QUARTERLY

OF

APPLIED MATHEMATICS

VOLUME XXXIII APRIL • 1975 NUMBER 1

QUARTERLY OF APPLIED MATHEMATICS

The QUARTERLY prints original papers in applied mathematics which have an intimate connection with applications. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\sqrt{\ }$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp\,[(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{(a^2+b^2)^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos\,(\pi x/2b)}{\cos\,(\pi a/2b)} \text{ is preferable to } \frac{\cos\dfrac{\pi x}{2b}}{\cos\dfrac{\pi a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u\, du \text{ is preferable to } \int \frac{\sin u}{u}\, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx) \cos t \text{ is preferable to } \cos t(a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{[a + (b + cx)^n] \cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n) \cos ky)^2.$$

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354–372(1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

CONTENTS

—BOOKS RECEIVED—

Notice in this section does not preclude later full review in the Book Reviews section.

Handbuch der Physik (*Encyclopedia of Physics*). Edited by S. Flügge. Volume VIa/3: *Mechanics of solids* III. Edited by C. Truesdell. Springer-Verlag, Berlin-Heidelberg-New York, 1973. xi + 647 pp. $95.20.

This volume contains the following articles: "The linear theory of viscoelasticity", by Marshall J. Leitman (Case Western Reserve University) and George M. C. Fisher (Bell Telephone Laboratories)—115 pp., "Theory of elastic stability", by R. J. Knops (Heriot-Watt University) and E. W. Wilkes (University of Newcastle-upon-Tyne)—174 pp., "Growth and decay of waves in solids", by Peter J. Chen (Sandia Laboratories)—100 pp., "Ideal plasticity", by Hilda Geiringer (Wheaton College)—132 pp., and "Topics in the mathematical theory of plasticity", by T. W. Ting (University of Illinois)—56 pp.

Numerical quadrature and solutions of ordinary differential equations, by A. H. Stroud. *Linear multivariable systems*, by W. A. Wolovich. *Optimal control theory*, by L. D. Berkovitz. *Similarity methods for differential equations*, by G. W. Bluman and J. D. Cole. Volumes 10–13 of the series "Applied Mathematical Sciences", Springer-Verlag, New York, 1974, $9.50 each.

The purpose of this series is to provide books suitable for the user of mathematics, the mathematician interested in applications, and the student scientist. The material is generally less formally presented than in finished texts and often based on lecture notes. In this manner, subjects of current interest can be presented speedily, attractively and inexpensively.

Linear and nonlinear waves. By G. B. Whitham, F.R.S. John Wiley & Sons, New York, 1974. xvi + 636 pp. $22.50.

This book provides a comprehensive and up-to-date treatment of linear and nonlinear wave propagation, including in-depth applications to traffic flow, flood waves, gas dynamics, sonic booms, shock dynamics, water waves, nonlinear optics, and related phenomena. About 50% of the material appears in book form for the first time.

Advances in applied mechanics, Volume 14. Edited by Chiu-Shun Yih. Academic Press, New York, 1974. x + 358 pp. $35.00.

This volume contains the following articles: Bernard Budiansky: Theory of buckling and post-buckling behavior of elastic structures. John W. Hutchinson: Plastic buckling. K. Stewartson: Multi-structured boundary layers on flat plates and related bodies. Daniel D. Joseph: Response curves for plane Poiseuille flow. S. C. Cowin: The theory of polar fluids.

Mathematical methods for the physical sciences. By K. F. Riley. Cambridge University Press, New York, 1974. xii + 533 pp. Cloth $26.00, paper $8.95.

This book is subtitled "An informal treatment for students of physics and engineering", and addressed to first- and second-year undergraduates; it stresses physical relevance of the mathematics presented, often by means of pictorial illustrations and qualitative arguments. Many worked examples are given.

Structural mechanics computer programs—surveys, assessments and availability. Edited by W. Pilkey, K. Saczalski and H. Schaeffer. University Press of Virginia, Charlottesville, 1974. xiii + 1105 pp. $20.00.

This volume contains critical reviews of structural mechanics computer programs in civil engineering, mechanical engineering, nuclear engineering, applied mechanics, marine engineering, and aerospace engineering, written by forty leading authorities. Each review contains a survey of available programs, summary of program capabilities, assessments by users, and details of availability.

Introduction to calculus and analysis, Volume 2. By Richard Courant and Fritz John, with the assistance of Albert A. Blank and Alan Solomon. John Wiley & Sons, New York, 1974. xxiii + 954 pp. $17.95.

This book (and Volume 1, published in 1965) is a revision of Richard Courant's famous *Differential and integral calculus* which very successfully introduced several generations of mathematicians to higher mathematics. Considerable additions and changes have been made in this revised edition to reflect more recent developments, but the work retains the arrangement of an informal introduction based on intuitive arguments together with applications, followed by rigorous proofs, for each subject.

The present volume deals with functions of several variables, containing also sections on exterior differential forms, indices of vector fields, Jordan measurable sets, multiple Fourier integrals, differential equations, calculus of variations, and functions of a complex variable.

The foundations of mechanics and thermodynamics. Selected papers by W. Noll, with a preface by C. Truesdell. Springer-Verlag, New York, 1974. x + 324 pp. $20.10.

The purpose of this volume of reprints is to put into general hands those of Noll's works which presently promise broadest service to students of the mechanics and thermodynamics of deformable bodies. It includes 16 papers, dating from 1958 to 1973.

Introduction to the theory and application of the Laplace transformation. By Gustav Doetsch. Translated by Walter Nader. Springer-Verlag, New York, 1974. vii + 326 pp. $27.90.

