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ON THE PRINCIPLES OF EQUIPRESENCE AND UNIFICATION*

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In 1960, Truesdell and Toupin [1] proposed that in formulating constitutive equations for phenomenological theories in which there is more than one constitutive equation, the following principle, which they named the Principle of Equipresence, must be satisfied: *A variable present as an independent variable in one constitutive equation should be so present in all.* In explaining the application of this principle, they state "Let it not be thought that this principle would invalidate the classical separate theories in the cases for which they are intended, or that no separation of effects remains possible. Quite the reverse: The various principles of invariance, stated above, when brought to bear upon a general constitutive equation have the effect of *restricting* the manner in which a particular variable, such as the spin tensor or the temperature gradient, may occur. The classical separations may always be expected, in one form or another, for small changes—not as assumptions, but as *proven consequences* of invariance requirements. The principle of equipresence states, in effect, that no restrictions *beyond* those of invariance are to be imposed in constitutive equations".

When applied to the constitutive equations for thermomechanics, this principle implies that the specific internal energy, stress, specific entropy and heat flux vector must all depend on the deformation gradients and the temperature gradient. Coleman and Mizel [2], starting from this assumption, purported to show from purely thermodynamic considerations that the internal energy and entropy cannot, in fact, depend on the temperature gradient. While one may question the significance of this result, depending as it does on many assumptions including the very doubtful one that the Clausius–Duhem inequality is both meaningful and valid, it may have influenced Truesdell and Noll[1] in their restatement of the Principle of Equipresence as: *A variable present as an independent variable in one constitutive equation should be so present in all,* unless, of course, its presence contradicts some law of physics or rule of invariance.

Before discussing the validity of the Principle of Equipresence, one must first consider what meaning is to be attached to the phrase "should be so present". Is a variable which is present with zero coefficient considered to be present or not? If the former, then the statement of the Principle has no meaning, since present and not present then have the same meaning. If, on the other hand, the variable is considered to be absent, we may conclude that the classical theory for the propagation of electromagnetic waves in free space or insulating materials is disallowed by the Principle. We recall that electromagnetic theory requires three constitutive equations—those for the magnetic induction field B, the electric displacement field D and electric current density vector J. In the case of propagation in free space or in insulating materials, the constitutive equation

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[1] Truesdell and Noll [3] describe the paper by Coleman and Mizel [2] as "a work of unusual depth".

for J is $J = 0$; neither of the independent variables E (the electric field) or H (the magnetic intensity field) is present.

The Principle of Equipresence may, of course, be criticized on many other grounds (see, for example, [4], [5]). While few would question that in formulating a set of constitutive equations for some phenomenological theory, it is worth considering whether the independent variables present in one constitutive equation should be included in the others, their inclusion or noninclusion must involve also some understanding of the physical system to which it is intended that the equations apply.

The relegation of physical considerations to a neglibible role in the formulation of physical theories in favor of arbitrary mathematical rules has, unfortunately, become too common a feature in modern nonlinear continuum theory. It has perhaps reached its ultimate expression in Eringen's [6] so-called *Principle of Unification* according to which different constitutive variables that characterize particular materials should be present in the constitutive equations of all materials. For example, a theory of the flow of viscous fluids based on the Navier–Stokes equation for the stress is not allowed by the Principle of Unification, because the constitutive equation for the stress does not depend on the displacement gradients, as does the stress in the theory of elasticity. Indeed, the Principle of Unification would appear to disallow any theory, since no matter how many independent variables are introduced into a constitutive equation, one can presumably find others which occur in some other theory and should therefore also be introduced.

It is perhaps not without significance that in his book [6] Eringen's statement of the Principle of Unification immediately follows a statement of the Principle of Equipresence and many other principles which allegedly must be satisfied by properly formulated phenomenological theories in continuum mechanics and physics.

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