:

1. The examples and applications should be in the field of economics, rather than physical or biological sciences.
2. The material should be presented at a simple enough analytical and mathematical level to be understood by students with virtually no college mathematics.
3. The topics covered should be those most relevant to the subject (considering the constraints of the previous point) and be well organized.
4. Whatever is covered should be presented rigorously, correctly, and clearly.

Guthrie's book seems typical of a number of texts designed for the undergraduate economist. Some of these criteria are met very well (the first and second) and others (the third and fourth) are not fulfilled. An author of such a text is faced with a problem as Guthrie points out in his preface. Students come to the course with a great variety of mathematical training and, even more important, sophistication. To make the book understandable to the least sophisticated group of students in the class, the level of mathematical analysis used in the exposition must be extremely low.

Unfortunately, this causes a dilemma he has not handled very satisfactorily and leads to his difficulties on the last two criteria. There seems to be a threshold of simplicity below which the discussion cannot fall if the essence of statistical inference is to be adequately explained to the students. If the student is to achieve a working knowledge of the reasoning behind inference procedures, he must be able to operate above the threshold. If he cannot, it may be that explanations of the background to statistical inference should not be attempted, as badly as one might feel about leaving this out of a student's education.

A little over 40 percent of the book is devoted to descriptive measures used in economics. These sections are well done. The only question which might be raised is about the time series chapter where the traditional methods of smoothing and cyclical analyses are described. Recent work, especially in spectral analysis, casts doubt on the validity of many of the old procedures.

The inference part of the book is less than fully satisfactory. The main trouble seems to lie in the choice of a mathematical level below the threshold needed for adequate understanding of statistical inference. I would judge that both sophisticated and unspohisticated students would have a hard time getting clear the relationships between probability, probability distributions, sampling, the search for good estimators, and hypothesis testing. For example, it is true that the binomial theorem, presumably covered in high school, can be used to derive the binomial probability distribution. But in the long tedious process (although simple mathematically), the idea of probability as a relative frequency and the use of counting rules to derive the relative frequency seem to get lost. Another example is in the bald statement that an unbiased estimator is one whose expected value equals the population parameter. A misleading and incomplete definition of expected value is given, no intuitive idea of what a biased estimator might mean is discussed, nor is any indication given of what other properties must be considered when choosing a good estimator and the relative importance of bias among these several properties. In fact, it is never made very clear that there is a real choice or problem in choosing point estimators.

In summary, the book may be useful for students with low analytical aptitude. They will learn the derivations of the descriptive measures used in economics. They will also get some feeling for the types of statistical hypothesis testing which might be possible and a few of the cookbook formulas which are used. It is doubtful, however, whether they will grasp the underlying philosophy and methods behind statistical inference methods. Given unprepared students, maybe this is all that can be expected.

MARK B. SCHUPACK (*Providence, R. I.*)

Plastomechanik der Umformung metallischer Werkstoffe. By H. Lippmann and O. Mahrenholtz. Volume I. Springer-Verlag, Berlin, 1967. xi + 406 pp. $17.25.

About two-thirds of this first volume are devoted to a most comprehensive discussion of the elementary theory of technological forming processes such as forging, rolling, drawing, and extruding. Numerous applications of the theory are presented in great detail. The remainder of the book is an introduction to mechanics of continua, which is to prepare the reader for the rigorous treatment of forming processes in the second volume. One chapter of this introduction is concerned with the mathematical, kinematic, and static foundations of continuum mechanics, the other chapter with the constitutive laws of fluid mechanics, elasticity and plasticity. The presentation is clear and does not require much mathematical background from the reader.

W. PRAGER (*Providence, R. I.*)

Perturbation theory for linear operators. By Tosio Kato. Springer-Verlag New York Inc., New York, 1966. xix + 592 pp. $19.80.

Perturbation theory refers to the behaviour of various spectral properties of linear operators when they undergo small changes. Professor Kato, who has played a major role in the development of this subject, has performed a great service in producing the first detailed and comprehensive monograph to appear on this subject. It is a superb book, very well written, well organized, and (in spite of the length and variety of topics treated) easy to read and understand. This last virtue is due, in large part, to the numerous examples given throughout the book which illustrate, in simple situations, the concepts involved. In short, this is an admirable example of how books should be written. I highly recommend it to any person interested in perturbation theory. It should also be remarked that Chapters 1, 3, and 5 provide an excellent introduction to the theory of linear operators (except for the spectral theorem, whose treatment given in Chapter 6 makes use of the theory of sesquilinear forms).

Chapter 1 gives a detailed introduction to operator theory in finite-dimensional vector spaces, and Chapter 2 gives an account of perturbation theory for this case (since all topologies are equivalent one can carry out the discussion for the most part without introducing the notion of norm or inner product). The remainder of the book is concerned with perturbation theory in infinite-dimensional Banach and Hilbert spaces. Chapter 3 gives a discussion of operator theory in Banach spaces (compact operators, closed operators, the closed graph theorem, the adjoint, the resolvent, etc.). Chapter 4 is concerned with the stability, under sufficiently "small" perturbations, of the spectrum of closed operators, and other spectral properties such as closedness and bounded invertibility. Of particular interest is the definition of a metric topology for unbounded closed operators which is a generalization of the uniform topology for bounded operators. Chapter 5 is concerned with operator theory in Hilbert spaces. Of particular interest to the physicist is the discussion of some typical Schrodinger and Dirac operators in potential theory in quantum mechanics (proofs of self-adjointness and some properties of the spectrum are given). Chapter 6 discusses the relation between sesquilinear forms and linear operators in Hilbert spaces, and the Friedrichs extension. The spectral theorem and perturbation of spectral families is also discussed in this chapter. Chapter 7 is concerned with analytic perturbation theory. Several definitions of when a family of operators is holomorphic are given, and perturbation series are discussed. One of the examples given is the quantum mechanical problem of an atomic system consisting of a fixed nucleus and two electrons. The interaction energy between the electrons is considered as a perturbation, and estimates of the radius of convergence of this perturbation series are given. Chapter 8 is concerned with asymptotic perturbation theory where one obtains asymptotic rather than analytic expansions for the perturbed eigenvalues etc. Chapter 9 is concerned with what the physicist calls time-dependent perturbation theory and what the mathematician calls perturbation theory for semigroups of operators. Chapter 10 discusses perturbation theory for continuous spectra, a problem relevant to scattering theory in quantum mechanics.

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