This is a translation of the second (1970) edition of the work which first appeared in 1958; in contrast to many books on the subject, it develops the applicable parts of the theory of the Laplace transformation in complete generality and with detailed, rigorous proofs. Applications are presented where the accessory tools have been developed. The book proceeds from easier to more difficult problems and is thus suitable to be used as a text.

An introduction to electromagnetic theory. By P. C. Clemmow. Cambridge University Press, New York, 1973. viii + 297 pp. Cloth $16.50, paper $6.95.

This book presents the subject on the undergraduate level, aimed particularly at the applied mathematician and physicist with theoretical leanings. It concentrates on field theory and omits extended description of experimental phenomena.

Introductory eigenphysics—an approach to the theory of fields. By C. A. Croxton. John Wiley & Sons, New York, 1974. vii + 275 pp. $19.95.

This book gives a physical introduction to the mathematical treatment of a large number of problems taken from physics, geophysics, astronomy and chemistry, with emphasis on the physical basis of the problems discussed. The author attempts to break down the division between quantum and classical mechanics, between physical optics and electron scattering, and between electrostatics and aerodynamics, by treating the relevant differential equations as a unifying basis.

—BOOK REVIEW SECTION—

Mathematical models of conception and birth. By Mindel C. Sheps and Jane A. Menken. University of Chicago Press, Chicago, 1973. xxiii + 428 pp. $18.50.

The models discussed in this book can be divided into three major areas: 1. Models of conception risks at each month of marriage or after the end of a previous pregnancy; 2. Family-building models, which describe the reproductive history of a couple as shown in the sequence and spacings of consecutive conceptions and births; 3. The distribution of intervals between successive births. The possibility of multiple births is ignored throughout.

The book is intended as a reference and a text, particularly for students of demography, statistics and biology. A comprehensive review of the techniques and results used in the models is included for readers without the necessary mathematical background, particularly in relevant aspects of mathematical statistics such as renewal theory (contributed by Walter L. Smith).

W. Freiberger (Providence)

Pattern classification and scene analysis. By Richard O. Duda and Peter E. Hart. John Wiley & Sons, New York, 1973. 482 pp. $22.50.

During the 1950s the number of textbooks in statistics started to multiply at a threatening rate. Now textbooks are still being offered in great numbers, most of them identical for all practical purposes, more or less the same content modulo permutations, the same examples, perhaps also the same tables. It is difficult to see any reasons for this phenomenon, except possibly commercial ones.

Pattern recognition is much younger than statistics, and the literature less extensive. In recent years we have seen, however, more and more textbooks appear, as well as monographs and collections of papers. This could be the beginning of another avalanche of books. It is again difficult to discover what the difference is between the books. They usually contain some Bayesian hypothesis testing, a few simple learning algorithms, perhaps something about potential functions, description of some software and often physiological and psychological speculations.

The present book differs to some extent from this description as far as Part II is concerned. Part I has the usual ingredients as above with a mixture of discriminant functions, supervised learning, clustering criteria and some discussion of unsupervised learning. This is quite standard fare.

Part II, called Scene Analysis, is a bit different. The authors present some material on the analysis of lines and shapes and on how perspective transformations operate. The last chapter describes how scenes can be described and analyzed through formal systems.

The mathematical level is elementary but adequate for the purpose. The style is clear and readable. A collection of exercises is given at the end of each chapter.

It is striking how much of the material in this book consists of direct applications, or sometimes modifications and extensions, of scientific methodology that existed before pattern recognition. This is certainly true of most of the statistical techniques in Part I and also of much of the elementary geometry in Part II. Some of this has led to useful and applicable results. Nevertheless, one cannot help feeling some disappointment that so little conceptual and theoretical knowledge has been generated during the 10–15 years that pattern recognition has existed as a recognized discipline.

Ulf Grenander (Providence)

Stresses in shells. By W. Flügge. 2nd Edition. Springer–Verlag, New York, Heidelberg, Berlin, 1973. xi + 525 pp. $19.10.

The fact that a second edition of this book has been prepared is itself evidence of the success that the original edition has enjoyed over the past fourteen years. In the first edition, the theory of shells

was presented by considering the various classes of shells, categorized according to configuration, type of loading and level of approximation, and developing the basic equations for each of them separately. The general equations of the theory were not derived. This fundamental viewpoint is maintained in the new edition.

Although most of the text is only slightly altered from the previous edition, several noteworthy changes have been made. For example, a brief separate chapter containing mostly new material on shallow shells has been added. A critical bibliography is again included, but references to many of the older papers have been omitted and references to several papers and books written during the past two decades have been added. The bibliography now also includes a list of Ph.D. theses on shell theory written at Stanford University under Professor Flügge's direction. Detailed discussion of methods of computation have been deleted in some places, but all of the numerical results are still included. In view of the fact that such computations can now be done quickly and simply by means of standard computer techniques, the author's choice is a reasonable one.

The remainder of the text is essentially unchanged from the familiar first edition. This book should continue to serve a central role in the teaching of the theory of shells, and in providing guidance in the analysis and design of shell structures.

L. B. Freund (Providence)

Multivariate analysis—a selected and abstracted bibliography, 1957–1972. By K. and K. Subrahmanian. Marcel Dekker, Inc., New York, 1973. xi + 265 pp. $19.75.

This work builds on the bibliography in T. W. Anderson's book of 1957. Its relation to the Anderson-Gupta-Styan bibliography of 1966 (in addition to its greater up-to-dateness) is this: A. -G. -S. covers all of multivariate analysis whereas S. -S. is limited to normal and related distributions; A. -G. -S. classifies articles whereas S. -S. also gives abstracts of the selected papers. The present work has 1189 references and abstracts, and a subject index.